Compressive Sensing Based Opportunistic Protocol for Exploiting Multiuser Diversity in Wireless Networks

Syed T. Qaseem[1], Tareq Y. Al-Naffouri[2], and Tamim M. Al-Murad[3]
[1]LCC, Riyadh, Saudi Arabia
[2]King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia
[3]King Abdullah University of Science & Technology, Jeddah, Saudi Arabia

***Abstract*—A key feature in the design of any MAC protocol is the throughput it can provide. In wireless networks, the channel of a user is not fixed but varies randomly. Thus, in order to maximize the throughput of the MAC protocol at any given time, only users with large channel gains should be allowed to transmit. In this paper, a compressive sensing based compressive sensing based opportunistic protocol for exploiting multiuser diversity in wireless networks is proposed. This protocol is based on the traditional protocol of R-ALOHA which allows users to compete for channel access before reserving the channel to the best user. We use compressive sensing to find the best user, and show that the proposed protocol requires less time for reservation and so it outperforms other schemes proposed in the literature. Also, as the proposed scheme requires less reservation time, it can be seen as an enhancement for R-ALOHA schemes in fast fading environment.**

***Index Terms*—Compressed sensing, opportunistic communications, protocols, random access, reservation ALOHA, scheduling, wireless networks.**

I. INTRODUCTION

The topic of multiple access schemes has drawn the attention of researchers and developers with every new technology in communications [1]. For wireless networks, random access protocols (ALOHA, slotted ALOHA..etc.) are very popular as they use shared medium for transmission and requires no coordination between participants with collisions as the main cause of low throughput. In order to reduce data packets collision, reservation ALOHA (R-ALOHA) was introduced where the time is divided into frames which are further divided into slots. Within each frame, the first few slots are used for reserving the frame for the best user for transmitting data [2]. In wireless networks, the duration of the frame is less than or equal to the coherence time of the channel. Also, several signal processing techniques have been suggested to resolve collisions by taking advantage of the fading nature of the wireless channel [3]. Recently, the topic of opportunistic scheduling has been widely investigated for both uplink and downlink communications in order to exploit multi-user diversity to improve random access protocols. In this situation the wireless channel is accessed by users who have the best channels [1], [4]-[8] and hence can support a larger transmission rate.

The process of exploiting the "multiuser diversity" i.e., selecting the user with the best channel gain, can be centralized where transmission decision requires knowledge of each user's channel gain as in [4] or distributed where transmission decisions are individually made by each user based on their local channel information as in [1], [7]-[8]. In the centralized approach, the time required to measure all the users channels grows linearly with the number of users [8]. Therefore, for systems with large number of users, this may not be possible as the time required to measure the channels may exceed the coherence time of the channel. On the other hand, in reciprocal networks (time-division duplex systems) distributed approach can be used where the base station broadcasts a pilot signal to all users, and each user measures its own channel using this pilot signal. In a time-division duplex (TDD) system, both uplink and downlink use the same carrier frequency, and therefore, downlink channel estimates can be used for the uplink too.

Combining reservation and opportunistic communication, Qin & Berry proposed a distributed algorithm for wireless networks where *strong* users (users whose channel gains are above a certain threshold) send reservation packet containing user ID and channel quality information (CQI)[1] and collisions in the reservation phase were resolved via splitting [7]-[8]. A splitting algorithm uses some tree-like mechanism where users involved in a collision are divided into several subsets and only the user or users in one of the subsets transmits at the next time slot so as to reduce the probability of collision [2]. Thus, splitting algorithm resolves a collision which eventually results in finding the user with the best channel gain out of all backlogged users. As the frame duration is limited by the coherence time of the channel, therefore reducing the reservation time will increase the time available for the best user to transmit data, thereby increasing the throughput. Qin and Berry showed that there scheme requires only 2.5 slots[2] (*on an average*) for reserving the frame for the best user and so the throughput is improved as the channel coherence time increases [7]-[8]. To the best of our knowledge, this is the best result in R-ALOHA literature.

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[1] We use the terms "CQI" and "channel gain" interchangeably throughout the rest of the paper.

[2] In [7]-[8], frames and slots are refereed as slots and mini-slots respectively.

