ITERATIVE BLIND DATA DETECTION IN CONSTANT MODULUS OFDM SYSTEMS

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ABSTRACT

In this paper, we consider blind data detection for OFDM transmission over block fading channels. Specifically, we show how constant modulus data of an OFDM symbol can be blindly detected using output symbol and associated cyclic prefix. Our approach relies on decomposing the OFDM channel into two subchannels (cyclic and linear) that share the same input and are characterized by the same channel parameters. This fact enables us to estimate the channel parameters from one subchannel and substitute the estimate into the other, thus obtaining a nonlinear relationship involving the input and output data only that can be searched for the maximum likelihood estimate of the input. This shows that OFDM systems are completely identifiable using output data only, irrespective of the channel zeros, as long as the channel delay spread is less than the length of the cyclic prefix. We also propose iterative methods to reduce the computational complexity involved in the maximum likelihood search of input.

1. INTRODUCTION

OFDM modulation has attracted considerable attention as it combines the advantages of high achievable data rates and relatively easy implementation. This is reflected by the many standards that considered and adopted OFDM [1]. For proper operation of an OFDM system, the receiver needs to estimate the channel and eliminate its effect. Many techniques have been proposed in the literature for this purpose (see, e.g., [1], [2], [3], [4], [5], and the references therein). These techniques fall into three distinct classes: 1) training based, 2) semiblind and 3) blind methods. Training/pilot based methods transmit symbols which are already known at the receiver (pilots) to estimate the channel. Blind methods rely only on a priori constraints for channel estimation and data recovery while semiblind methods make use of both pilots and additional channel/input data constraints to perform channel identification and equalization.

In this paper, we perform channel identification and equalization from output data only (i.e. OFDM output symbol and associated cyclic prefix (CP)), without the need for a training sequence or a priori channel information. The advantage of our approach is three fold:

1. The method provides a blind estimate of the data from one output symbol without the need for training or averaging (contrary to the common practice in blind methods where averaging over several symbols is required). Thus, the method lends itself to block fading channels.
2. Data detection is done without any restriction on the channel (as long as the delay spread is shorter than the (CP)). In fact, data detection can be performed even in the presence of zeros on the FFT grid.[1]
3. The fact that we use two observations (the OFDM symbol and CP) to recover the input symbol enhances the diversity of the system as can be seen from simulations.

Our approach is based on the transformation of the OFDM channel into two parallel subchannels due to the presence of a cyclic prefix at the input. One is a cyclic channel that relates the input and output OFDM symbols and thus is free of any intersymbol interference (ISI) effects and is best described in the frequency domain. The other one is a linear channel that carries the burden of ISI and that relates the input and output prefixes through linear convolution. This channel is best studied in the time domain.

It can be shown that the two subchannels are characterized by the same set of parameters (or impulse response(IR)) and are driven by the same stream of data. They only differ in the way in which they operate on the data (i.e. linear vs circular convolution). This fact enables us to estimate the IR from one subchannel and eliminate its effect from the other, thus obtaining a nonlinear least squares relationship that involves the input and output data only. This relationship can in turn be optimized for the ML data estimate, something that can be achieved through exhaustive search (in the worst case scenario).

Exhaustive search is very computationally expensive. We suggest two approaches to reduce the computational complexity. The first approach is based on approximating the nonlinear least squares problem with a linear one. In the second approach, we use the Particle Swarm Optimization (PSO) and the Genetic Algorithm (GA) [6], [7], [8] to directly solve the nonlinear problem. The estimate obtained by the linear approximation approach can be used to kick start these search algorithms.

This paper is organized as follows. After introducing our notation in the next section, we perform a careful study in section 3 of the elements of an OFDM channel decomposing it into a cyclic subchannel described in the frequency domain and a linear subchannel described in the time domain. In section 4, we show how this characterization can be used to

[1] This comes contrary to the common belief that OFDM using CP cannot be equalized for channels with zeros on the FFT grid [1] and [9]

construct a nonlinear objective function that can be exhaustively searched for the data. In section 5, we describe two approaches for reducing the computational complexity by linearizing the nonlinear function and by using the PSO and GA. Simulation results are discussed in section 6 with conclusion in section 7.

2. NOTATION

We denote scalars with small-case letters, vectors with small-case boldface letters, and matrices with uppercase boldface letters. Calligraphic notation (e.g. $\boldsymbol{\mathcal{X}}$) is reserved for vectors in the frequency domain. The individual entries of a vector like $\boldsymbol{h}$ are denoted by $h(l)$. A hat over a variable indicates an estimate of the variable (e.g., $\hat{\boldsymbol{h}}$ is an estimate of $\boldsymbol{h}$). When any of these variables become a function of time, the time index i appears as a subscript.

