Wavelet Transform for JPG, BMP & TIFF

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ABSTRACT

This paper presents briefly both wavelet transform and inverse wavelet transform for three different images format; JPG, BMP and TIFF. After brief historical view of wavelet, an introduction will define wavelets transform. By narrowing the scope, it emphasizes the discrete wavelet transform (DWT). A practical example of DWT is shown, by choosing three images and applying MATLAB code for 1st level and 2nd level wavelet decomposition then getting the inverse wavelet transform for the three images.

KEYWORDS: Wavelet Transform, JPG, BMP, TIFF, MATLAB.

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INTRODUCTION

In 1807, theories of frequency analysis by Joseph Fourier were the main lead to wavelet. However, wavelet was first appeared in an appendix to the thesis of A. Haar in 1909 [1].

Compact support was one property of the Haar wavelet which means that it vanishes outside of a finite interval. Unfortunately, Haar wavelets have some limits, because they are not continuously differentiable. In 1930, work done separately by scientists for representing the functions by using scale-varying basis functions was the key to understanding wavelets. In 1980, Grossman and Morlet, a physicist and an engineer, provided a way of thinking for wavelets based on physical intuition through defining wavelets in the context of quantum physics [3].

In 1985, a jump-start was given to wavelets by Stephane Mallat through his work in digital signal processing. He discovered relationships between quadrature mirror filters, pyramid algorithms and orthonormal wavelets bases. Later, Ingrid Daubechies constructed a set of orthonormal basis functions which become the start of wavelet applications today [1].

The main idea that describes wavelets is simply, scaling can lead to analyzing. It depends on mathematical principles for dividing data into different frequency components, then studying every individual component according to its scale. The scale is very important factor in wavelet analysis, since wavelets process data at different scales and resolutions. High level features can be noticed if the signal is studied in large windows, than if it is studied in small one. However, the wavelet analysis goal is to see both high level features and low level features of the signals. Since sines and cosines have infinite limits, then they are very poor in approximating choppy signals. This forces the scientists to look for an alternative functions for this type of approximation. The answer was wavelets, which are very suitable for approximating sharp spikes [1].

The wavelet analysis starts with adopting a wavelet prototype function called the mother wavelet. The frequency analysis is carried out with a low frequency version of the wavelet, while the temporal analysis is achieved with the high frequency version of the mother wavelet [1].

1. WAVELET TRANSFORM

There are many different types of wavelets transform. Most of data analysis applications are using continuous-time wavelet transform (CWT). However, the most famous type which affected the properties of many real signals is discrete wavelet transform (DWT) [4].

Scale parameter, measure of

Normalization constant

Signal to be analyzed.

$$CWT_x^{\Psi}(\tau,s) = \Psi_x^{\Psi}(\tau,s) = \frac{1}{\sqrt{|s|}}\int x(t)\Psi^*\left(\frac{t-\tau}{s}\right)dt \quad (1.1)$$

Continuous wavelet transform of the signal x(t) using the

The mother wavelet, all kernels are obtained by shifting

Discrete wavelet transform with k&j are integers

Ψ is the mother wavelet, τ is translation factor

$$\Psi_{j,k}(t) = \frac{1}{\sqrt{s_0^j}}\Psi\left[\frac{t-k_{\tau_0 s_0^j}}{s_0^j}\right]$$

(1.2)#

s_0 > 0 is dilation

2. DISCRETE WAVELET TRANSFORM

A discrete wavelet transform represents a time domain signal into time-frequency domain and the signals are called wavelet coefficients. If we have a signal f(t) in space Vj:

$$f(t) = \sum_{k\in Z} a_k^j\,\varphi(2^j t - k) \in V_j \quad (2.1)$$

Then, the signal next level decomposition can be written as:

$$f(t) = \sum_{k\in Z} a_k^{j-1}\ \varphi(2^{j-1}t - k) + \sum_{k\in Z} d_k^{j-1}\ \psi(2^{j-1}t - k) \qquad (2.2)$$

Where, a^j_k's are smoothed part approximation coefficients and d^j_k s are the detail coefficients.

To ensure that high and low frequencies disturbance are extracted, two scale signal decomposition are performed. The wavelet transform output consists of two decomposed signals, with different levels of resolution. The range of frequencies for the first and second scaled signal are (f/2-f/4) and (f/4-f/8) respectively, where f is the sampling frequency of the time domain signal [5].

