Compressive Sensing for Feedback Reduction in MIMO Broadcast Channels

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***Abstract*—We propose a generic feedback channel model, and compressive sensing based opportunistic feedback protocol for feedback resource (channels) reduction in MIMO Broadcast Channels under the assumption that both feedback and downlink channels are noisy and undergo block Rayleigh fading. The feedback resources are shared and are opportunistically accessed by users who are *strong* (users above a certain fixed threshold). Strong users send same feedback information on all shared channels. They are identified by the base station via compressive sensing. The proposed protocol is shown to achieve the same sum-rate throughput as that achieved by dedicated feedback schemes, but with feedback channels growing only logarithmically with number of users.**

I. Introduction

Recently, it has been shown that dirty paper coding (DPC) achieves the sum-rate capacity of the multiple-input multiple-output (MIMO) broadcast channel. However, it requires a great deal of feedback as the base station (BS) needs perfect channel state information for all users and is computationally expensive [1]. Since then, many works have attempted to achieve the same sum-rate capacity with imperfect channel state information (reduced feedback load) at the BS. This was done by applying opportunistic communication in the downlink [2]-[5]. Components that are generally fed back are i) Channel Direction Information (CDI), e.g., beam index, channel quantization index etc. ii) Channel Quality Information (CQI), e.g., SNR, SINR etc. iii) User identity. User selection is based on either one or a combination of these components [2]-[7].

While the downlink is opportunistic in nature, almost all feedback schemes are non-opportunistic. Here, each user has a dedicated feedback channel. Since interaction or cooperation among the competing users is not allowed, hence defying opportunism, there is a linear increase in the feedback resources (channels) with the number of users. Even if thresholding is applied, there is no reduction in the number of feedback channels. This is because the channels are reserved even when users are not sending any CQI feedback information.

Recently, some works [6]-[7] have started to consider opportunistic feedback schemes where feedback resources (time slots) are shared and are opportunistically accessed by strong users. While the scheme proposed in [7] works for MIMO systems, the scheme proposed in [6] is limited to the single-input single-output (SISO) case. Feedback to the BS is successful if only one user is attempting to feed back in a slot otherwise collision occurs. Although the schemes requires only an integer feedback per slot, both these schemes require accurate timing-synchronization to avoid collisions which is difficult to achieve in practice. Moreover, the two schemes only work for digital feedback but not when the designer is interested in analog feedback. In all the feedback schemes discussed above, the feedback links were assumed ideal when the downlink was subjected to both fading and noise. This asymmetry in the way the two links are treated is unrealistic.

The paper proposes generic feedback channel model, and compressive sensing (CS) [8]-[9] based opportunistic feedback protocol for feedback resource reduction. We consider a broadcast scenario where the downlink and the feedback links are symmetric in the sense that they are both i) non-ideal and ii) opportunistic or shared. Thus, both links (downlink and feedback links) undergo Rayleigh fading and are subjected to additive Gaussian noise. Moreover, the channels in both links are shared and are opportunistic in the sense that channels are dominated by strong users. Finally, the feedback links can be used for both analog and digital feedback. In this paper, we discuss the digital feedback only. The proposed protocol is shown to achieve the sum-rate capacity as that achieved by dedicated feedback schemes but with feedback channels growing only logarithmically with number of users. .

The remainder of the paper is organized as follows. In Section II, generalized feedback model is introduced. In Section III we discuss the proposed feedback strategy. In Section IV, performance evaluation of the proposed feedback scheme is presented followed by numerical results and conclusions in Sections VI and VII respectively.

Notation: We use bold upper and lower case letters for matrices and vectors, respectively. $\mathbf{A}^*$ refers to Hermitian conjugate of $\mathbf{A}$. $\Re(\mathbf{A})$ & $\Im(\mathbf{A})$ represents real and imaginary part of $\mathbf{A}$ respectively. $\mathbb{E}(\cdot)$ denotes the expectation operator, and $\mathbb{P}[\ ]$ is the probability of a given event. The natural logarithm is referred to as $\log(\cdot)$, while the base 2 logarithm is denoted as $\log_2(\cdot)$. $f(x) = O(g(x))$ is equivalent to $f(x) = cg(x)$ where c is a constant. $|A|$ denotes the size of a set A.