Now consider a length-N vector $\boldsymbol{x}_i$. We deal with three derivatives associated with this vector. The first two are obtained by partitioning $\boldsymbol{x}_i$ into a lower (trailing) part $\underline{\boldsymbol{x}}_i$ (known as the cyclic prefix) and an upper vector $\tilde{\boldsymbol{x}}_i$. The third derivative, $\overline{\boldsymbol{x}}_i$, is created by concatenating $\boldsymbol{x}_i$ with a copy of CP i.e. $\underline{\boldsymbol{x}}_i$. Thus, we have

$$\overline{\boldsymbol{x}}_i = \begin{bmatrix} \underline{\boldsymbol{x}}_i \\ \boldsymbol{x}_i \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{x}}_i \\ \tilde{\boldsymbol{x}}_i \\ \underline{\boldsymbol{x}}_i \end{bmatrix} \tag{1}$$

Thus, in line with the above notation, a matrix $\boldsymbol{Q}$ having N rows will have the natural partitioning

$$\boldsymbol{Q} = \begin{bmatrix} \tilde{\boldsymbol{Q}} \\ \underline{\boldsymbol{Q}} \end{bmatrix} \tag{2}$$

where the number of rows in $\tilde{\boldsymbol{Q}}$ and $\underline{\boldsymbol{Q}}$ are understood from the context and when it is not clear, the number of rows will appear as a subscript.

3. SYSTEM OVERVIEW

In an OFDM system, data is transmitted in symbols $\boldsymbol{\mathcal{X}}_i$ of length N each. The symbol undergoes an IFFT operation to produce the time domain symbol $\boldsymbol{x}_i$, i.e.

$$\boldsymbol{x}_i = \sqrt{N}\boldsymbol{Q}\boldsymbol{\mathcal{X}}_i \tag{3}$$

where $\boldsymbol{Q}$ is the $N \times N$ IFFT matrix. When juxtaposed, these symbols result in the sequence $\{x_k\}$. [2] We assume a non-ideal channel $\underline{\boldsymbol{h}}$ of maximum length $L+1$. To avoid ISI caused by passing through the channel, a cyclic prefix (CP) $\underline{\boldsymbol{x}}_i$ (of length L) is appended to $\boldsymbol{x}_i$, resulting finally in super-symbol $\overline{\boldsymbol{x}}_i$ as defined in (1). The concatenation of these symbols produces the underlying sequence $\{\overline{x}_k\}$.

When passed through the channel $\underline{\boldsymbol{h}}$, the sequence $\{\overline{x}_k\}$ produces the output sequence $\{\overline{y}_k\}$ i.e.

$$\overline{y}_k = \underline{h}_k * \overline{x}_k + \overline{n}_k \tag{4}$$

[2]The time indices in the sequence $\boldsymbol{x}_i$ and the underlying sequence $\{x_k\}$ are dummy variables. Nevertheless, we chose to index the two sequences differently to avoid any confusion that might arise from choosing identical indices.

where $\overline{n}_k$ is the additive white Gaussian noise and $*$ stands for linear convolution. Motivated by the symbol structure of the input, it is convenient to partition the output into symbols of length $M = N + L$, i.e.

$$\overline{\boldsymbol{y}}_i = \begin{bmatrix} \underline{\boldsymbol{y}}_i \\ \boldsymbol{y}_i \end{bmatrix} \tag{5}$$

This is a natural way to partition the output because the prefix $\underline{\boldsymbol{y}}_i$ actually absorbs all ISI that takes place between the adjacent symbols $\overline{\boldsymbol{x}}_{i-1}$ and $\overline{\boldsymbol{x}}_i$. Moreover, the remaining part $\boldsymbol{y}_i$ of the symbol depends on the ith input OFDM symbol $\boldsymbol{x}_i$ only. These facts allow us to partition the total OFDM channel described by (4) into two subchannels that we describe next.