3. DISCRETE HAAR TRANFORM

In image processing, the best way for analysis of an object is to represent it as a matrix. Given J< N and a matrix $c_0 = \{a(n,m)\}_{n,m=0}^{2^N-1}$ and defining # $2^{N-J} \times 2^{N-J}$ matrices $c_J,\ d_J^{(1)}, d_J^{(2)}, d_J^{(3)}$ by

$$c_J = H^{col} H^{row} c_{J-1} \qquad (3.1)$$

$$d_J^{(1)} = G^{col} H^{row} c_{J-1} \qquad (3.2)$$

$$d_j^{(2)} = H^{col} G^{row} c_{j-1} \tag{3.3}$$

$$d_j^{(3)} = G^{col} G^{row} c_{j-1} \tag{3.4}$$

Where, $H^{col}, G^{col}, H^{row}, and\ G^{row}$ are the $2^{N-j-2} \times 2^{N-j-1}$ matrices. The

DHT of the original matrix c0 is the collection of matrices

$$\left\{d_j^{(1)}, d_j^{(2)}, d_j^{(3)}\ \right\}_{j=1}^{J} \cup \{c_J\}. \tag{3.5}$$

However, applying the adjoint for the matrices H &G for both columns wise and row wise is the way to get the inverse DHT for the matrices which can be written as:

$$c_{j-1} = H^{row*} H^{col*} c_j + H^{row*} G^{col*} d_j^{(1)} + G^{row*} H^{col*} d_j^{(2)} + G^{row*} G^{col*} d_j^{(3)} \tag{3.6}$$

Where H* and G* are the adjoint of H and G. [6].

4. HORIZONTAL, VERTICAL AND DIAGONAL EGDES

Any pixel exist in an image has eight neighbors. Four located in diagonal direction, two located in horizontal direction and two located in vertical direction. The pixel is said a horizontal edge point, if the variation in the horizontal direction is small compared to vertical direction. Moreover, the pixel is said a vertical edge point, if the variation in the vertical direction is small compared to the horizontal direction. Also, the pixel is said that is a diagonal edge point, if the variation is large in both directions [6].

5. THE PICTURES FORMAT

There are many pictures format, but I selected three of my personal pictures as JPG, BMP and TIFF. Output of Matlab code.

After running the Matlab code for the three different pictures format, the output goes through two different levels of approximation, level1 and level2. Moreover, each level has three different types of details, horizontal, vertical and diagonal. After this process, the inverse wavelet transform for the pictures are produced.

5.1 JPG 1^{st} level approximation output

The first level of decomposition was done by reading the image in Figure1 (Jaroudi1.JPG) and using a Matlab function “dwt2” which depends on bior3.7 wavelet. It generates 1^{st} level of approximation (cA1) and the other three details; horizontal (cH1), vertical (cV1) and diagonal (cD1). Then, the approximation and details (A, H, V, and D) are constructed by using Matlab function “upcoef2” which get red off the “c” coefficient. Finally, the output is displayed by using “plot” and “image” Matlab function; as shown in Figure 5.

5.2 JPG 2nd level approximation output

The second level of decomposition was done by using the Matlab function wavedec2, which depends again on bior3.7 wavelet. The coefficients of the second level decomposition are return back into one vector. Then, the second level approximation and details are constructed by using Matlab functions “appcoef2” and “detcoef2” respectively as shown in Figure 6.

Finally, the original JPG image is reconstructed from multi-level of decomposition by using Matlab function “waverec2” as shown in Figure 7.

5.3 BMP 1st level approximation output

By using the same procedures which were used in the JPG first level of approximation, but using different image format (BMP) which is provided in Figure 2 (Jaroudi2.BMP). A different first level approximation can be produced, as can be seen in Figure 8.

5.4 BMP 2nd level approximation output

The same procedures of second level of decomposition and inverse wavelet transform which was used for JPG format, is applied for BMP picture format.

Also, Another second level of approximation and original BMP image can be seen in Figure 9 and Figure 10 respectively.

5.5 TIFF 1st level approximation output

Tag image file format (TIFF) is another picture format which exposed to first level of decomposition. By using the picture in figure 3 (Jaroudi3.TIFF), and following the same procedures which were used for both JPG and BMP pictures the output can be seen in Figure 11.

5.6 TIFF 2nd level approximation output

The second level of decomposition and inverse wavelet transform can also be applied for TIFF format by following the same procedures which were used for both JPG and BMP and output can be seen in Figure 12 and Figure 13 respectively.

6. Conclusions

Wavelet transform is a transform which provides the time-frequency representation. The are many types of wavelet transform, but the most important one is called DWT because it is affecting the properties of many real signals. Wavelet transform for 3 different pictures types, JPG, BMP and TIFF were tested by applying Matlab code of first level and second level of decomposition. The original pictures were reconstructed by using inverse wavelet transform. However, there are slight differences between original signal and reconstructed signal, because some information are lost during horizontal, vertical and diagonal transformation.