II. System Model

A. Downlink Transmission Model

We consider a single cell multi-antenna broadcast channel with p antennas at the BS (transmitter) and n users (receivers) each having one antenna. The channel is described by a propagation matrix which is constant during the coherence interval and is known completely at the receiver. Let $\mathbf{u} \in \mathbb{C}^{p\times 1}$ be the transmit symbol vector and let x_i be the received signal by the i-th user, the received signal by the i-th user can then be written as

$$x_i = \sqrt{\rho_i}\mathbf{h}_i\mathbf{u} + w_i, \qquad i = 1, \ldots, n \tag{1}$$

where $\mathbf{h}_i \in \mathbb{C}^{1\times p}$ is the channel gain vector between the transmitter and the user, and w_i is the additive noise. The entries of $\mathbf{h}_i$ and w_i are i.i.d. complex Gaussian with zero mean and unit variance, $\mathcal{CN}(0, 1)$. Moreover, $\mathbf{u}$ satisfies an average transmit power constraint $\mathbb{E}\{\mathbf{u}^*\mathbf{u}\} = 1$ and ρ_i is the SNR of the i-th user. A homogeneous network is considered, in which all users have the same SNR, i.e. $\rho_i = \rho$ for $i = 1, \ldots, n$. We also assume that the number of users is greater than or equal to the number of transmit antennas, i.e., $n \geq p$, and that the BS selects p out of n users to transmit to.

B. Generic Multi-antenna Feedback Channel

We present here a generic model for the multiuser feedback channel with r feedback channels (possibly shared) among n users, in which users report channel quality information (CQI) to the base station in order to exploit multiuser diversity. As we shall soon see, this model encompasses the existing feedback models. The feedback channels are described by a propagation matrix $\mathbf{A}$ which is constant during the coherence interval and is assumed to be perfectly known at the BS (receiver), and are to be independent of the downlink channel. Let $\mathbf{v} \in \mathbb{C}^{n\times 1}$ be transmit feedback vector and let y_i be the signal received via the i-th feedback channel. The signal received through the i-th feedback channel is mathematically described as

$$\begin{bmatrix} y_1 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{r1} & \cdots & a_{rn} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_r \end{bmatrix}$$

or

$$\mathbf{y} = [\mathbf{a}_1 \cdots \mathbf{a}_n]\mathbf{v} + \mathbf{w}$$

or equivalently

$$\mathbf{y} = \mathbf{A}\mathbf{v} + \mathbf{w} \tag{2}$$

where $r \ll n$. The entries of $\mathbf{A}$ are i.i.d. complex Gaussian with zero mean and unit variance, $a_{ij} \sim \mathcal{CN}(0, 1)$. Note that in contrast to the majority of existing feedback reduction techniques, a noisy feedback channel is assumed. The entries of $\mathbf{w}$ represent the additive noise and are assumed to be i.i.d. complex Gaussian with zero mean and variance σ^2, $w_i \sim \mathcal{CN}\left(0, \sigma^2\right)$.

This model subsumes the non-opportunistic feedback model [2]-[5] as a special case. Here, each user is allocated its own feedback channel, thus the feedback channel matrix $\mathbf{A}$ becomes diagonal and of size n (equal to the number of users). If no fading is considered ($\mathbf{A}$ is deterministic), all entries of $\mathbf{A}$ are equal to a constant. Specifically, for the opportunistic models proposed in [6]-[7], the feedback channel matrix $\mathbf{A}$ becomes diagonal of size $r \times r$, where r is the number of feedback slots and is usually less than n. $\mathbf{v}$ represents feedback data in each slot, and when a collision in a particular slot takes place, the corresponding entry of $\mathbf{v}$ is not valid. In all these schemes, the additive noise $\mathbf{w}$ is set to zero.