3.1. Circular Convolution (Subchannel)

Due to the presence of the cyclic prefix, the input and output OFDM symbols $\boldsymbol{x}_i$ and $\boldsymbol{y}_i$ are related by circular convolution (denoted by $\circledast$), i.e.

$$\boxed{\boldsymbol{y}_i = \boldsymbol{h}_i \circledast \boldsymbol{x}_i + \boldsymbol{n}_i} \tag{6}$$

where $\boldsymbol{h}_i$ is a length-N zero-padded version of $\underline{\boldsymbol{h}}_i$. In the frequency domain, the cyclic convolution (6) reduces to the element-by-element operation

$$\boxed{\boldsymbol{\mathcal{Y}}_i = \boldsymbol{\mathcal{H}}_i \odot \boldsymbol{\mathcal{X}}_i + \boldsymbol{\mathcal{N}}_i} \tag{7}$$

where $\boldsymbol{\mathcal{H}}_i$, $\boldsymbol{\mathcal{X}}_i$, $\boldsymbol{\mathcal{N}}_i$, and $\boldsymbol{\mathcal{Y}}_i$, are the DFT's of $\boldsymbol{h}$, $\boldsymbol{x}_i$, $\boldsymbol{n}_i$, and $\boldsymbol{y}_i$ respectively

$$\begin{gathered} \boldsymbol{\mathcal{H}}_i = \boldsymbol{Q}^*\boldsymbol{h}_i, \quad \boldsymbol{\mathcal{X}}_i = \frac{1}{\sqrt{N}}\boldsymbol{Q}^*\boldsymbol{x}_i, \\ \boldsymbol{\mathcal{N}}_i = \frac{1}{\sqrt{N}}\boldsymbol{Q}^*\boldsymbol{n}_i, \text{ and } \boldsymbol{\mathcal{Y}}_i = \frac{1}{\sqrt{N}}\boldsymbol{Q}^*\boldsymbol{y}_i \end{gathered} \tag{8}$$

Since $\underline{\boldsymbol{h}}_i$ corresponds to the first $L+1$ elements of $\boldsymbol{h}_i$, we can show that

$$\boldsymbol{\mathcal{H}}_i = \boldsymbol{Q}^*_{L+1}\underline{\boldsymbol{h}}_i \quad \text{and} \quad \underline{\boldsymbol{h}}_i = \boldsymbol{Q}_{L+1}\boldsymbol{\mathcal{H}}_i \tag{9}$$

where $\boldsymbol{Q}^*_{L+1}$ consists of the first $L+1$ columns of $\boldsymbol{Q}^*$ and $\boldsymbol{Q}_{L+1}$ consists of first $L+1$ rows of $\boldsymbol{Q}$. This allows us to rewrite (7) as

$$\boxed{\boldsymbol{\mathcal{Y}}_i = \text{diag}(\boldsymbol{\mathcal{X}}_i)\boldsymbol{Q}^*_{L+1}\underline{\boldsymbol{h}}_i + \boldsymbol{\mathcal{N}}_i} \tag{10}$$

3.2. Linear Convolution (Subchannel)

From (4), we can also deduce that the cyclic prefixes at the input and output are related by linear convolution. Specifically, if we concatenate all cyclic prefixes at the input into a sequence $\{\underline{x}_k\}$ and the cyclic prefixes at the output into the corresponding sequence $\{\underline{y}_k\}$, then we can show that the two sequences are related by linear convolution [10]

$$\boxed{\underline{y}_k = \underline{h}_k * \underline{x}_k + \underline{n}_i} \tag{11}$$

From this we deduce that the cyclic prefix of OFDM symbol $\boldsymbol{y}_i$ is related to the input cyclic prefixes $\underline{x}_{i-1}$ and $\underline{x}_i$ by

$$\boxed{\underline{\boldsymbol{y}}_i = \underline{\boldsymbol{X}}_i \underline{\boldsymbol{h}}_i + \underline{\boldsymbol{n}}_i} \tag{12}$$

where $\underline{\boldsymbol{X}}_i$ is constructed from $\underline{x}_{i-1}$ and $\underline{x}_i$ according to

$$\underline{\boldsymbol{X}}_i = \begin{bmatrix} \underline{x}_i(0) & \underline{x}_{i-1}(L-1) & \cdots & \underline{x}_{i-1}(0) \\ \underline{x}_i(1) & \underline{x}_i(0) & \cdots & \underline{x}_{i-1}(1) \\ \vdots & \ddots & \ddots & \vdots \\ \underline{x}_i(L-1) & \cdots & \underline{x}_i(0) & \underline{x}_{i-1}(L-1) \end{bmatrix} \tag{13}$$

This fact together with the FFT relationship (9) yields the desired time-frequency form

$$\underline{\boldsymbol{y}}_i = \underline{\boldsymbol{X}}_i \boldsymbol{Q}_{L+1} \boldsymbol{\mathcal{H}}_i + \underline{\boldsymbol{n}}_i \tag{14}$$

