How Much Does Transmit Correlation Affect the Sum-Rate of MIMO Downlink Channels?

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Abstract— **This paper considers the effect of spatial correlation between transmit antennas on the sum-rate capacity of the MIMO broadcast channel (i.e., downlink of a cellular system). Specifically, for a system with a large number of users n, we analyze the scaling laws of the sum-rate for the dirty paper coding and for different types of beamforming transmission schemes. When the channel is i.i.d., it has been shown that for large n, the sum rate is equal to $M \log\log n + M \log \frac{P}{M} + o(1)$ where M is the number of transmit antennas, P is the average signal to noise ratio, and $o(1)$ refers to terms that go to zero as $n \to \infty$. When the channel exhibits some spatial correlation with a covariance matrix R (non-singular with $tr(R) = M$), we prove that the sum rate of dirty paper coding is $M \log\log n + M \log \frac{P}{M} + \log\det(R) + o(1)$. We further show that the sum-rate of various beamforming schemes achieves $M \log\log n + M \log \frac{P}{M} + M \log c + o(1)$ where $c \leq 1$ depends on the type of beamforming. We can in fact compute c for random beamforming proposed in [12] and more generally, for random beamforming with precoding in which beams are pre-multiplied by a fixed matrix. Simulation results are presented at the end of the paper.**

I. INTRODUCTION

Multiple input multiple output (MIMO) communication has been the focus of a lot of research which basically demonstrated that the capacity of a point to point MIMO link increases linearly with the number of transmit and receive antennas. Research focus has shifted recently to the role of multiple antennas in multiuser systems, especially broadcast scenarios (i.e., *one to many* communication) as downlink scheduling is the major bottleneck for future broadband wireless networks. An overview of the research on this problem can be found in [13], [4], [1].

In these scenarios, when multiple users are present, one is usually interested in 1) quantifying the maximum possible sum rate to all users and 2) devising computationally efficient algorithms for capturing most of this rate. The first question was settled recently by using dirty paper coding (DPC) [8]. While DPC solves the broadcast problem optimally, it is computationally expensive and requires a great deal of feedback as the transmitter needs perfect channel state information for all users [13], [1].

There has been increased interest recently to devise simple techniques that utilize multiuser diversity and achieve a sum-rate close to the sum-rate capacity of the MIMO broadcast channel (see, e.g., [9], [12], [8], [3], [4]). The scheme proposed in [12], known as opportunistic multiple beamforming (or random beamforming), has been proved to asymptotically maximize the sum-rate (or throughput) of the downlink of single antenna cellular systems by transmitting to the users with the best channel conditions for a given set of random beams. The gain of this and other beamforming schemes can be attributed to multiuser diversity– each user experiences a different channel and therefore the transmitter can exploit this variation and choose the users that have the best channel conditions. Clearly, the multiuser gain would be specially magnified when the channels between the transmitter and the users are changing independently.

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In this paper we focus on a multi-antenna downlink channel in the presence of correlation between transmit antennas. This correlation is caused by local scatterers around the base station or the fact that the transmit antennas in the base station are not spaced far enough to create independent channels. The overriding question then is to analyze the effect of this correlation on the sum-rate of DPC and various beamforming scheduling techniques.

Specifically, we consider three variations of random beamforming, namely, random beamforming with channel whitening, beamforming with general precoding, and deterministic beamforming. In the first, the transmitter spatially whitens the channel and then uses random beamforming. In random beamforming with precoding, the transmitter employs a more general precoding matrix. In both of these transmission schemes, the transmitted signal needs to be scaled properly to maintain the average power constraint. Finally, in deterministic beamforming, as its names suggests, we use a fixed beamformer for all channel uses in place of the randomly varying one.