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REFERENCES

1. An Introduction to Wavelets, http://www.amara.com/IEEEwave/IEEEwavelet.html
2. Iran W. Selesnick “Wavelet Transforms – A Quick Study”. Polytechnic University, September 27, 2007.
3. Taylor and Francis Group, LLC “The Image Processing Handbook”. 6th Ed 2011.
4. Alfredo Mertins “Signal Processing; Wavelets, Filter Banks, Times Frequency Transforms and Applications”. Wollongong University, 1999.
5. A. Manjunath and H.M. Ravikumar “Comparison of Discrete Wavelet Transform (DWT), Lifting Wavelet Transform (LWT), Stationary Wavelet Transform (SWT) and S-Transform in Power Quality Analysis”. European Journal of Scientific Research, 2010.
6. David F. Walnut “An Introduction to wavelet Analysis” George Mason University, 2001.

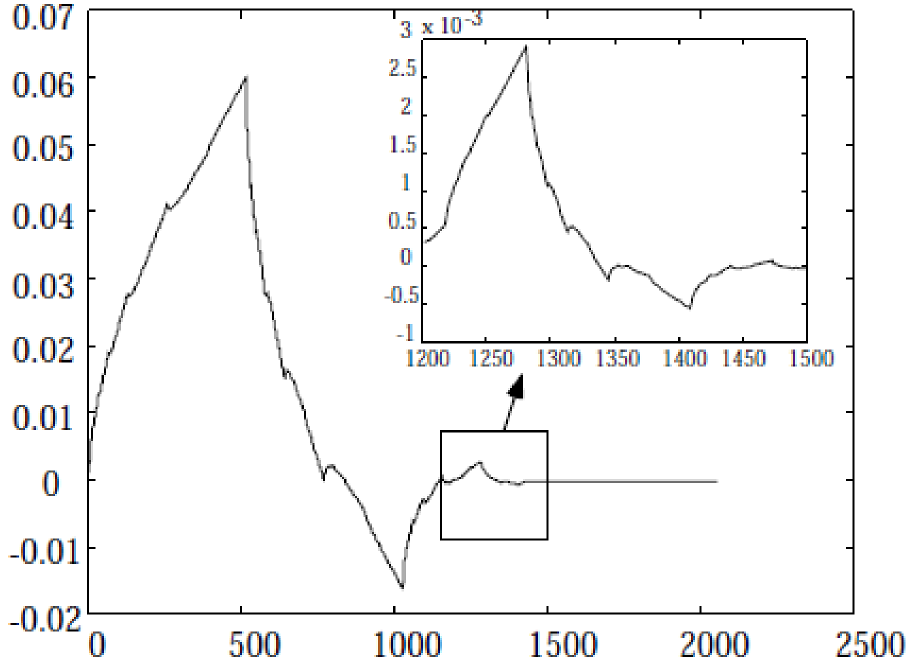


Figure1. Daubechies mother wavelet and its fractal similarity.