4. MAXIMUM-LIKELIHOOD ESTIMATION

Consider the frequency domain description of the cyclic subchannel (7). To obtain the ML estimate of $\boldsymbol{\mathcal{H}}_i$, we assume that the sequence $\boldsymbol{\mathcal{X}}_i$ is deterministic and perform an element-by-element division of (7) by $\boldsymbol{\mathcal{X}}_i$ to get

$$\boldsymbol{D}_{\mathcal{X}}^{-1} \boldsymbol{\mathcal{Y}}_i = \boldsymbol{\mathcal{H}}_i + \boldsymbol{\mathcal{N}}_i^{'} \tag{15}$$

where

$$\boldsymbol{D}_{\mathcal{X}} = \text{diag}(\boldsymbol{\mathcal{X}}_i) \tag{16}$$

and $\boldsymbol{\mathcal{N}}_i^{'}$ is Gaussian distributed with zero mean and autocorrelation matrix

$$\boldsymbol{R}_{n'} = \sigma_n^2 \boldsymbol{D}_{\mathcal{X}}^{-1} \boldsymbol{D}_{\mathcal{X}}^{-*} = \sigma_n^2 |\boldsymbol{D}_{\mathcal{X}}|^{-2} \tag{17}$$

The maximum-likelihood estimate of $\boldsymbol{\mathcal{H}}$ can now be obtained by solving the system of equations (15) in the least-squares (LS) sense subject to the constraint

$$\tilde{\boldsymbol{Q}}_{N-L-1} \boldsymbol{\mathcal{H}}_i \triangleq \tilde{\boldsymbol{Q}} \boldsymbol{\mathcal{H}}_i = 0 \tag{18}$$

We can show that the ML estimate is given by [11]

$$\begin{aligned} \hat{\boldsymbol{\mathcal{H}}}_i^{ML} &= \left[\boldsymbol{I} - \boldsymbol{R}_{n'} \tilde{\boldsymbol{Q}}^* \left(\tilde{\boldsymbol{Q}} \boldsymbol{R}_{n'} \tilde{\boldsymbol{Q}}^* \right)^{-1} \tilde{\boldsymbol{Q}} \right] \boldsymbol{D}_{\mathcal{X}}^{-1} \boldsymbol{\mathcal{Y}}_i \\ &= \left[\boldsymbol{I} - |\boldsymbol{D}_{\mathcal{X}}|^{-2} \tilde{\boldsymbol{Q}}^* \left(\tilde{\boldsymbol{Q}} |\boldsymbol{D}_{\mathcal{X}}|^{-2} \tilde{\boldsymbol{Q}}^* \right)^{-1} \tilde{\boldsymbol{Q}} \right] \boldsymbol{D}_{\mathcal{X}}^{-1} \boldsymbol{\mathcal{Y}}_i \end{aligned} \tag{19}$$

The ML estimate (19) was obtained solely from the circular convolution subchannel. In the case of constant modulus data, we have

$$|\boldsymbol{D}_{\mathcal{X}}|^{-2} = \frac{1}{\mathcal{E}_X} \boldsymbol{I} \quad \text{and} \quad \boldsymbol{D}_{\mathcal{X}}^{-1} = \frac{1}{\mathcal{E}_X} \boldsymbol{D}_{\mathcal{X}}^* \tag{20}$$

where $\mathcal{E}_X$ stands for energy of data. Upon substitution, equation (19) becomes

$$\hat{\boldsymbol{\mathcal{H}}}_i^{ML} = \frac{1}{\mathcal{E}_X} \left[\boldsymbol{I} - \tilde{\boldsymbol{Q}}^* \left(\tilde{\boldsymbol{Q}} \tilde{\boldsymbol{Q}}^* \right)^{-1} \tilde{\boldsymbol{Q}} \right] \boldsymbol{D}_{\mathcal{X}}^* \boldsymbol{\mathcal{Y}}_i \tag{21}$$

$$= \frac{1}{\mathcal{E}_X} \left[\boldsymbol{I} - \tilde{\boldsymbol{Q}}^* \tilde{\boldsymbol{Q}} \right] \boldsymbol{D}_{\mathcal{X}}^* \boldsymbol{\mathcal{Y}}_i \tag{22}$$

where in (22), we used the fact that $\tilde{\boldsymbol{Q}}$ is a left-inverse of $\tilde{\boldsymbol{Q}}^*$ - a consequence of the unitary nature of $\boldsymbol{Q}$

$$\boldsymbol{I} = \boldsymbol{Q}\boldsymbol{Q}^* = \begin{bmatrix} \boldsymbol{Q}_{L+1} \\ \tilde{\boldsymbol{Q}}_{N-L-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{Q}_{L+1}^* & \tilde{\boldsymbol{Q}}_{N-L-1}^* \end{bmatrix} \tag{23}$$

