An Iterative Receiver for Coded OFDM Systems Over Time-Varying Wireless Channels

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Abstract— **This paper presents a low-complexity iterative receiver for coded OFDM systems. We present an EM-based iterative algorithm for combined channel estimation and decoding that makes collective use of the available data and system constraints. Minimum number of pilots are sent only in the first symbol of the packet to acquire the channel; then the iterative algorithm is used to track the channel time variation, which is assumed to follow a state-space model, using an EM-based Kalman filter. Data recovery can be achieved within a single OFDM symbol. We also propose the use of an optional outer LDPC code in serial concatenation to offer a trade-off between latency and performance, especially for multi-amplitude modulations, without affecting the complexity of the core iterative algorithm.**

Keywords— **Multicarrier transmission, Iterative receiver, Channel estimation and tracking.**

I. Introduction

OFDM is an effective multicarrier modulation technique for mitigating intersymbol interference (ISI) on frequency-selective wireless channels. Estimating the channel at the receiver enables coherent detection, which saves 3 dB compared to differential detection and allows the use of more efficient multi-amplitude signaling. Reference pilot symbols can be used to acquire the channel initially; and then data decisions can be used to track the channel over a number of subsequent symbols as in [1], where the channel variation was assumed to be very slow. OFDM systems usually use coding and interleaving across subchannels to exploit frequency diversity in frequency-selective channels. It is natural then to attempt to use this coding information to aid in estimating the channel as in [2], in which hard estimates of the decoded symbols were used. Iterative channel estimation and decoding algorithms have been suggested to cope with fast channel time variation [3], [4]. These algorithms, however, fail to make a collective use of the data and system constraints offered by the rich structure of the coded OFDM system.

This paper presents an EM-based iterative channel estimation and decoding algorithm that exploits the data and system constraints inherent in the coded OFDM system to improve the quality of the channel estimate and/or accelerate convergence. Data constraints include: coding, finite alphabet, cyclic-prefix, and pilots, if any. Channel constraints include: finite delay spread, frequency correlation, and time correlation.

To eliminate error flooring caused by occasional loss of tracking when multi-amplitude modulation is used at high Doppler frequency, we propose using a rate-1/2 outer Low-Density Parity-Check code [5] in a serial concatenation. This approach, which also exploits the time diversity and achieves a higher coding gain, offers a trade-off between latency and performance and does not increase the complexity of the receiver significantly.

Section II introduces the system model and notation used in this paper. The proposed iterative channel estimation and tracking algorithm is presented in Section III. The optional use of the outer LDPC code is illustrated in Section IV. Section V presents the simulation results, and concluding remarks are given in Section VI.

II. System Model

Fig. 1 shows the system model and the notation used in this paper. The total bandwidth of the system is divided into N subchannels, and a square M-QAM modulation is assumed to be used on each subchannel. The encoder in Fig. 1 is assumed to be a rate-1/2 4-state recursive systematic convolutional (RSC) encoder with the generator matrix $G(D) = [1 \ \frac{1+D^2}{1+D+D^2}]$, where D is a delay operator. A minimum-state code is used to minimize the complexity of the iterative algorithm. The interleaver is assumed to be a random interleaver.

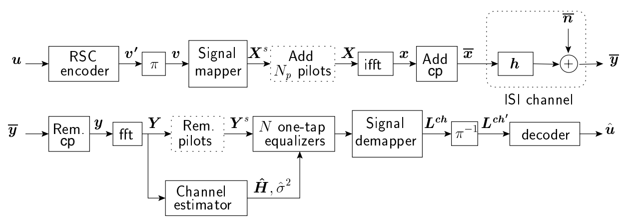

Fig. 1. Coded OFDM system model.

The output of the encoder can be written as

$$\left[\ u(D) \quad p(D)\ \right] = u(D) \cdot \left[\ 1 \quad \frac{1+D^2}{1+D+D^2}\ \right], \tag{1}$$

where $u(D)$ and $p(D)$ represent the sequences of systematic and parity bits, respectively. Each of these sequences has a length $K = r \cdot [b \cdot (N - N_p)]$, where $r = 1/2$ is the rate of the code, $b = \log_2 M$ is the number of bits per constellation point, and N_p is the number of pilots in an OFDM symbol. Let $\boldsymbol{v'}$ be the multiplexed output vector of length $b \cdot (N - N_p)$ with $v'_{2k} = u_k$, and $v'_{2k+1} = p_k$, where $k = 0, 1, \ldots, K-1$. The vector $\boldsymbol{v}$ is the interleaved version of $\boldsymbol{v'}$. In our notation, $\boldsymbol{X^s}$ represents the vector of $N - N_p$ signal QAM symbols, $\boldsymbol{X^p}$ is the vector of the