When the number of users is large and there is no correlation, the sum rate for DPC and random beamforming asymptotically coincide [12]

$$R = M \log\log n + M \log \frac{P}{M} + o(1) \tag{1}$$

where n is the number of users, M is the number of transmit antennas, and P is the average signal to noise ratio, and $o(1)$ represents terms that go to zero as $n \to \infty$. It turn out that this is not the case for the channel with transmit correlation. In this case, the sum-rate can be written as

$$M \log\log n + M \log \frac{P}{M} + M \log c + o(1) \tag{2}$$

where the constant $0 < c \leq 1$ (which refers to the sum-rate loss due to correlation) depends on the scheduling scheme and the eigenvalues of the covariance matrix $\mathcal{R}$.

II. Channel Model and Problem Formulation

In this paper we consider a multi-antenna Gaussian broadcast channel with n receivers equipped with one antenna and a transmitter (base station) with M antennas. Let $S(t)$ be the $M \times 1$ vector of the transmit symbols at time slot t, and let $Y_i(t)$ be the received signal at the i'th receiver. We can then write the received signal at the i'th user as

$$Y_i(t) = \sqrt{P} H_i S(t) + W_i, \qquad i = 1, \ldots, n, \tag{3}$$

where W_i is the additive noise which is complex Gaussian with zero mean and unit variance, $CN(0,1)$. Moreover, $S(t)$ is the transmit symbol satisfying the power constraint $E\{S^*S\} = 1$. Here P is the transmit power (or equivalently the average SNR considering the normalization for the noise and channel variances).

The channel H_i is a $1 \times M$ complex channel vector, known perfectly to the receiver, and distributed as $CN(0, \mathcal{R})$. The $M \times M$ covariance matrix $\mathcal{R}$ is a measure of the spatial correlation and is assumed to be non-singular with $tr(\mathcal{R}) = M$ [1]. We also assume that H_i follows a block fading model, i.e., it remains constant during a coherence interval T and varies independently from one such interval to the next. We finally note that the channel is identically distributed across users but is independent from one user to another.

III. Review of Transmission Schemes in the Downlink

A. Dirty Paper Coding (DPC)

The capacity region of the multi-antenna broadcast channel is achieved by dirty paper coding when full channel state information (CSI) is available to the transmitter and users. Intuitively, if the transmitter knows the channels of all users, it can use DPC to pre-subtract the interference for each user while preserving the average power constraint. More precisely, the sum rate capacity, R_{DPC}, can be written as (see [8] and the references therein),

$$R_{DPC} = E\left\{ \max_{\{P_1,\ldots,P_n, \sum tr(P_i) \le P\}} \log\det\left(1 + \sum_{i=1}^{n} H_i^* P_i H_i\right)\right\} \tag{4}$$

In a system with a large number of users n, and for fixed M and P, it has been shown that the sum-rate of DPC behaves as in (1), when there is no spatial correlation, i.e., $\mathcal{R} = I$ [12]. Scaling of the sum rate capacity has also been investigated for other regions of n, M, and P (see [6], [4], [5] for details).

There are two major drawbacks of this scheme. First, it is very computationally complex, both at the receivers and transmitter. Moreover, it requires full CSI feedback from all active users to the transmitter of the base station (this feedback requirement increases with the number of antennas and users and with the decrease of the coherence time of the system).

[1]We assume that the spatial correlation is invariant across users. This assumption is realistic because this is effectively the transmit correlation among antennas at the base station.

B. Random Beamforming

Given these drawbacks of DPC, research has focused on devising algorithms for multiuser broadcast channels that have less computational complexity and/or less feedback and still achieve most of the sum-rate promised by DPC such as random beamformig [11] and zero forcing [3] (see also [7], [2]). A random beamforming scheme was proposed in [12] where the transmitter sends multiple (in fact M) random orthonormal beams chosen to users with the best signal to interference ratio (SINR). In this scheme the only feedback required from each user is the SINR of the best beam and the corresponding index.