Upon replacing $\boldsymbol{\mathcal{H}}_i$ that appears in the time-frequency form (14) (corresponding to the linear subchannel) with its ML estimate (22), we obtain

$$\underline{\boldsymbol{y}}_i = \frac{1}{\mathcal{E}_X} \underline{\boldsymbol{X}}_i \boldsymbol{Q}_{L+1} \left[\boldsymbol{I} - \tilde{\boldsymbol{Q}}^* \tilde{\boldsymbol{Q}} \right] \boldsymbol{D}_{\mathcal{X}}^* \boldsymbol{\mathcal{Y}}_i + \underline{\boldsymbol{n}}_i \tag{24}$$

This is an input/output relationship that does not depend on any channel information whatsoever. Since the data is assumed deterministic, maximum-likelihood estimation is the optimum way to detect it, i.e. we minimize

$$\hat{\boldsymbol{\mathcal{X}}}_i^{ML} = \arg\min_{\mathcal{X}_i} \left\| \underline{\boldsymbol{y}}_i - \frac{1}{\mathcal{E}_X} \underline{\boldsymbol{X}}_i \boldsymbol{Q}_{L+1} \left[\boldsymbol{I} - \tilde{\boldsymbol{Q}}^* \tilde{\boldsymbol{Q}} \right] \boldsymbol{\mathcal{Y}}_i \odot \boldsymbol{\mathcal{X}}_i^* \right\|^2 \tag{25}$$

From (23), we can also deduce that

$$\boldsymbol{Q}_{L+1} \tilde{\boldsymbol{Q}} \; = \; \boldsymbol{Q}_{L+1} \tilde{\boldsymbol{Q}}_{N-L-1} \; = \; 0$$

So, the ML estimate of $\boldsymbol{\mathcal{X}}_i$, for the constant modulus case, is obtained by performing the minimization

$$\hat{\boldsymbol{\mathcal{X}}}_i^{ML} = \arg\min_{\mathcal{X}_i} \left\| \underline{\boldsymbol{y}}_i - \frac{1}{\mathcal{E}_X} \underline{\boldsymbol{X}}_i \boldsymbol{Q}_{L+1} \boldsymbol{\mathcal{Y}}_i \odot \boldsymbol{\mathcal{X}}_i^* \right\|^2 \tag{26}$$

Notice that the only unknowns in this minimization are $\underline{\boldsymbol{X}}_i$ and $\boldsymbol{\mathcal{X}}_i$, i.e. the input data sequence. This minimization is nothing but a *nonlinear least-squares* problem in the data. In the worst case scenario, we can obtain the ML estimate through an exhaustive search.

5. METHODS TO REDUCE COMPUTATIONAL COMPLEXITY

The search for the optimal $\boldsymbol{\mathcal{X}}_i$ in (26) is computationally very complex. In the following, we describe two approaches to reduce this complexity:

5.1. Linearization Approach

One way to reduce the computational complexity is to transform the nonlinear into a linear least squares problem. To do so, note first that the $\underline{\boldsymbol{X}}_i$ involved in equation (26) is composed of an upper and lower triangle formed by the CP of previous (known) and current (unknown) symbol respectively as shown in equation (13). Thus, we can write,

$$\underline{\boldsymbol{X}}_i = \underline{\boldsymbol{X}}_{\mathsf{U}i-1} + \underline{\boldsymbol{X}}_{\mathsf{L}i} \tag{27}$$

where $\underline{\boldsymbol{X}}_{\mathsf{U}i-1}$ is the upper triangle part of $\underline{\boldsymbol{X}}_i$ and $\underline{\boldsymbol{X}}_{\mathsf{L}i}$ is its lower triangular part. Thus equation (26) can be rewritten as