N_p pilot symbols, and $\boldsymbol{X}$ is the combined vector of all N signal and pilot symbols. The bits of the nth b-tuple of $\boldsymbol{v}$ are mapped to $X_n^s = A_m$ according to a specific mapping function, where A_m is a complex number drawn from a square M-QAM constellation, and $m = 0, 1, \ldots, M-1$. The first $b/2$ bits of the nth b-tuple are mapped to $Re\{X_n^s\}$, and the second $b/2$ bits are mapped to $Im\{X_n^s\}$, where $n = 0, 1, \ldots, N - N_p - 1$. A cyclic-prefix extension is then added to the vector $\boldsymbol{x} = \boldsymbol{Q}^* \boldsymbol{X}$, where $\boldsymbol{Q}$ is an $N \times N$ DFT matrix, to obtain the vector $\overline{\boldsymbol{x}}$ which is eventually transmitted through the channel.

A. Channel model

The channel $\boldsymbol{h}$ is assumed to be an ISI channel with at most $L = \nu + 1$ non-zero complex taps. We assume that the channel state is fixed over the duration of a single OFDM symbol. Moreover, we approximate the channel variation from symbol to symbol with an autoregressive model of order one AR(1) [6], which can be written in state-space form as

$$\boldsymbol{h}_i = \boldsymbol{F}\boldsymbol{h}_{i-1} + \boldsymbol{G}\boldsymbol{u}_i, \tag{2}$$

where $\boldsymbol{u}_i$ is a zero-mean i.i.d. circular complex Gaussian vector process with correlation matrix $\boldsymbol{R}_{uu}(j) = \boldsymbol{I}_L \delta(j)$ for every lag j. The matrices $\boldsymbol{F}$ and $\boldsymbol{G}$ are assumed to be known to the receiver.

Although the presented algorithm is more general, to address the specific cases commonly encountered in practice, we further assume that the channel taps follow the WSSUS model [7] and change according to Rayleigh fading. The time autocorrelation of the taps is assumed to follow Jakes' model [8] and is governed by the Doppler rate $f_D T$, where f_D is the maximum Doppler spread. In this case, all that the receiver needs to know to determine the diagonal matrices $\boldsymbol{F}$ and $\boldsymbol{G}$ uniquely are the Doppler rate and the power profile of the channel. The diagonal elements of $\boldsymbol{F}$ are given by [6]

$$a_k(1) = \mathcal{J}_o(2\pi f_D^{(k)} T), \tag{3}$$

where $\mathcal{J}_o(\cdot)$ is the zero-order Bessel function of the first kind, $k = 0, 1, \ldots, \nu$, and $f_D^{(k)} T$ is the Doppler rate of the kth tap. Given the diagonal channel covariance matrix $\boldsymbol{\Pi}$ with diagonal elements as $E\left\{|h^{(k)}|^2\right\}$, the variance of the k'th channel tap, we can determine the diagonal elements of $\boldsymbol{G}$ as

$$g_k = \sqrt{(1 - a_k^2(1)) \cdot E\left\{|h^{(k)}|^2\right\}} \tag{4}$$

for $k = 0, 1, \ldots, \nu$.

B. Input output relation

For the ith OFDM symbol, let $\overline{\boldsymbol{y}}_i^T = \left[\begin{array}{cc} \underline{\boldsymbol{y}}_i^T & \boldsymbol{y}_i^T \end{array}\right]$ be the output of the channel of length $N + \nu$, where $\underline{\boldsymbol{y}}_i$ is the cyclic-prefix observation of length ν, and $\boldsymbol{y}_i$ is the remaining part of length N, which can be obtained through the following cyclic convolution:

$$\boldsymbol{y}_i = \boldsymbol{h}_i \otimes \boldsymbol{x}_i + \boldsymbol{n}_i, \tag{5}$$

where $\boldsymbol{n}_i$ is a complex additive white Gaussian noise (AWGN) vector with the covariance matrix $\boldsymbol{R}_{nn} = 2\sigma^2 \boldsymbol{I}_N$, i.e., σ^2 is the noise power per dimension. We can then write

$$\boldsymbol{Y}_i = \mathrm{diag}(\boldsymbol{H}_i)\boldsymbol{X}_i + \boldsymbol{N}_i, \tag{6}$$