Despite the low number of slots required to reserve the best user, two major drawbacks associated with this scheme are i) the length of a slot must be greater than the round-trip time (RTT), and ii) channel coherence time (CCT) must be large in order to achieve near-optimal performance [7]-[8]. It is important to note that there is little that one can do regarding reducing RTT or increasing CCT as these are the fundamental limitations of any practical wireless network. This is because CCT depends on the physical environment and RTT is limited by the cell size.

In this paper, we consider a TDD system and distributed approach based random access for the uplink. Here also within each frame, we reserve the first few slots for sending reservation packets. However, in our case the number of reservation slots is fixed and is decided beforehand. All users whose CQI is above a particular threshold contend for reservation (send reservation packet) and remains silent otherwise. To this end, strong users multiply "1" with a random binary chip sequence (consisting of ±1 each with probability 0.5)[3] of length equal to the number of reservation slots. This creates an undetermined independent system of equations in a sparse vector of users. We use the emerging compressive sensing (CS) technique to identify users who have fed back and to estimate the fedback CQI.

From the reservation standpoint, our scheme differs from [7]-[8] in the following:

1) Slots duration (T_s) are not limited by the RTT, in fact each slot is only one bit long.
2) No user ID is fedback which grows logarithmically in the number of users.
3) More reservation slots may be required.

The remainder of the paper is organized as follows. In Section II, the system model is introduced. In Section III we discuss the proposed scheme. In Section IV, we present the throughput obtained by the proposed scheme. In Section V, performance evaluation of the proposed scheme is presented followed by numerical results and conclusions in Sections VI and VII respectively.

II. SYSTEM MODEL

We consider a multi-access model where all n users are backlogged and always have data to send. The total time is divided into frames with each frame duration (T_c) equal to CCT. We assume that at the start of each frame, each transmitter has knowledge of its own channel gain, but not the gain of any of the other transmitters. In reciprocal networks (as in TDD) this knowledge could be gained by having the receiver broadcast a pilot signal at the start of each frame. The channel gains of the users are assumed to be i.i.d. gaussian random variables (with zero mean and unit variance). Within each uplink frame, first few slots (T_r) are used for selecting the best user while the rest of the frame (T_d) is used by the selected user for sending data. Pictorial representation of the frame structure is shown in Fig. 1. Thus, $T_c = T_r + T_d$. The details of Fig. 1 will be discussed later in the paper.

We present here a compressive sensing model for reserving the frame for the best user i.e, the user with the best channel gain. The number of slots r used for reservation are fixed and are shared among all n users, in which strong users (users with CQI above a certain threshold) report CQI to the base station (BS) in order to exploit multiuser diversity. Note that each strong user multiples "1" with a random binary chip sequence of length equal to the number of reservation slots r before sending it over the multiple access shared channels, e.g. if user i is strong, it will multiply $v_i = 1$ with a random binary sequence $\mathbf{a}_i$ of length r. A weak user remains silent or effectively sends a "0" multiplied by a random binary sequence of length r. In a nutshell, the model can be described as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots & \mathbf{a}_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$$

or

$$\mathbf{y} = \mathbf{A}\mathbf{v} \tag{1}$$

where $\mathbf{A}$ is a $r \times n$ Bernoulli matrix [4] with $r \ll n$ and where $\mathbf{v}$ is a sparse vector having "1" as non zero entries with $\|\mathbf{v}\|_0 = s$, where $\|\cdot\|_0$ is the combinatorial norm ℓ_0.

The BS receives the users' requests and finds the strong users via compressive sensing. Ideally a threshold should be set such that there is only one strong user who sends reservation packet. However because of the random nature of the channel there may be multiple users who are strong. In this case, the BS randomly selects one of the strong users. To reduce the number of users s (obvious choice of s is 1) who send reservation packets, we pursue a thresholding strategy where the user will send reservation packets if his CQI is greater than a threshold ζ to be determined. Noting that the users' CQI are i.i.d., we can choose ζ to produce a sparsity level s. This happens provided that

$$\bar{F}(\zeta) = \arg \max_{u \in (0,1)} \binom{n}{s} u^s (1-u)^{n-s} \tag{2}$$

where $\bar{F}(\zeta)$ is the complementary cumulative distribution function (CCDF) of the channel gain defined as

$$\bar{F}(\zeta) = \mathbb{P}[\text{channel gain} > \zeta] = \exp(-\zeta), \zeta \geq 0$$

Lemma 1: The threshold that maximizes (2) is given by $\zeta = \bar{F}^{-1}\left(\frac{s}{n}\right)$

Proof: Let $\psi = \binom{n}{s} u^s (1-u)^{n-s}$. Differentiating ψ w.r.t u and setting the derivative to 0, and solving for u yields $u = s/n$, which is confirmed to be a maxima by the second derivative test. Thus, $\bar{F}(\zeta) = s/n$, or $\zeta = \bar{F}^{-1}(s/n)$.