$$\begin{aligned} \hat{\boldsymbol{\mathcal{X}}}_i^{ML} &= \arg\min_{\mathcal{X}_i} \left\| \underline{\boldsymbol{y}}_i - \frac{1}{\mathcal{E}_X} \left(\underline{\boldsymbol{X}}_{\mathsf{U}i-1} + \underline{\boldsymbol{X}}_{\mathsf{L}i} \right) \boldsymbol{Q}_{L+1} \boldsymbol{D}_{\mathcal{Y}} \boldsymbol{\mathcal{X}}_i^* \right\|^2 \\ &= \arg\min_{\mathcal{X}_i} \left\| \underline{\boldsymbol{y}}_i - \left(\underline{\boldsymbol{X}}_{\mathsf{U}i-1} + \underline{\boldsymbol{X}}_{\mathsf{L}i} \right) \boldsymbol{A} \boldsymbol{\mathcal{X}}_i^* \right\|^2 \\ &= \arg\min_{\mathcal{X}_i} \left\| \underline{\boldsymbol{y}}_i - \boldsymbol{B} \boldsymbol{\mathcal{X}}_i^* - \boldsymbol{C} \boldsymbol{\mathcal{X}}_i^* \right\|^2 \end{aligned}$$

where

$$\boldsymbol{A} = \frac{1}{\mathcal{E}_X}\boldsymbol{Q}_{L+1}\boldsymbol{D}_{\mathcal{Y}}, \qquad \boldsymbol{B} = \underline{\boldsymbol{X}}_{\mathsf{U}_{i-1}}\boldsymbol{A}$$

and hence are completely known and where

$$\boldsymbol{C} \;=\; \underline{\boldsymbol{X}}_{\mathsf{L}_i}\boldsymbol{A} \tag{28}$$

Thus, the elements of $\boldsymbol{C}$ are linear in the input $\boldsymbol{\mathcal{X}}_i$ making $\boldsymbol{C}\boldsymbol{\mathcal{X}}_i^*$ quadratic in $\boldsymbol{\mathcal{X}}_i$. In fact, each element of $\boldsymbol{c} = \boldsymbol{C}\boldsymbol{\mathcal{X}}_i^*$ can be written as

$$c(j) = \|\boldsymbol{\mathcal{X}}_i\|^2_{\boldsymbol{W}_j} \tag{29}$$

for some weighted matrix $\boldsymbol{W}_j$ that is independent from input $\boldsymbol{\mathcal{X}}_i$. Thus, the nonlinear minimizing problem can be written as

$$\hat{\boldsymbol{\mathcal{X}}}_i^{ML} \;=\; \arg\min_{\boldsymbol{\mathcal{X}}_i}\left\|\underline{\boldsymbol{y}}_i - \boldsymbol{B}\boldsymbol{\mathcal{X}}_i^* - \boldsymbol{c}\right\|^2 \tag{30}$$

The linear approximation is obtained by replacing the matrix $\boldsymbol{W}_j$ by its diagonal, i.e.

$$\begin{aligned} c\,(j) &= \|\boldsymbol{\mathcal{X}}_i\|^2_{\boldsymbol{W}_j} \\ &\simeq \|\boldsymbol{\mathcal{X}}_i\|^2_{\mathsf{diag}(\boldsymbol{W}_j)} \\ &= \mathcal{E}_X \;\; tr\,(\boldsymbol{W}_j) \\ &= z\,(j) \end{aligned}$$

where the third line follows from the fact that the elements of $\boldsymbol{\mathcal{X}}_i$ have constant modulus. The input dependent vector $\boldsymbol{c}$ is thus replaced by the constant vector $\boldsymbol{z}$, and the objective function becomes linear in $\boldsymbol{\mathcal{X}}_i$

$$\arg\min_{\boldsymbol{\mathcal{X}}_i}\left\|\left(\underline{\boldsymbol{y}}_i - \boldsymbol{z}\right) - \boldsymbol{B}\boldsymbol{\mathcal{X}}_i^*\right\|^2 \tag{31}$$

One way to solve equation (31) is by using least squares

$$\hat{\boldsymbol{\mathcal{X}}}_i^* = (\boldsymbol{B}^*\boldsymbol{B} + \delta\boldsymbol{I})^{-1}\boldsymbol{B}^*\left(\underline{\boldsymbol{y}}_i - \boldsymbol{z}\right) \tag{32}$$

where δ is a small constant.

This estimated data is fed again into equation (30) and now the complete $\boldsymbol{W}_j$ matrices are used to obtain $\boldsymbol{z}$ (as opposed to approximating them by their diagonal). Least squares (32) is used again to estimate $\hat{\boldsymbol{\mathcal{X}}}_i^*$. This procedure is repeated for a desired number of iterations.