Specifically, the transmitter chooses M random orthonormal beam vectors ϕ_m (of size $M \times 1$) generated according to an isotropic distribution. Now these beams are used to transmit the symbols $s_1(t), s_2(t), \ldots, s_M(t)$ by constructing the transmitted vector $S(t) = \sum_{m=1}^{M} \phi_m(t) s_m(t)$, for $t = 1, \ldots, T$. After T channel uses, the transmitter independently chooses another set of orthogonal vectors $\{\phi_m\}$ (or the beamforming matrix $\Phi = [\phi_1, \ldots, \phi_M]$) and constructs the signal vector and so on. From now on and for simplicity, we will drop the time index t. The signal Y_i at the i'th receiver is given by

$$Y_i = \sqrt{P} H_i S + W_i = \sqrt{P} \sum_{m=1}^{M} H_i \phi_m s_m + W_i, \tag{5}$$

for $i = 1, \ldots, n$ and where $E(SS^*) = \frac{1}{M} I$ since the s_i's are assumed to be i.i.d. and assigned to different users. The i'th receiver uses its knowledge of the effective channel gain $H_i \phi_m$, something that can be arranged by training, to calculate M SINR's, one for each transmitted beam

$$\mathrm{SINR}_{i,m} = \frac{|H_i \phi_m|^2}{\frac{M}{P} + \sum_{k \ne m} |H_i \phi_k|^2}, \quad m = 1, \ldots, M \tag{6}$$

Each receiver then feeds back its maximum SINR, i.e. $\max_{1 \le m \le M} \mathrm{SINR}_{i,m}$, along with the maximizing index m. Thereafter, the transmitter assigns s_m to the user with the highest corresponding SINR, i.e. $\max_{1 \le i \le n} \mathrm{SINR}_{i,m}$. If we do the above scheduling, the throughput for large n can be written as [14], [12] [2],

$$R_{RBF} = M E \log\left(1 + \max_{1 \le i \le n} \mathrm{SINR}_{i,m}\right) + o(1) \tag{7}$$

where the term $o(1)$ accounts for the small probability that user i may be the strongest user for more than one signal s_m [12].

In [12], it is shown that the sum-rate of random beamforming for a channel with no spatial correlation, i.e., $\mathcal{R} = I$, scales exactly the same as the sum-rate capacity for large n as in (1).

C. Other Beamforming Schemes

In the presence of channel correlation, one may think of other types of beamforming as follows:

1-Random beamforming with channel whitening: One may first whiten the channel and then use random beamforming

[2]The proof follows from the fact the when n is large the maximum SINR and the M'th maximum SINR behave quite similarly.

scheduling. In this case, and instead of using Φ as the beamforming matrix[3], we would use $\sqrt{\alpha}\mathcal{R}^{-1/2}\Phi$ where α is a constant to make sure that the transmit symbol has an average power of 1.

2-Random beamforming with general precoding: More generally, we can precode with a general matrix $\sqrt{\alpha}A^{-1/2}$ before beamforming, i.e. we use $\sqrt{\alpha}A^{-1/2}\Phi$ to transmit the information symbols. The scaling of this scheme follows directly from the scaling of random beamforming over correlated channels and so is considered in Sections V-B and V-D.

3-Deterministic beamforming: Finally, by fixing the beamforming matrix Φ, we obtain deterministic beamforming, a scheme analyzed by Park and Park [10] (for the two antenna case) and which we further analyze in Section V-C.

IV. Effect of Transmit Correlation on the Sum-Rate of DPC

In this section, we derive the scaling laws of DPC for correlated channels. As mentioned the sum-rate capacity (achieved by DPC) is given by (4) and its behavior when n is large is given by (1) for i.i.d. channels. It turns out that when the number of users is large, the sum-rate capacity will be decreased by a constant which depends on the covariance matrix of the channel.

The next theorem proves this statement. The proof is along the same line as the proof for the i.i.d. case (see [12]) with the major difference that the lower bound, rather than being achieved with random beamforming, is achieved with a spacial type of deterministic beamforming. We first give the lower bound in the following lemma.