In our case, there are different ways to interpret the system of equations (cf. (2)). One possibility is to assume that each user is equipped with one antenna and the BS is equipped with r antennas. In this case a_{ij} represents the gain from the j-th user to the i-th antenna (spatial feedback channels). Another possibility is to assume that each single-antenna user is going to feedback the same information over r frequency bands shared with the other users. Thus, a_{ij} represents the gain of the j-th user in the i-th band (frequency feedback channels).

III. Proposed Feedback Strategy

Before we discuss the proposed feedback strategy, we present important compressive sensing results used in our work.

A. Compressive Sensing Results

Compressive sensing refers to the recovery of sparse signals $\mathbf{v}$ from a small number of compressive measurements $\mathbf{y} = \mathbf{A}\mathbf{v}$. Two approaches for recovering the sparsity pattern S (with $|S| = s$) of signal $\mathbf{v} \in \mathbb{R}^n$ ($S = \{i \in \{1, \ldots, n\} | v_i \neq 0\}$) in the noisy setting (cf. (2)) are discussed here. These results were derived for the case when the entries of $\mathbf{A}$ and $\mathbf{w}$ are i.i.d. real Gaussians i.e., $a_{ij} \sim \mathcal{N}(0, 1)$ and $w_i \sim \mathcal{N}\left(0, \sigma^2\right)$.

1) Sparsity Pattern Recovery Results: A recent paper by Fletcher et. al. [8] shows that it is surprisingly possible to recover the sparsity pattern of signals accurately from limited measurements in a noisy setting using maximum correlation provided the number of measurements (or channels) satisfies $r \sim O(s \log(n - s))$. Similar results for sparsity pattern recovery from limited measurements in a noisy setting using LASSO are derived by Wainwright in [9].

2) Estimating or Refining Sparse Signal: Once the sparsity pattern S is known, least squares can be used to estimate or refine $\mathbf{v}_S$ (non-zero entries of $\mathbf{v}$) as follows [10]:

$$\mathbf{v}_S^{ls} \;\; = (\mathbf{A}_S^*\mathbf{A}_S)^{-1}\mathbf{A}_S^*\mathbf{y} \tag{3}$$

where $\mathbf{A}_S$ denotes the sub-matrix formed by the columns $\{\mathbf{a}_j | j \in S\}$, indexed by the sparsity pattern S.

B. General Strategy of Using Compressive Sensing for Feedback

Any feedback scheme has two components, a direction component and a magnitude component. The transmitter usually has certain pre-determined directions for which it seeks user feedback. Thus, the BS announces that it is seeking feedback for a particular direction. At this instant, the strong users, users whose channels lie at or are close to this direction, feedback their CQI. Now a limited number of users will feedback on the set of shared feedback channels according to input/output equation (2).

Thus the vector $\mathbf{v}$ in (2) is sparse with sparsity level s determined by the number of users who feedback. CS can now be used to recover the sparsity pattern of $\mathbf{v}$ [8]-[9] (i.e. which user prefer that particular direction) and could also recover the vector $\mathbf{v}$ itself [9] (i.e. users' feedback CQI). A factor that enhances the level of recovery is how sparse the vector $\mathbf{v}$ as compared to the number of feedback channels available. We need at least one strong user (i.e. $s \geq 1$) for each beam or direction in order to achieve full multiplexing gain which implies that small values of s are sufficient. To reduce s, we pursue a thresholding strategy where the user will feedback if his CQI is greater than a threshold ζ to be determined.

Now consider a particular beam (CDI) (all beams will behave in an identical manner as the users are i.i.d. and the beams are equi-powered). Noting that the users' CQI (SINR) are i.i.d., we can choose ζ to produce a sparsity level s. This happen by requiring that

$$F(\zeta) = \arg \max_{u \in (0,1)} \binom{n}{s} u^s (1-u)^{n-s} \tag{4}$$

where $F(\zeta) = \mathbb{P}[\text{SINR} > \zeta]$ = $\frac{\exp(-\zeta/\rho)}{(1+\zeta)^{p-1}}$, $\zeta \geq 0$ [2].