III. THE PROPOSED PROTOCOL

Before we discuss the proposed protocol, we present important compressive sensing results used in our work.

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[3] There are two ways of assigning chip sequences to the users: preprogrammed in users' device or by sending it over the air.

[4] CS can be applied as Bernoulli matrices are shown to satisfy the RIP [11].

A. Compressive Sensing Results

Compressive sensing refers to the recovery of sparse signal $\mathbf{v} \in \mathbb{R}^n$ where $\|\mathbf{v}\|_0 = s$ and $s << n$ accurately from compressed (limited) measurements. The sparsest solution to underdetermined systems of linear equations is given by

$$\min_{\mathbf{v}\in\mathbb{R}^n} \|\mathbf{v}\|_0 \qquad \text{s. t.} \qquad \mathbf{y} = \mathbf{A}\mathbf{v} \tag{3}$$

The solution to this problem is in general NP hard [9]. This computational intractability has recently led researchers to develop alternatives to (3).

Recently, ℓ_1-minimization (Basis Pursuit) has been proposed as a convex alternative to the ℓ_0 [9] (see also [10])

$$\min_{\mathbf{v}\in\mathbb{R}^n} \|\mathbf{v}\|_1 \qquad \text{s. t.} \qquad \mathbf{y} = \mathbf{A}\mathbf{v}. \tag{4}$$

A recent paper by Candes, Romberg & Tao [11] shows that when $\mathbf{A}$ is a random matrix with i.i.d. entries from a suitable distribution, all sparse signals $\mathbf{v}$ with sparsity level s can be recovered using (4) with very high probability provided the number of measurements (or channels) satisfy $r_1 = c_1 s \log(n/s)$, where c_1 is a constant.

Similar results are obtained using matching pursuit algorithms e.g., maximum correlation [12] and CoSaMP [13], a much simpler method compared to ℓ_1-minimization. The number of measurements (or channels) required for recovering sparse signal satisfy $r_2 = c_2 s \log(n/s)$, where c_2 is a constant.

B. Compressive Sensing based Reservation Protocol

Users send reservation packet only when their CQI is above a particular threshold, and remain silent otherwise. Thus the vector $\mathbf{v}$ in (1) is sparse with sparsity level determined by the number of users who send reservation packet. To increase the reservation granularity, we let the users compare their CQI to a set of thresholds, not just one. Thus, suppose that we want to set k thresholds $\zeta_1 < \zeta_2 < ......, < \zeta_k$ such that the number of users whose CQI lie between the two consecutive thresholds $Q_i = [\zeta_i, \zeta_{i+1})$ is equal to s. Note that the last interval is $[\zeta_k, \infty)$ as $\zeta_{k+1} = \infty$. Using Lemma 1, we can set the lowermost threshold as

$$\bar{F}(\zeta_1)n = sk, \qquad \text{or,} \qquad \zeta_1 = \bar{F}^{-1}\left(\frac{sk}{n}\right)$$

Continuing in the same way, we get

$$\zeta_2 = \bar{F}^{-1}\left(\frac{s(k-1)}{n}\right), \cdots\cdots, \zeta_k = \bar{F}^{-1}\left(\frac{s}{n}\right).$$

The reservation procedure is thus as follows:

1) **Threshold Determination:** The BS decides on thresholding levels $\zeta_1, \zeta_2, ..., \zeta_k$ based on the sparsity level that can be recovered.
2) **Reservation:** Repeat the following steps for each threshold interval $[\zeta_i, \zeta_{i+1})$, $i = 1, \cdots, k$:
   - CQI Determination: Each user determines his CQI.
   - Reservation: All users whose CQI lies in threshold interval $[\zeta_i, \zeta_{i+1})$, send reservation packet according to input/output equation (1). Otherwise, the user remains silent.
   - Compressive Sensing: BS finds the strong users using Compressive Sensing.
3) **User Selection:** BS randomly selects one of strong users of the highest active threshold interval, where active threshold interval here means that there is at least one user sending reservation packet in the interval. Here, CQI is the lower limit of the highest active threshold interval. Thus, it is evident that with more number of threshold levels, higher accuracy of CQI will be achieved.