Lemma 1. *Consider a Gaussian broadcast channel with channel covariance matrix $\mathcal{R}$ which is non-singular and $tr(\mathcal{R}) = M$. Let there be one transmitter with M antennas and n users with single antennas that have access to the CSI and the transmitter knows the CSI perfectly. We assume the transmitter uses the deterministic beamforming matrix $\Phi = U^*$ where U is the unitary matrix consisting of the eigenvectors of $\mathcal{R}$. Then for large n, the sum-rate of this scheduling is*

$$R = M \log\log n + M \log \frac{P}{M} + M \log \sqrt[M]{\det \mathcal{R}} + o(1). \quad (8)$$

Proof: See Section V-C for the proof. ∎

Clearly (8) is a lower bound for the sum-rate capacity. In the next theorem we show that (8) is indeed an upper bound for the sum-rate as well.

Theorem 1. *Consider a Gaussian broadcast channel with channel covariance matrix $\mathcal{R}$ defined in Lemma 1. Let there be one transmitter with M antennas and n users with single antennas that have access to the CSI. Assume further that the transmitter knows the CSI perfectly. The sum-rate capacity (achieved by DPC) scales like*

$$R_{DPC} = M \log\log n + M \log \frac{P}{M} + M \log \sqrt[M]{\det \mathcal{R}} + o(1), \quad (9)$$

for large n.

[3]Note that Φ is an orthonormal matrix composed of the beam (column) vectors $\phi_1, \ldots, \phi_M$.

Proof: The result of Lemma 1 can serve as a lower bound for the sum-rate. As for the upper bound, we can bound the sum-rate capacity in (4) by first defining $\tilde{H}_i = \mathcal{R}^{-1/2}H_i$, and then using the geometric mean-arithmetic mean inequality. We omit the details of the proof for brevity and refer the reader to [15].

V. Effect of Transmit Correlation on Random Beamforming

The deterministic beamforming scheme of Lemma 1 asymptotically achieves the DPC sum-rate. However it has the drawback that, unless the H_i's change very rapidly over different channel uses, it will often transmit to a fixed set of users. To make the scheduling more short-term fair, it is useful to further randomize the user selection by random beamforming (see [11], [12] for more details). In this section, we analyze the effect of correlation on the sum-rate of random beamforming. We start by the simplest case in which the beamforming matrix is multiplied by $R^{-1/2}$ in order to whiten the channel. We then turn our attention to the random beamforming scheme and finally use it to deduce the sum rates of deterministic beamforming and beamforming with general precoding.

A. Random Beamforming with Channel Whitening

To whiten the channel, we multiply all the beams with $\sqrt{\alpha}\mathcal{R}^{-1/2}$ where α is a normalization factor. The transmit symbol is therefore equal to

$$S(t) = \sum_{m=1}^{M} \sqrt{\alpha}\mathcal{R}^{-1/2}\phi_m(t)s_m(t) \quad (10)$$

We choose α to satisfy the power constraint– that the transmit symbol average power is bounded by unity,

$$E\{\alpha S^*\mathcal{R}^{-1}S\} = \alpha E\{tr(S\mathcal{R}^{-1}S^*)\} = \alpha\frac{tr(\mathcal{R}^{-1})}{M} \quad (11)$$

Thus, the constraint $E\{\alpha S^*\mathcal{R}^{-1}S\} \leq 1$ implies that $\alpha \leq \frac{M}{tr(\mathcal{R}^{-1})}$. We can therefore write the SINR as

$$\text{SINR}_{i,m} = \frac{|H_i^w\phi_m|^2}{\frac{M}{P\alpha} + \sum_{k\neq m}|H_i^w\phi_k|^2}, \quad m = 1, \ldots, M \quad (12)$$

where $H_i^w = H_i\mathcal{R}^{-1/2}$ has covariance of I and therefore has i.i.d. Gaussian entries with zero mean and unit variance. Therefore we can apply the random beamforming result of [12] to obtain the sum rate of random beamforming with channel whitening. This is summarized in the following Theorem.