Lemma 1: Threshold ζ that maximizes (4) is given by $\zeta = F^{-1}\left(\frac{s}{n}\right)$

Proof: Let $\psi = \binom{n}{s} u^s (1-u)^{n-s}$. Differentiating ψ w.r.t. u and setting the derivative equal to 0, and solving for u yields $u = s/n$. Thus, $F(\zeta) = s/n$, or $\zeta = F^{-1}(s/n)$.

C. Feedback Protocol

In the digital feedback scenario, users above threshold feed back "1" and remains silent otherwise. To increase the feedback granularity, we let the users compare his CQI to a set of thresholds, not just one. Thus, suppose that we want to set k thresholds $\zeta_1 < \zeta_2 <$ ......, $< \zeta_k$ such that the number of users whose CQI lie between the two consecutive thresholds $[\zeta_i,\ \zeta_{i+1})$ is equal to s. Note that the last interval is $[\zeta_k,\ \infty)$. Following our discussion in subsection III-B, we can set the lowermost threshold as

$$F(\zeta_1)n = sk, \qquad \text{or,} \qquad \zeta_1 = F^{-1}\left(\frac{sk}{n}\right)$$

Continuing in the same way, we get

$$\zeta_2 = F^{-1}\left(\frac{s(k-1)}{n}\right), \cdots\cdots, \zeta_k = F^{-1}\left(\frac{s}{n}\right).$$

The feedback procedure is as follows:

1) **Threshold Determination:** BS decides on thresholding levels $\zeta_1, \zeta_2, ..., \zeta_k$ based on the sparsity level that can be recovered. For each threshold interval $[\zeta_i,\ \zeta_{i+1})$, repeat the *User Feedback* step.
2) **User Feedback:** Repeat the following steps for each beam.
   - CQI Determination: Each user determines his best beam (corresponding to the highest CQI value).
   - CQI Feedback: Each user feeds back his CQI if it lies in threshold interval $[\zeta_i,\ \zeta_{i+1})$ on all shared channels. Otherwise, the user remains silent.
   - Compressive Sensing: BS finds the strong users using Compressive Sensing.
   - Least-squares estimation/refining: BS estimates or refines results obtained via CS using least-squares.
3) **User Selection:** For each beam, BS randomly selects one of strong users of the highest active threshold interval, where active threshold interval here means that there is at least one user sending feedback data in the interval. Here, CQI is the lower limit of the highest active threshold interval.

D. Throughput in the RBF Case

The sum-rate throughput achieved by p beams in the multiple thresholds (k in number) based digital feedback case for RBF is given below.

$$\mathcal{R} \approx p\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq k} \zeta_i)\right]$$

where $\max_{1 \leq i \leq k} \zeta_i$ is the lower limit of the CQI of the highest active threshold interval.

Alternatively, the same throughput can be derived analytically as follows. The throughput achieved for any transmit beam m is given as follows:

$$\mathcal{R}_m = \sum_{i=1}^{k} \log_2(1+\zeta_i)\mathbb{P}(\text{selected user in the threshold interval}) \times \mathbb{P}(\text{threshold interval})$$

The probability of the threshold interval (denoted as Q_i) is given by $\mathbb{P}(Q_i) = [\bar{F}(\zeta_{i+1}) - \bar{F}(\zeta_i)]$, where $\bar{F}(\zeta)$ is the cumulative distribution function (CDF) of CQI (SINR) defined as: $\bar{F}(\zeta) = \mathbb{P}[\text{SINR} \leq \zeta] = 1 - \frac{\exp(-\zeta/\rho)}{(1+\zeta)^{p-1}}, \zeta \geq 0$ [2]. Capitalizing on the work of [11], we calculate the probability that selected user is in threshold interval Q_i as follows:

$$\mathbb{P}(\text{selected user is in } Q_i) = \sum_{j=0}^{n-1} \frac{1}{j+1} \binom{n-1}{j} \mathcal{P}_1 \mathcal{P}_2$$

where

$$\begin{aligned}\mathcal{P}_1 &= \mathbb{P}(j \text{ users other than the selected user are in } Q_i) \\ &= [\bar{F}(\zeta_{i+1}) - \bar{F}(\zeta_i)]^j, \text{and}\end{aligned}$$

$$\begin{aligned}\mathcal{P}_2 &= \mathbb{P}((n-j-1) \text{ users lies below the interval } Q_i) \\ &= [\bar{F}(\zeta_i)]^{(n-j-1)}\end{aligned}$$