5.2. Using Search Algorithms

We can use the search algorithms like Particle Swarm Optimization (PSO) and the Genetic Algorithm (GA) to directly solve the nonlinear problem (equation (26)). PSO and GA are widely used algorithms to solve nonlinear problems. PSO and GA are motivated by the evolution of nature. Depending on the number of variables in the problem, a population of individuals is generated. The rule of survival of the fittest is used to manipulate the population by cooperation and competition within the individuals in case of PSO, and by using genetic operators like mutation, crossover and reproduction in case of GA. The best solution is selected from the generations.

The data estimated by using the linearization approach can be used to initialize PSO or GA. This initialization, with close to optimal solution, will help to kick start them for better results.

6. SIMULATIONS AND RESULTS

We consider an OFDM system with $N = 16$ and cyclic prefix of length $L = 4$. The OFDM symbol consists of BPSK or 4-QAM symbols. The channel IR consists of 5 iid Rayleigh fading taps. We compare the BER performance of three methods: (i) Perfectly known channel, (ii) Channel estimated using $L+1$ pilots and (iii) Blind based estimation using exhaustive search.

In Figure 1, we compare the three mentioned approaches of signal estimation for BPSK modulated data over a Rayleigh fading channel. As expected, the best performance is achieved by the perfectly known channel, followed by that obtained by training based estimated channel.

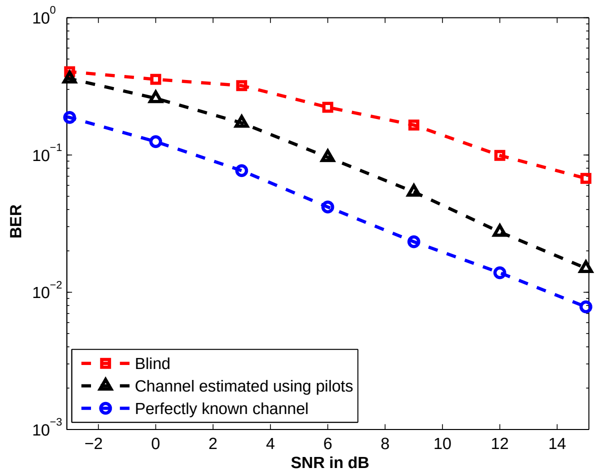

Fig. 1. BER vs SNR for BPSK-OFDM over a Rayleigh channel

The same conclusion can be made for the 4-QAM input (see Figure 2). Note however that in the high SNR region, the BER curve of blind based estimation exhibits steeper slope (higher diversity) which can be explained from the fact that the two channels (linear and cyclic) are used to detect the data in the blind case when only the linear channel is used in the known data case. The presence of occasional nulls in the channel also make the blind channel case better. This happens also in BPSK modulated data (Figure 1) when it is simulated for high SNR.

In Figure 3, the three approaches are compared for BPSK modulated data when the channel IR has persistent zeros on the FFT grid. We note that at high SNR, the BER for perfectly known channel and that of the estimated channel reach an error floor. Our blind method does not suffer from this problem and thus blind case outperforms the other two cases.

The low complexity algorithms proposed in Section 5 (linearization approach, PSO and GA) have been compared in Figure 4 for BPSK modulated data. We can observe that PSO and GA, initialized with the data detected by linearization approach, perform quite close to the blind exhaustive search especially at low SNR.