Theorem 2. *Consider the setting of Lemma 1. Let there be one transmitter with M antennas and n users with single antennas that have access to the CSI. If the transmitter knows the channel autocorrelation perfectly, then the sum rate capacity for random beam forming with channel whitening (denoted by R_{BF-W}) is given by*

$$R_{BF-W} = M \log\log n + M \log \frac{P}{M} - M \log \frac{tr(\mathcal{R}^{-1})}{M} + o(1) \quad (13)$$

for sufficiently large n.

When the the channel is i.i.d, Theorem 2 reduces to the already known result of [12]. It is also worth mentioning that (13) is less than the sum-rate achieved by DPC in (9).

B. Sum-Rate of Random Beamforming

In this section, we study the effect of transmit correlation on random beam-forming. To do this, we need to derive the CDF and PDF of the SINR defined in (6).

The sum rate capacity of random beamforming is given in (7). The expectation in (7) over H_i and Φ can be done as follows,

$$R_{RBF} = E_{\Phi}\left\{E_{H_i's|\Phi}\log\left(1+\max_{1\le i\le n}\mathrm{SINR}_{i,m}\right)|\Phi\right\}+o(1), \tag{14}$$

i.e., we evaluate the expectation by first conditioning on Φ and calculating the expectation over H_i and we subsequently average over Φ. The advantage of doing so is that Φ is common among all users and so, by conditioning over Φ, all the SINR's, $\mathrm{SINR}_{1,m},\ldots,\mathrm{SINR}_{n,m}$ remain iid. This in turn allows us to evaluate $\max\limits_{1\le i\le n}\mathrm{SINR}_{i,m}$ using extreme value theory provided we can evaluate the CDF (and pdf) of the SINR.

It turns out that the main challenge lies in calculating the CDF of SINR given Φ. When the channel is i.i.d., calculating the CDF is straightforward as the SINR numerator and denominator are independent [12]. This ceases to be the case in the presence of correlation and in evaluating the CDF. Instead, we use a contour integral representation of the unit step and find the CDF using the Gaussian integral. Once the CDF is available, we appeal to results in extreme value theory to obtain the behavior of $\max\limits_{1\le i\le n}\mathrm{SINR}_{i,m}$ when n is large and proceed to calculate the expectation in (14)

With the scaling law for random beamforming at hand, it becomes straightforward to obtain the scaling laws of random beamforming with precoding and of deterministic beamforming.

1) Distribution of $\mathrm{SINR}_{i,1}$ *Given* Φ *:* Let $U^*\Lambda^{-1}U$ be the eigenvalue decomposition of $\mathcal{R}^{-1}$ and define the matrix A as,

$$A=(1+x)\Lambda^{1/2}\overline{\phi}_m\overline{\phi}_m^*\Lambda^{1/2}-x\Lambda \tag{15}$$

where $\overline{\phi}_m=\phi_m U$. Then, the CDF of $\mathrm{SINR}_{i,1}$ can be written as,

$$\boxed{F(x)=1-\tfrac{1}{2\pi^M\det(R)}\lambda_M\prod_{i=1}^{M-1}\tfrac{\lambda_i'\lambda_M'}{x(\lambda_i'-\lambda_M')}e^{-\frac{M}{P}\frac{x}{\lambda_M'}}} \tag{16}$$

where λ_i' is the i'th eigenvalue of A. We would like to emphasize that the eigenvalues of A (i.e., λ_i') are functions of x.