Substituting these values of $\mathcal{P}_1$ and $\mathcal{P}_2$, and after some manipulations, one can show that

$$\mathbb{P}(\text{selected user is in } Q_i) = \frac{[\bar{F}(\zeta_{i+1})]^n - [\bar{F}(\zeta_i)]^n}{[\bar{F}(\zeta_{i+1}) - \bar{F}(\zeta_i)]}$$

Thus,

$$\mathcal{R}_m = \sum_{i=1}^{k} \log_2(1+\zeta_i)([\bar{F}(\zeta_{i+1})]^n - [\bar{F}(\zeta_i)]^n)$$

As, in our case there are p beams and all of them are identical, so the sum-rate throughput is given as

$$\mathcal{R} = \sum_{m=1}^{p} \mathcal{R}_m = p \sum_{i=1}^{k} \log_2(1+\zeta_i)([\bar{F}(\zeta_{i+1})]^n - [\bar{F}(\zeta_i)]^n)$$

IV. Performance Evaluation

We consider following metrics for the performance evaluation of the proposed feedback scheme.

A. Feedback Resources Reduction

There is a significant reduction in number of feedback channels required for carrying feedback information. The proposed scheme requires only $O(\log(n))$ feedback channels (shown in the Lemma given below) as opposed to n feedback channels required in the dedicated feedback case.

Lemma 2: The number of multiple access feedback channels required for our scheme is $\frac{c}{2}(s\log(n))$, where c is a constant.

Proof: Specifically, let's assume that there are r channels shared between users over which feedback can take place. We can represent these channels using the system of equations (2). As already mentioned, (2) is similar to ones considered in [8]-[9], except that in our case the measurement matrix $\mathbf{A}$, and the noise vector $\mathbf{w}$ are complex instead of real. So, we replace the complex-valued model in (2) by its real-valued equivalent which upon simplification can be written as

$$\begin{bmatrix} \Re(\mathbf{y}) \\ \Im(\mathbf{y}) \end{bmatrix} = \begin{bmatrix} \Re(\mathbf{A}) \\ \Im(\mathbf{A}) \end{bmatrix} \begin{bmatrix} \mathbf{v} \end{bmatrix} + \begin{bmatrix} \Re(\mathbf{w}) \\ \Im(\mathbf{w}) \end{bmatrix}.$$

or

$$\underline{\mathbf{y}} = \underline{\mathbf{A}}\mathbf{v} + \underline{\mathbf{w}} \tag{5}$$

The entries of $\underline{\mathbf{A}}$ are i.i.d. $\mathcal{N}(0, 1/2)$, and the entries of $\underline{\mathbf{w}}$ are i.i.d. $\mathcal{N}\left(0, \sigma^2/2\right)$. The above model (5) gives us $2r$ real measurements, so the sparsity pattern recovery techniques discussed in Section III-A can be applied. Also, from Section III-B, we know that small values of s are sufficient, therefore

$$2r = O(s\log(n-s) \approx O(s\log(n)) = cs\log(n) \tag{6}$$
$$\Rightarrow r = \frac{c}{2}(s\log(n)). \tag{7}$$

Lemma 3: In the RBF case when $n \to \infty$, the minimum number of multiple access feedback channels required is $p(\log\log\log(n))\log(n)$.

Proof: From Lemma 2, we have $r = \frac{c}{2}(s\log(n))$ and for $n \to \infty$, $c = 2$ [9]. For RBF systems with large number of users ($n \to \infty$), the minimum value of s (the number of users who should feedback) required to achieve the sum-rate capacity is given by $p\log\log\log(n)$ [12]. Substituting these value of c and s in $r = \frac{c}{2}s\log(n)$, the desired result is achieved.