Figure 2. Jaroudi1.JPG

Figure 3.Jaroudi2.bmp

Figure 4.Jaroudi3.Tiff

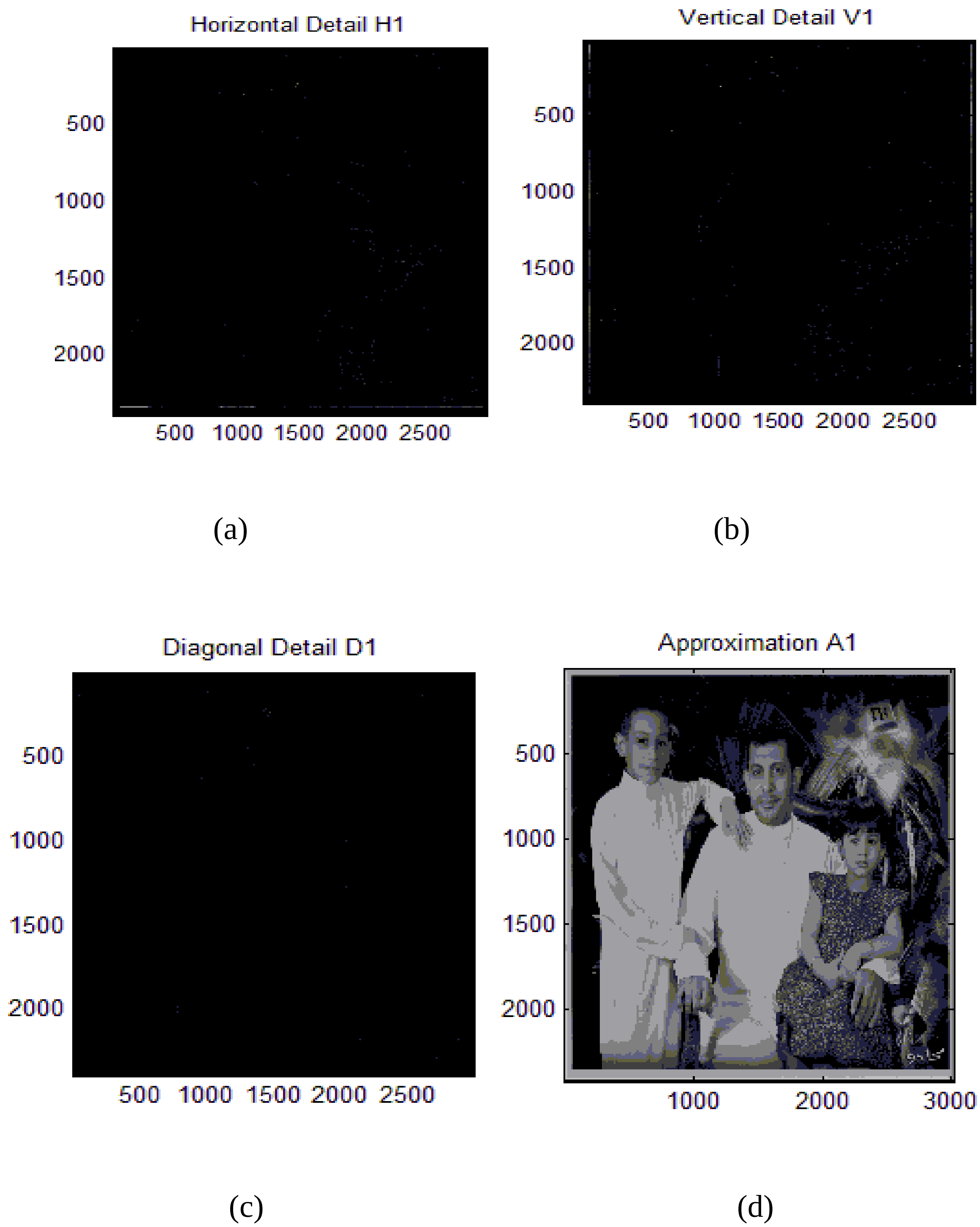
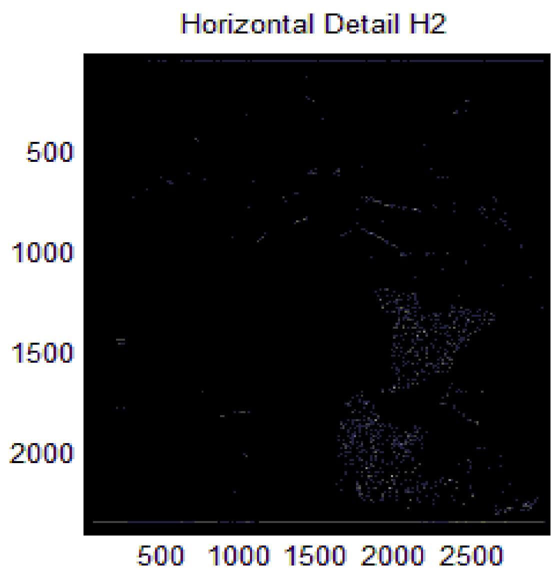
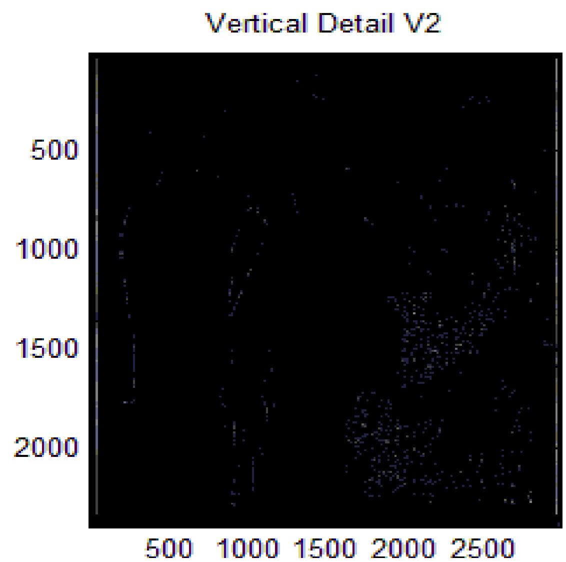