IV. THROUGHPUT

From Fig. 1, we see that the portion of the frame used for sending data is $\left(1 - \frac{T_r}{T_c}\right)$. Thus, the throughput achieved by the proposed scheme is given by

$$\mathcal{C} = \left(1 - \frac{T_r}{T_c}\right)\mathcal{R}$$

where $\mathcal{R}$ is the maximum possible rate at which data can be transmitted and is given by

$$\mathcal{R} \approx \mathbb{E}\left[\log_2(1 + \max_{1\leq i\leq k} \zeta_i)\right]$$

and where $\max_{1\leq i\leq k} \zeta_i$ is the lower limit of the CQI of the highest active threshold interval.

Alternatively, the same throughput $\mathcal{R}$ can be derived analytically as follows [14]:

$$\mathcal{R} = \sum_{i=1}^{k} \log_2(1+\zeta_i)\mathbb{P}(\text{selected user in the threshold interval}) \times \mathbb{P}(\text{threshold interval})$$

The probability of a user falling in the threshold interval Q_i is given by $\mathbb{P}(Q_i) = [F(\zeta_{i+1}) - F(\zeta_i)]$, where $F(\zeta)$ is the cumulative distribution function (CDF) of CQI (channel gain) defined as: $F(\zeta) = \mathbb{P}[\text{Channel Gain} \leq \zeta] = 1 - \exp(-\zeta), \zeta \geq 0$. The probability that selected user is in the threshold interval Q_i is given as follows:

$$\mathbb{P}(\text{selected user is in } Q_i) = \sum_{j=0}^{n-1} \frac{1}{j+1}\binom{n-1}{j}\mathcal{P}_1\mathcal{P}_2$$

where

$$\begin{aligned}\mathcal{P}_1 &= \mathbb{P}(j \text{ users other than the selected user are in } Q_i)\\ &= [F(\zeta_{i+1}) - F(\zeta_i)]^j, \text{and}\\ \mathcal{P}_2 &= \mathbb{P}((n-j-1) \text{ users lies below the interval } Q_i)\\ &= [F(\zeta_i)]^{(n-j-1)}\end{aligned}$$

Substituting these values of $\mathcal{P}_1$ and $\mathcal{P}_2$, and after some manipulations, one can show that

$$\mathbb{P}(\text{selected user is in } Q_i) = \frac{[F(\zeta_{i+1})]^n - [F(\zeta_i)]^n}{[F(\zeta_{i+1}) - F(\zeta_i)]}$$

Thus,

$$\mathcal{R} = \sum_{i=1}^{k} \log_2(1+\zeta_i)([F(\zeta_{i+1})]^n - [F(\zeta_i)]^n).$$

V. PERFORMANCE EVALUATION: RESERVATION TIME (T_r)

Here, we compare the reservation time required in our scheme with that of Qin & Berry [7]-[8].

The scheme proposed in [7]-[8] requires each strong user to send a reservation packet (containing his ID and CQI) in each slot to the BS and then to wait for the base station to tell whether the slot was idle, contained a successful transmission or contained a collision. In case of unsuccessful transmission, splitting is done and this process is continued until the best user is found or there are no more slots in the frame. Note that $\log_2(n)$ bits are required for unique representation of users as there are n users in the system. For comparison purposes, we quantize the channel gain information to q bits. Thus, the time required for reserving the frame for the best user for the scheme proposed in [7]-[8] is (see Fig. 2)

$$T_r = \eta(\text{RTT} + (q + \log_2(n))T_b) \tag{5}$$

where η is the number of slots required to find the best user (average number of slots is 2.5), and T_b is the time required to transmit one bit.