B. Feedback Load Reduction

In addition to the feedback resources reduction, there is a reduction in the amount of feedback. In RBF scheme with dedicated feedback channel, n real values and n integer values, as there are n users in the system. However in our case, only pkr bits are to be fedback. This is because there are r shared channels and the scheme is repeated for each beam & threshold. Note that the feedback load reduction is more dominant in systems with large number of users, as $r \sim O(\log(n))$ and p & k are small.

C. Trade-off

Given a budget of bits that can be fedback, using intuition, it was shown in [13] that trade-off exists between the multi-user diversity and feedback accuracy. In our context, multi-user diversity is related to the number of shared channels r whereas feedback accuracy is related to the number of thresholds k, and so a similar trade-off may exist. The number of shared channels and thresholds must be chosen such that the throughput is maximized. This is explored using simulation in section VI.

V. Feedback Channel Training

In the previous sections, we assumed that the channel $\mathbf{A}$ estimation is given to the system with the aid of a "genie" at no cost. In this section, we present how the feedback channel training can be accomplished and explore ways to reduce it. Here, we assume that (2) represents frequency feedback channels i.e., the entries of $\mathbf{A}$, a_{ij} represents the gain of the j-th user in the i-th frequency band.

A. Channel Matrix is Full

The optimal number of symbols required for channel training is equal to the number of transmit antennas [14]. So we need p training symbols for the downlink channel and n training symbol for the uplink channel (as there are n users each having one transmit antenna). Training for each user in the uplink can be performed one by one, i.e., the first symbol of the coherence interval is reserved for user 1 to perform training for all shared channels, and second symbol reserved for user 2, and so on. Continuing in this way, we need n symbol time to accomplish training for all users. Also, it is important to note that as there is little data to be sent for feedback purposes, so much of the uplink coherence time can be used for feedback training. Coherence time is typically of the order of few thousand symbols, so training would not be an issue for systems with moderate number of users. However, a method for reducing in the amount of feedback channel training time is discussed in the next subsection.

B. Channel matrix is Block Diagonal

In order to reduce the feedback training time, we divide the users into groups with each group being allowed to feedback only on a set of feedback channels, thereby reducing the full channel matrix to a block diagonal one

$$\mathbf{A}_{BD} = \begin{bmatrix} \mathbf{A}_1 & & & \\ & \mathbf{A}_2 & & \\ & & \ddots & \\ & & & \mathbf{A}_l \end{bmatrix}.$$

Compressive sensing is applied in the same way as discussed in Section III, the only difference being that it is now applied on each block. Strong users in each block (or group) are found and a user among the strong users is randomly selected. As the users are i.i.d., so we divide the feedback resource equally among the l groups. Thus, training can now be performed for each block simultaneously. This approach reduces the feedback training time considerably, e.g. if we divide the total number of users into two groups, then the training will require $n/2$ symbol time as opposed to n symbol time required for the case when the channel matrix is full.

The flip side of this approach is that compressive sensing is now applied on the group of users instead of all users as one block. Thus, for same sparsity level s (overall), with block diagonalization, the number of feedback channels required is given below

$$\begin{aligned} f_{\mathbf{A}_{BD}} &= f_{\mathbf{A}_1} + \cdots + f_{\mathbf{A}_l} \\ &= k f_{\mathbf{A}_1} \\ &= k\left[\frac{1}{2}\left(c'\left(\frac{s}{l}\right)\log\left(\frac{n}{l}\right)\right)\right]. \end{aligned}$$

Note that from the above equation it may first appear that the number of channels have reduced as the quantity inside the logarithm is reduced by a factor of l, however, it is the other way round. This is because now c' has increased as the problem dimension (n) is reduced by a factor of l [9]. Thus, there is a trade-off between the reduction in the amount of feedback training and the number of feedback channels. Also, note that there is now an additional constraint requiring s/l to be an integer.