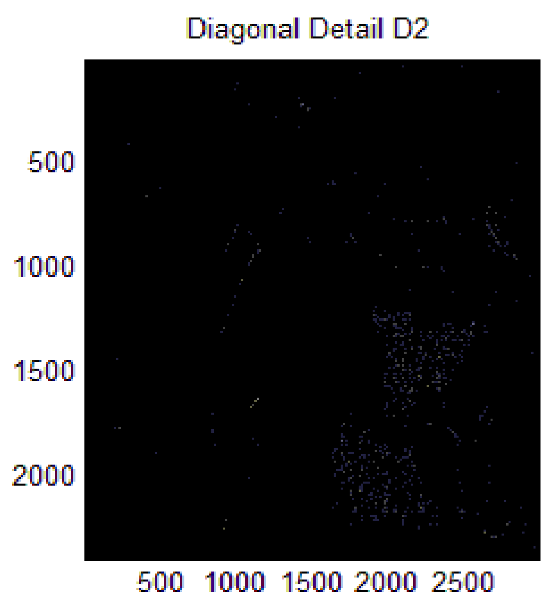
Figure 5. (a) Horizontal detail, (b) Vertical detail, (c) Diagonal detail, (d) 1st level approximation.



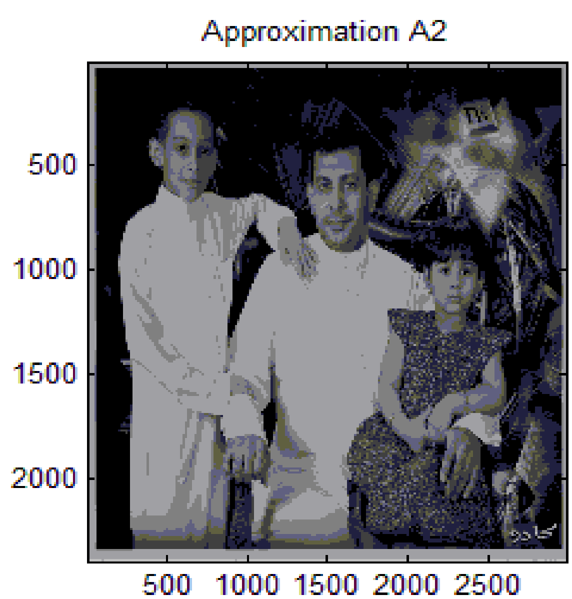
(a)



(b)



(c)



(d)

Figure 6. (a) Horizontal detail, (b) Vertical detail, (c) Diagonal detail, (d) 2nd level approximation.