We can further show that the CDF of SINR satisfies,

$$\lim_{x\to\infty}\frac{1-F(x)}{f(x)}=\frac{P}{M\|\overline{\phi}_m\|^2_{\Lambda^{-1}}}$$

where $f(x)$ is the PDF of the SINR and $\|A\|_\Lambda = A^*\Lambda A$. Using extreme value theory, and the lemma above, we know that $\max\limits_{1\le i\le n}\mathrm{SINR}_{i,m}$ behaves like $\frac{P}{M\|\overline{\phi}_m\|^2_{\Lambda^{-1}}}\log n$. Upon substituting this in (14) and noting that the $\overline{\phi}$'s are identically distributed, we can write

$$\begin{aligned}R &= \sum_{m=1}^{M}E_{\phi_m}\log\left(\frac{P}{M\|\overline{\phi}_m\|^2_{\Lambda^{-1}}}\log n\right)+o(1)\\ &= M\log\log n+M\log\frac{P}{M}+ME_{\phi_m}\log(\frac{1}{\|\overline{\phi}_m\|^2_{\Lambda^{-1}}})+o(1)\end{aligned}$$

It thus remains to calculate the above expectation for which we need to derive the CDF of $\frac{1}{\|\overline{\phi}_m\|^2_{\Lambda^{-1}}}$ where $\overline{\phi}_m$ is a vector uniformly distributed over the complex sphere of radius one. Here is the result.

Lemma 2. *The CDF of* $y=\frac{1}{\|\overline{\phi}_m\|^2_{\Lambda^{-1}}}$ *is given by*

$$\boxed{G(x)=1-\textstyle\sum_i\eta_i\left(\frac{1}{x}-\frac{1}{\lambda_i}\right)^{M-1}u\left(1-\frac{x}{\lambda_i}\right)}$$

where λ_i*'s are the diagonal entries of* Λ, $\eta_i=\frac{1}{\prod_{j\neq i}(\frac{1}{\lambda_j}-\frac{1}{\lambda_i})}$ *and* $u(\cdot)$ *is the unit step function.*

Therefore the sum-rate of beamforming can be written as,

$$\begin{aligned}R_{RBF} \;=\;& M\log\log n+M\log\frac{P}{M}+\log\lambda_1+o(1)+\\ &\sum_{i=1}^{M}\eta_i\log\left(\frac{\lambda_i}{\lambda_1}\right)\sum_{k=1}^{M-1}\frac{1}{k+2}\left(\frac{-1}{\lambda_i}\right)^{M-1-k}\times\\ &\left\{\frac{1}{(\lambda_i)^{k+2}}-\frac{1}{(\lambda_1)^{k+2}}\right\}\end{aligned} \tag{17}$$

C. Sum-Rate of Deterministic Beamforming

Here we consider the case where the beamforming matrix Φ is fixed over all channel uses. In this case, we can use the same analysis as we done in the case of random beamforming with the only exception that we do not need to take expectation over the beamforming matrix. Therefore, we may write the sum-rate for the deterministic beamforming matrix Φ as,

$$R_{BF-D}=M\log\log n+M\log\frac{P}{M}+\sum_{i=1}^{M}\log\left(\frac{1}{\phi_i^*U^*\Lambda^{-1}U\phi_i}\right)+o(1)$$

where $U^*\Lambda^{-1}U$ is the eigenvalue decomposition of the correlation matrix $\mathcal{R}^{-1}$.

One interesting spacial case would be the case where the $U\phi_i$'s are the columns of the identity matrix. In this case, the beamforming matrix is in fact equal to U^* and the argument in the logarithm would therefore reduce to λ_i. Thus, when n is large, the sum-rate is given by

$$M\log\log n+M\log\frac{P}{M}+M\log\sqrt[M]{\det\mathcal{R}}+o(1). \tag{18}$$

Keeping in mind that the eigenvalues of Λ are such that $\sum_{i=1}^{M}\lambda_i=M$, it is clear that the geometric mean of λ_i's would be less than 1. This in fact proves Lemma 1. It should be also mentioned that this result is obtained in [10] for $M=2$. As mentioned before, this actually coincides with the upper bound obtained in Theorem 1 for the sum-rate of DPC.