However, in our scheme, strong users send their reservation packets to the BS where it uses compressive sensing to find the best user and then informs the selected user of its decision. So the base station communicates only once with the user during the reservation time. Thus, the time required for reserving the frame for the best user for the proposed scheme is (see Fig. 1)

$$T_r = krT_b + \text{RTT}. \tag{6}$$

where $r = cs\log(n/s)$ is the number of slots per threshold, & k is the number of thresholds. Thus, our scheme is relatively more efficient when

$$\eta(\text{RTT} + (q + \log_2(n))T_b) > krT_b + \text{RTT} \tag{7}$$

$$\Rightarrow T_b < \frac{(\eta - 1)\text{RTT}}{kr - \eta q - \eta\log_2(n)} \tag{8}$$

Note that (8) applies to the case when $kr > (\eta q + \eta\log_2(n))$. For the case when $kr \leq (\eta q + \eta\log_2(n))$, our scheme is always better regardless of the value of T_b. Also, note that reduction in the reservation time allows more time for data transmission, thereby improving the throughput.

VI. NUMERICAL RESULTS

In this section, we present numerical results for the proposed CS-based reservation scheme. For calculating the throughput, we set the threshold according to the sparsity level s (which we set to 1 for each threshold interval), and use the maximum correlation technique for compressive sensing as this is computationally much more efficient than ℓ_1-minimization. Also, we use the following data for simulation:

1) $n = 100$ users
2) $T_c = 30 \times 10^{-6} sec$
3) data rate supported by MAC device (or link speed) is 1 Gbps, 100 Mbps and 10 Mbps
4) distance between the BS and the user is 500 m
5) $q = 4, 8$ or 16 bits

Thus, the propagation delay between the user and BS (assuming speed of signal is $3 \times 10^8 m/sec$) is $500/(3 \times 10^8)$ = $1.666 \times 10^{-6} sec$, which implies RTT = $3.3333 \times 10^{-6} sec$. Also, note that $T_b = 10^{-9} sec$, $T_b = 10^{-8} sec$, or $T_b = 10^{-7} sec$ for 1 Gbps, 100 Mbps or 10 Mbps respectively (reciprocal of the supported data rate by MAC devices).

Based on the above data, in Fig. 3 - Fig. 5, we present the throughput versus number of slots (or bits) used for reservation per threshold, for different number of thresholds. Also, we plot the throughputs achieved by Qin & Berry's scheme for $q = 4, 8$ and 16 bits and the maximum capacity that can be achieved (corresponding to zero reservation time). These figures differ only in the data rate supported by MAC device.

As we see from (8), our scheme is more efficient for MAC devices that support higher data rate as this reduces T_b thereby reducing the total reservation time and eventually resulting in larger throughput. This very fact can be observed from Fig. 3 - Fig. 5 where all the parameters except T_b kept unchanged. When T_b is relatively much smaller that RTT it is good to have large number of thresholds as reservation time is primarily dominated by RTT (see Fig. 3). From the figures, we can also note that with a link speed of 100 Mbps and above, the dominant part in the reservation phase is the RTT.

However, it is not always a good idea to increase the number of thresholds (beyond a point) and that too when T_b is large as is clearly evident from Fig. 5. This is because when T_b is relatively not much smaller that RTT, the reservation time corresponding to kr bits is either at par with RTT or dominates the total reservation time. Note that the same applies to the number of quantization bits in the Qin & berry's case.

Also, it is evident that the proposed scheme outperforms Qin & Berry's scheme in all three cases considered in this paper.

VII. CONCLUSIONS

In this paper, compressive sensing based opportunistic protocol for exploiting multi-user diversity is proposed. We have shown that the proposed protocol requires less time for reservation and so it enhances the performance of R-ALOHA, i.e. achieve better throughput than other R-ALOHA schemes proposed in the literature.

Also, as the proposed scheme requires less reservation time, it can be seen as an enhancement for R-ALOHA schemes in fast fading environment.