Figure 7. JPG inverse transform

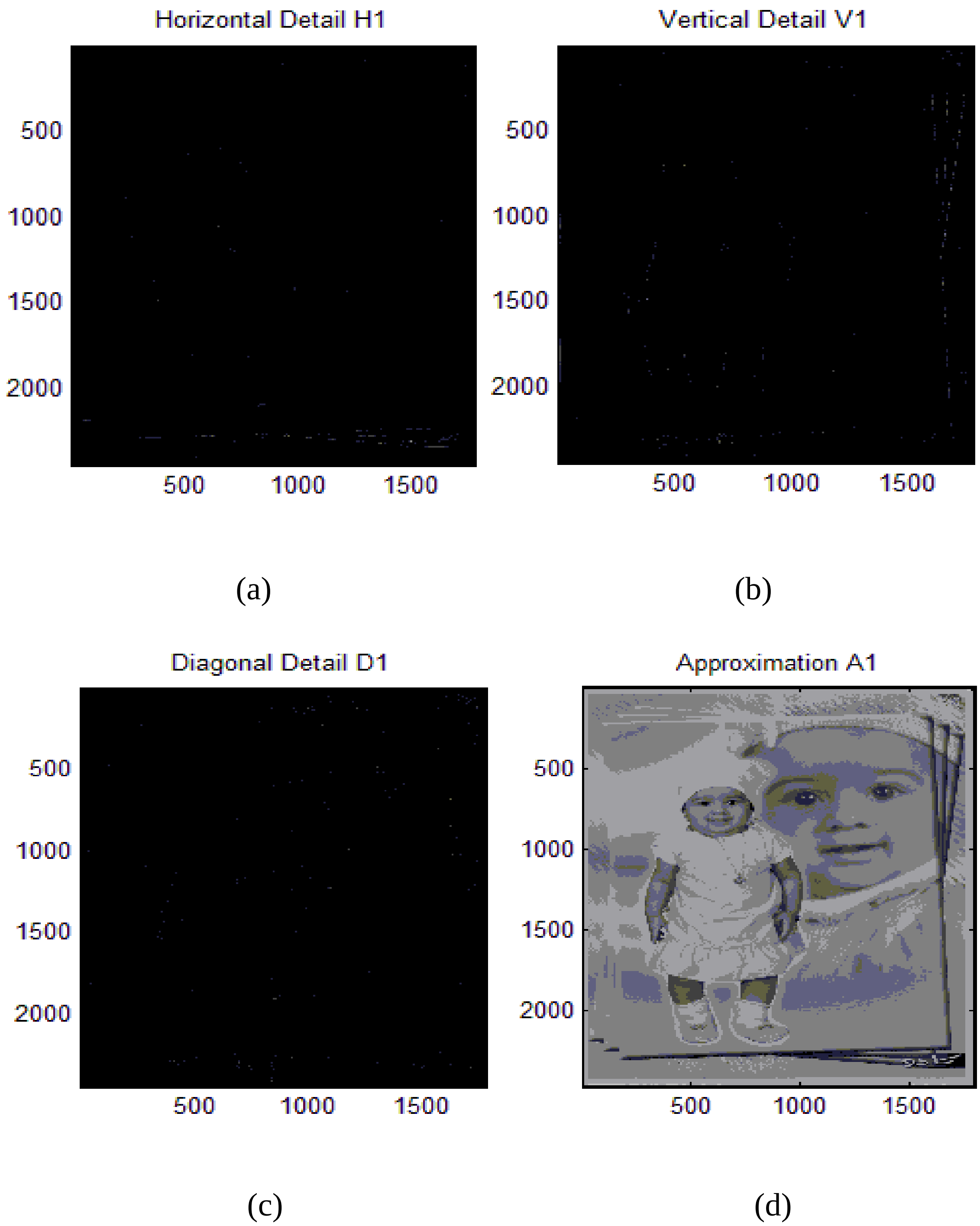


Figure 8. (a) Horizontal detail, (b) Vertical detail, (c) Diagonal detail, (d) 1st level approximation.

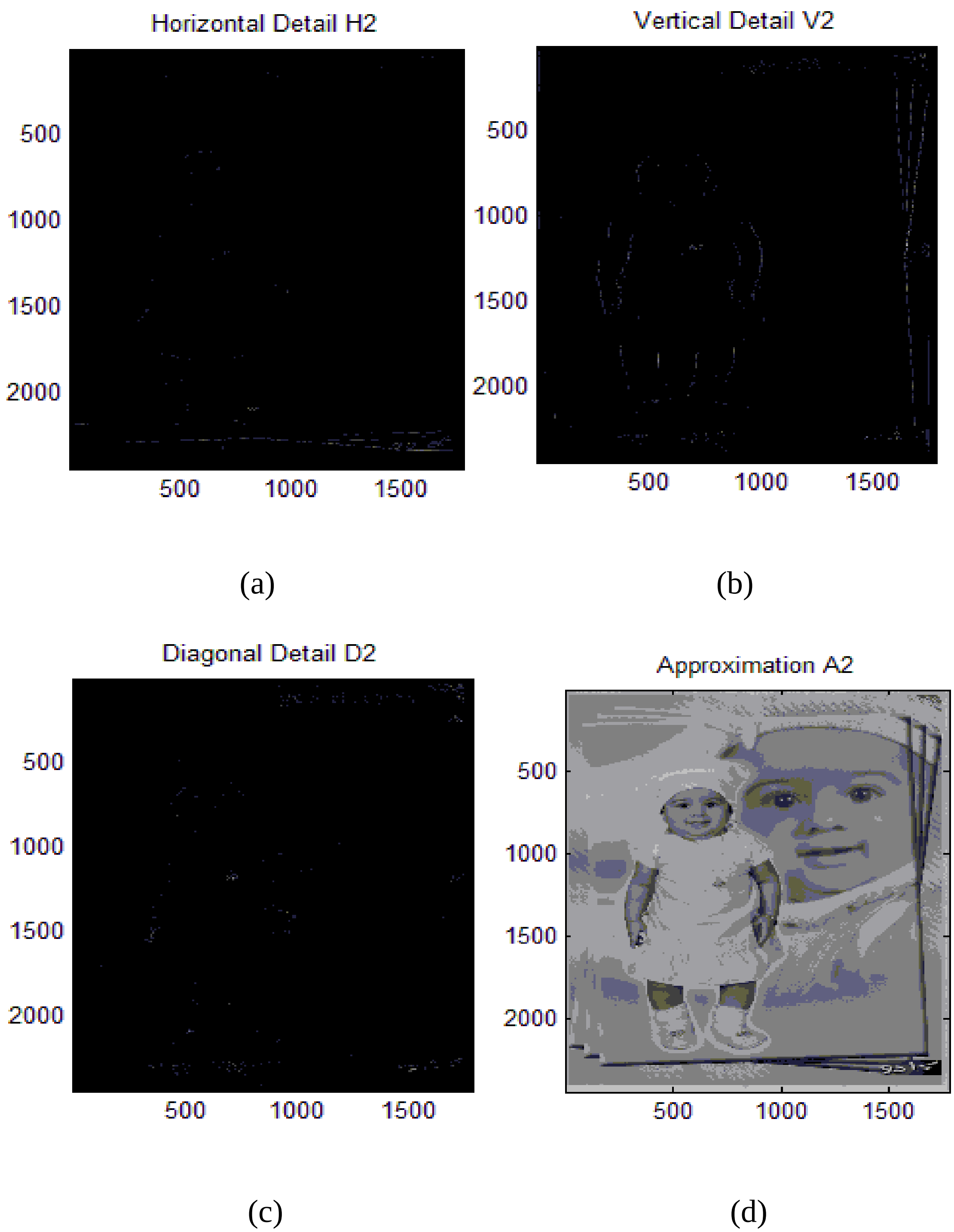
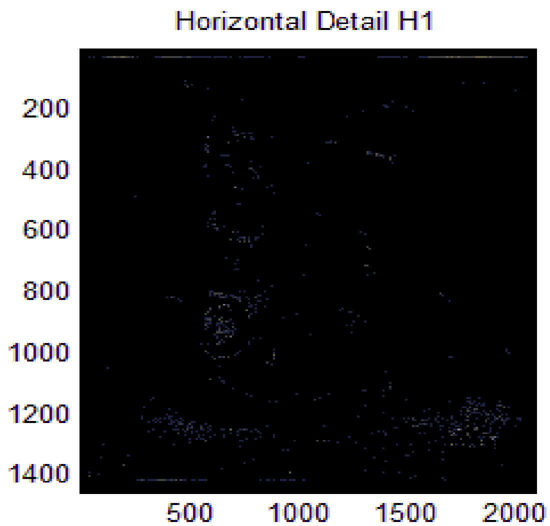
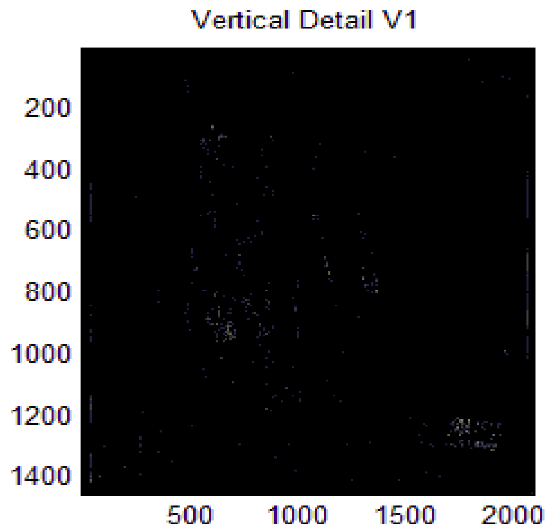