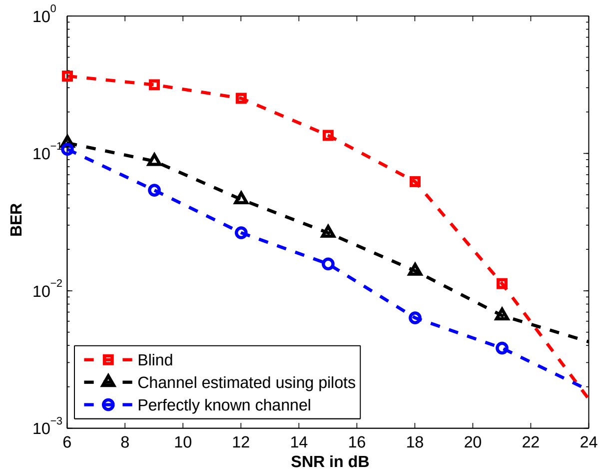

Fig. 2. BER vs SNR for 4QAM-OFDM over a Rayleigh channel

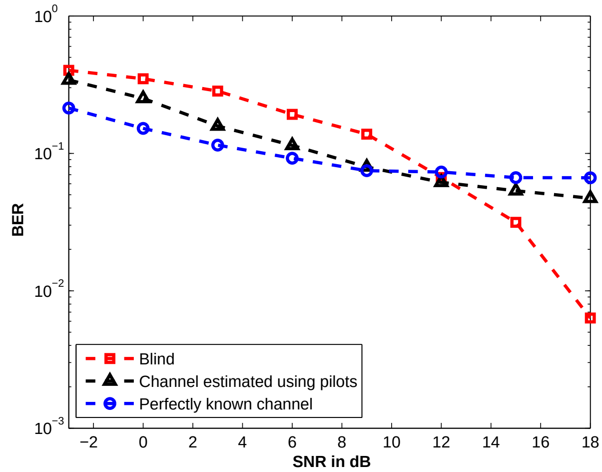

Fig. 3. BER vs SNR for BPSK-OFDM over channel with zeros on FFT grid

7. CONCLUSION

In this paper, we demonstrated how to perform blind ML data recovery in OFDM transmission. This is done using a single output OFDM symbol and associated CP. In particular, it was shown that the ML data estimate is the solution of an integer nonlinear-least squares problem. This proves that the data recovery is possible from output data only, irrespective of the channel zero locations and irrespective of the quality of the channel estimates or of its exact order. Iterative methods have been proposed to reduce the exponential complexity entailed in the algorithm developed in the paper.

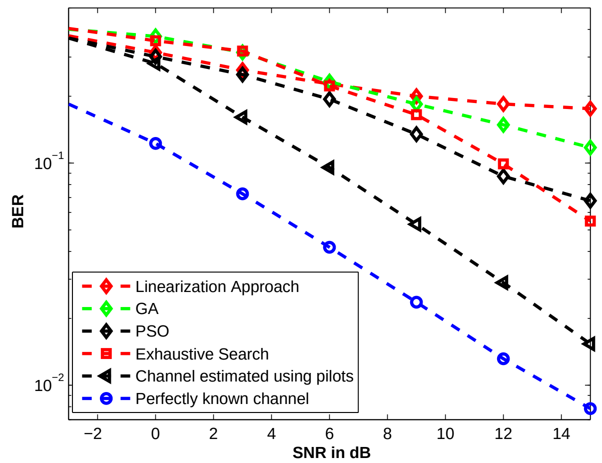

Fig. 4. Comparison of low complexity algorithms for BPSK-OFDM over a Rayleigh channel

8. REFERENCES

[1] Z. Wang and G. B. Giannakis, “Wireless Multicarrier Communications: Where Fourier Meets Shannon,” IEEE Signal Processing Mag., vol. 17, May 2000.

[2] T. Y. Al-Naffouri, “An EM-based forward-backward Kalman for the estimation of time-variant channels in OFDM,” IEEE Trans. Signal Process., vol. 55, Jul. 2007.

[3] M. Chang and Yu T. Su, “Blind and semiblind detections of OFDM signals in fading channels,” IEEE Transactions on Communications, vol. 52, Issue 5, May 2004.

[4] R. W. Heath and G. B. Giannakis, “Exploiting input cyclostationarity for blind channel identification in OFDM systems,” IEEE Trans. Signal Process., vol. 47, Mar. 1999.

[5] B. Muquet, M. de Courville, P. Duhamel, and V. Buzenae, “A subspace based blind and semi-blind channel identification method for OFDM systems,” Proc. IEEE Workshop for Signal Process., May 1999.

[6] J. Kennedy and R. Eberhart, “Particle swarm optimization,” IEEE International Conference on Neural Networks, vol. 4, Nov. 1995.

[7] Y. Shi and R. Eberhart, “A modified particle swarm optimizer,” IEEE International Conference on Evolutionary Computation Proceedings, May 1998.

[8] K. C. Sharman and G. D. McClurkin, “Genetic algorithms for maximum likelihood parameter estimation,” IEEE International Conference Acoustics, Speech, and Signal Processing, vol. 4, May 1989.

[9] E. Chen, R. Tao, and X. Zhao, “Channel Equalization for OFDM System Based on the BP Neural Network,” The 8th International Conference on Signal Processing, 2006.

[10] X. Wang and R. Liu, “Adaptive channel estimation using cyclic prefix in multicarrier modulation system,” IEEE Comm. Lett., vol. 3, no. 10, Oct. 1999.

[11] Ali H. Sayed, “Fundamentals of Adaptive Filtering,” John Wiley and Sons, Inc., 2003.