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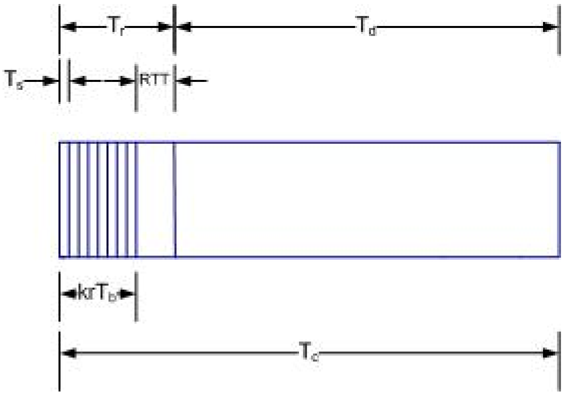

Figure 1. An uplink frame of the proposed scheme

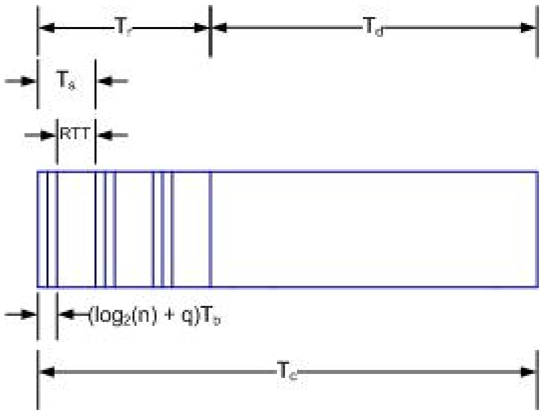

Figure 2. An uplink frame of Qin & Berry's scheme

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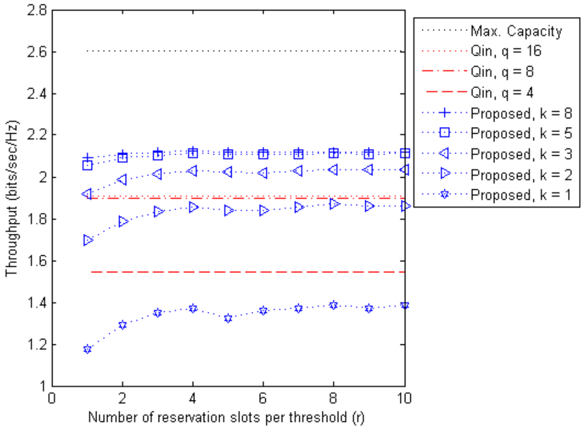

Figure 3. Throughput vs. r, $n = 100$, $T_b = 10^{-9} sec$, $q = 4, 8$ and 16 bits, and $T_c = 30 \times 10^{-6} sec$ for different values of k.

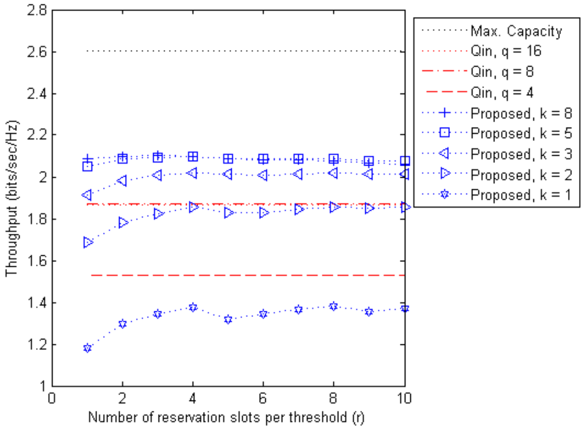

Figure 4. Throughput vs. r, $n = 100$, $T_b = 10^{-8} sec$, $q = 4, 8$ and 16 bits, and $T_c = 30 \times 10^{-6} sec$ for different values of k.

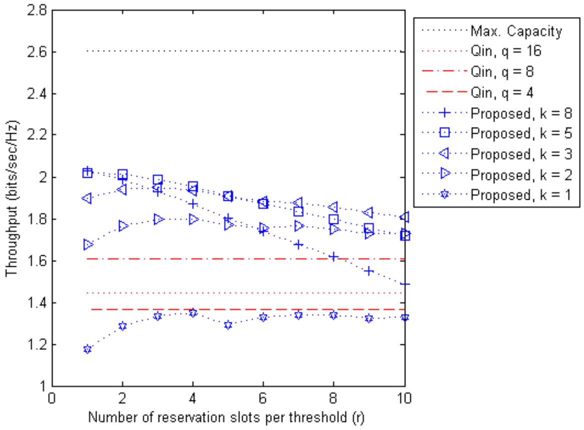

Figure 5. Throughput vs. r, $n = 100$, $T_b = 10^{-7} sec$, $q = 4, 8$ and 16 bits, and $T_c = 30 \times 10^{-6} sec$ for different values of k.