Figure 9. (a) Horizontal detail, (b) Vertical detail, (c) Diagonal detail, (d) 2nd level approximation.

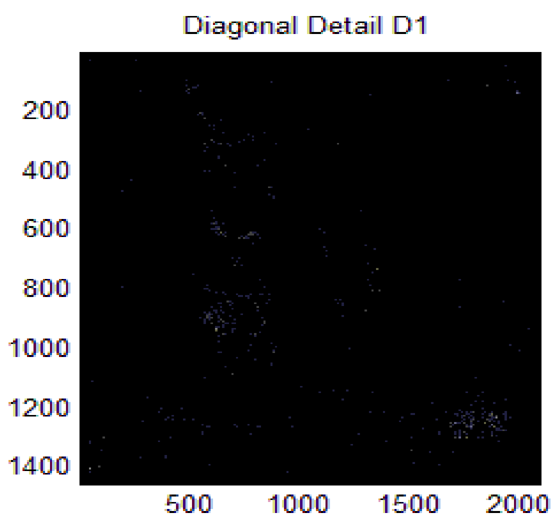
Figure10. BMP inverse transform



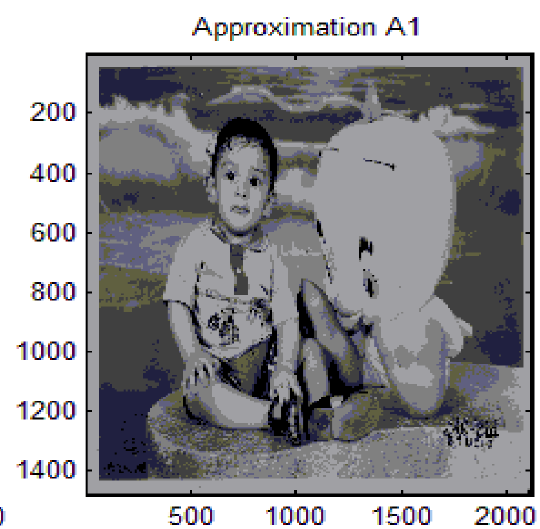
(a)



(b)