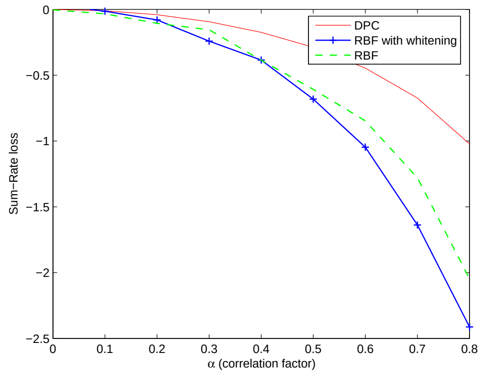

Fig. 1. Sum-rate loss versus the correlation factor for a system with $M = 2$ and $n = 100$.

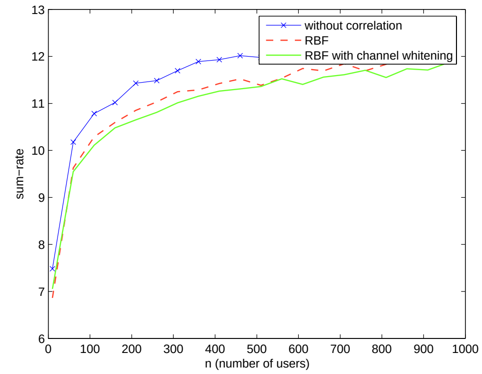

Fig. 2. Sum-rate versus the number of users in a system with $M = 2$ and $\beta = 0.5$

D. Sum-Rate of Random Beamforming with Precoding

We can consider a generalization of the random beamforming by using precoding. In this scheme the new beamforming matrix is $\sqrt{\alpha}A^{-1/2}\Phi$ where A is a positive definite matrix and α is just a normalization factor to adjust the transmit power. Again similar to Section V-B, we can state that α has to be less than $\frac{M}{tr(A^{-1})}$.

In order to analyze the sum-rate, we can proceed along the same line as what we did for the analysis of the random beamforming with the only exception that the covariance matrix of the channel is replaced with $\tilde{\mathcal{R}} = A^{-*/2}\mathcal{R}A^{-1/2}$. Here is the main result.

Corollary 1. *Considering the random beamforming scheduling with beamforming matrix $\sqrt{\alpha}A^{-1/2}\Phi$ where Φ is a random unitary matrix, the sum-rate of this scheme can be written as*

$$\begin{aligned} R_{BF-Prec} &= M\log\log n + M\log\frac{P}{M} + o(1) \\ &+ \sum_{i=1}^{M} E\log\left(\frac{M}{tr(\Lambda^{-1})}\frac{1}{\phi_i^* U^*\Lambda^{-1}U\phi_i}\right), \end{aligned}$$

for large n, where $U^*\Lambda^{-1}U$ represents the eigenvalue decomposition of $\tilde{\mathcal{R}}^{-1}$.

VI. Simulation Results

In this section we present the simulation results for the sum-rate of beamforming schemes and DPC. In the first example, we consider a system with two transmit antennas, i.e., $M = 2$, and 100 users. The covariance matrix is assumed to be like

$$\mathcal{R} = \begin{bmatrix} 1 & \beta \\ \beta & 1 \end{bmatrix} \tag{19}$$

where β is the correlation. Fig. 1 shows the sum-rate loss (compared to the case of no correlation) for DPC, RBF and RBF with whitening. It is clear that RBF outperforms the one with channel whitening for not too small value of β. In Fig. 2, we show the sum-rate versus the number of users in system with $M = 2$, $\beta = 0.5$, $P = 10$ for beamforming scheme and it is compared to the case of having no correlation. The non-smooth behavior of the sum-rate is due to the averaging of the rates over 1000 channel realizations.

VII. Conclusion

This paper considers the effect of spatial correlation on various multiuser scheduling schemes for MIMO broadcast channels. Specifically, we considered dirty paper coding and various (random, deterministic, and channel whitening) beamforming schemes. The rate loss due to correlation has been obtained for the aforementioned transmission schemes and when the number of users is large.

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