VI. Numerical Results

In this section, we present numerical results for CS-based feedback schemes by applying it in RBF context. We use $p = 4$ base station antennas, $n = 100$ users, and SNR = 10 dB for both downlink and feedback link for calculating the sum-rate throughput. We use the maximum correlation technique for compressive sensing as this is much more computationally efficient than LASSO. We chose $s = 1$ (the minimum possible value) and set multiple thresholds as discussed in section III-C. This is because for the proposed scheme, $s = 1$ will allow us to set the highest possible uppermost threshold thereby ensuring a higher throughput.

In Fig. 1, we present the sum-rate throughput achieved with shared channel feedback in the digital feedback case. It is evident form the figure that the proposed scheme in a noisy scenario achieves the throughput obtained in a noiseless dedicated feedback scenario (dedicated feedback with ideal

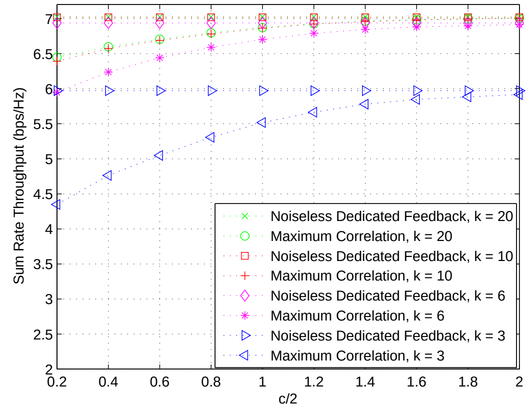

Figure 1. Digital Shared Channel Feedback: Throughput versus c/2 for RBF, $p = 4$, $n = 100$ and SNR = 10 dB (both downlink & feedback link) for different values of k.

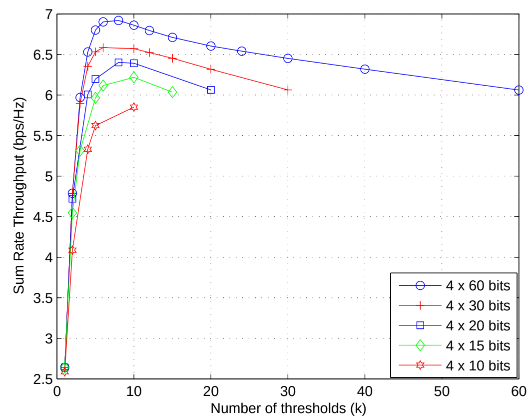

Figure 2. Digital Shared Channel Feedback: Throughput versus k for RBF, $p = 4$, $n = 100$ and SNR = 10 dB (both downlink & feedback link) for different budgets of bits that are to be fedback.

feedback links). Also, we see that the throughput increases with the increase in the number of shared channels & thresholds. Taking the pessimistic view, we need only 10 feedback channels (corresponds to $c/2 = 2$ and $s = 1$ according to (7)). However, it is important to note that beyond a certain number of shared channels or thresholds, the throughput either becomes stagnant or increases marginally.

In Fig. 2, we consider fixed budgets of $p \times kr$ bits that can be fedback. From the figure, we note that such a trade-off exists and for a given fixed budget there is an optimum number of thresholds and shared feedback channels that maximizes the throughput.

VII. Conclusions

In this paper, a generic feedback channel model and compressive sensing based opportunistic feedback scheme are proposed. We have shown that the proposed opportunistic feedback scheme achieves the same sum rate throughput as that achieved by dedicated feedback schemes, but with feedback channels growing only logarithmically with number of users. Also, it has been shown that in a noisy scenario, the proposed scheme comes close to achieving the throughput obtained in the case of noiseless dedicated feedback.

Although the results presented here only show the performance of the the proposed scheme for digital feedback in the RBF case, the scheme can easily work with analog feedback and other beamforming methods.

VIII. Acknowledgment

This work was supported by King Abdul Aziz City for Science and Technology (KACST), Project no. AR-29-79.

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