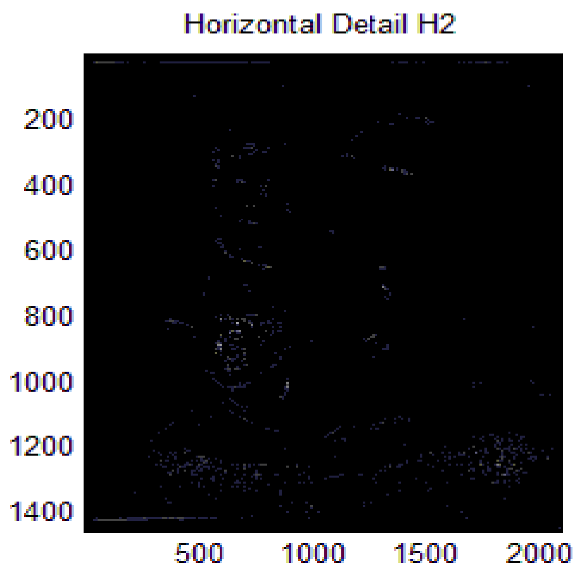


(c)

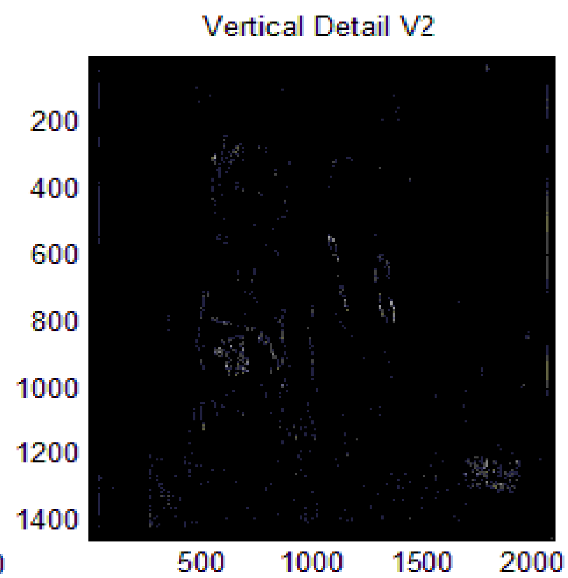


(d)

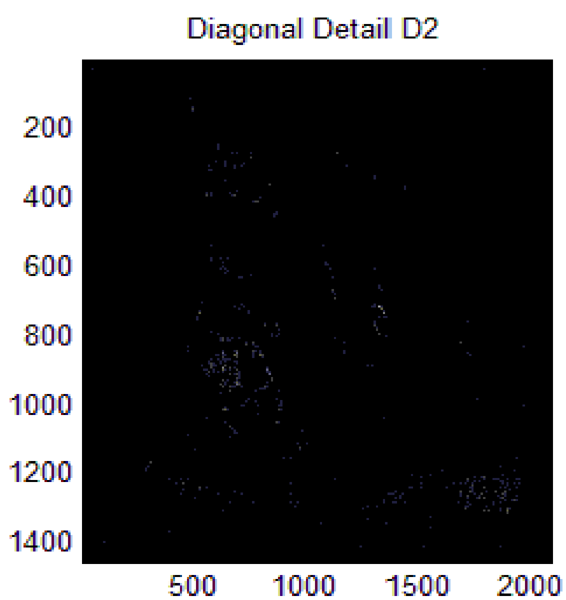
Figure 11. (a) Horizontal detail, (b) Vertical detail, (c) Diagonal detail, (d) 1st level approximation.



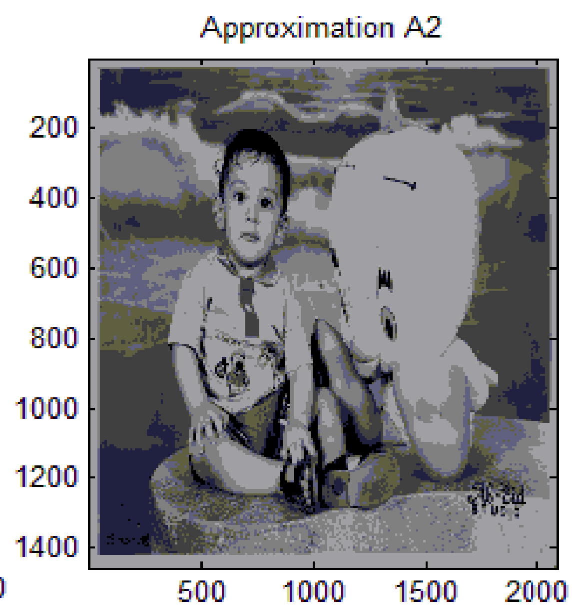
(a)



(b)



(C)



(d)

Figure 12. (a) Horizontal detail, (b) Vertical detail, (c) Diagonal detail, (d) 2nd level approximation.

Figure 13. TIFF inverse transform