where $\boldsymbol{X}_i = \boldsymbol{Q}\boldsymbol{x}_i, \boldsymbol{Y}_i = \boldsymbol{Q}\boldsymbol{y}_i, \boldsymbol{N}_i = \boldsymbol{Q}\boldsymbol{n}_i$, and $\boldsymbol{H}_i = \boldsymbol{V}\boldsymbol{h}_i$, where $\boldsymbol{V}$ is an $N \times L$ Vandermonde matrix with elements given by $V_{n,l} = e^{-j\frac{2\pi}{N}nl}$ for $n = 0, 1, \ldots, N-1$ and $l = 0, 1, \ldots, L-1$. Equation (6) can be rewritten as

$$\begin{aligned} \boldsymbol{Y}_i &= \mathrm{diag}(\boldsymbol{X}_i)\boldsymbol{H}_i + \boldsymbol{N}_i, && (7) \\ &= \mathrm{diag}(\boldsymbol{X}_i)\boldsymbol{V}\boldsymbol{h}_i + \boldsymbol{N}_i. && (8) \end{aligned}$$

The cyclic-prefix observation of the ith OFDM symbol can be written as

$$\underline{\boldsymbol{y}}_i = \underline{\boldsymbol{x}\boldsymbol{x}}_i\ \boldsymbol{h}_i + \underline{\boldsymbol{n}}_i, \tag{9}$$

where $\underline{\boldsymbol{x}\boldsymbol{x}}_i$ is the following toeplitz matrix of the cyclic-prefix parts of $\overline{\boldsymbol{x}}_i$ and $\overline{\boldsymbol{x}}_{i-1}$

$$\underline{\boldsymbol{x}\boldsymbol{x}}_i = \left[\begin{array}{ccccc} x_0^i & x_{\nu-1}^{i-1} & x_{\nu-2}^{i-1} & \cdots & x_0^{i-1} \\ x_1^i & x_0^i & x_{\nu-1}^{i-1} & \cdots & x_1^{i-1} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ x_{\nu-1}^i & x_{\nu-2}^i & \cdots & x_0^i & x_{\nu-1}^{i-1} \end{array}\right]. \tag{10}$$

Equations (8) and (9) can be combined as

$$\left[\begin{array}{c} \underline{\boldsymbol{y}}_i \\ \boldsymbol{Y}_i \end{array}\right] = \left[\begin{array}{c} \underline{\boldsymbol{x}\boldsymbol{x}}_i \\ \mathrm{diag}(\boldsymbol{X}_i)\boldsymbol{V} \end{array}\right] \boldsymbol{h}_i + \left[\begin{array}{c} \underline{\boldsymbol{n}}_i \\ \boldsymbol{N}_i \end{array}\right], \tag{11}$$

which can be written in matrix form as

$$\boldsymbol{\mathcal{Y}}_i = \boldsymbol{A}_i \boldsymbol{h}_i + \boldsymbol{\mathcal{N}}_i. \tag{12}$$

We next present a low-complexity iterative algorithm for finding a good-quality approximate solution to the following joint maximum-likelihood (ML) channel/data estimation problem:

$$\left(\hat{\boldsymbol{X}}_i, \hat{\boldsymbol{H}}_i\right) = \arg \max_{\tilde{\boldsymbol{X}}_i, \tilde{\boldsymbol{H}}_i} \left\{ p\left(\boldsymbol{\mathcal{Y}}_i | \tilde{\boldsymbol{X}}_i, \tilde{\boldsymbol{H}}_i\right)\right\}. \tag{13}$$

III. Iterative Joint Decoding and Channel Estimation/Tracking

In practice, an OFDM system usually operates in a burst or packet mode, where a packet consists of p OFDM symbols. We assume that L pilots are used in the first OFDM symbol of the packet, and that no more pilots are used in the remaining $p-1$ symbols. We propose an iterative algorithm that iterates between ML soft decoding and ML channel estimation over each symbol individually and, therefore, has the minimum latency of a single OFDM symbol. The L pilots in the first symbol are used to acquire a good initial estimate of the channel. Then the channel time variation from symbol to symbol is tracked through the proposed iterative algorithm, which employs an EM-based Kalman filter to exploit the time statistics of the channel, which are assumed to be known to the receiver.

Fig. 2 shows a block diagram of the proposed iterative algorithm. For each OFDM symbol, the algorithm is initialized by an initial estimate of the channel. Extrinsic soft information for the coded bits are then iteratively exchanged between the soft decoder and the EM-based channel estimator, which takes the first and second moments

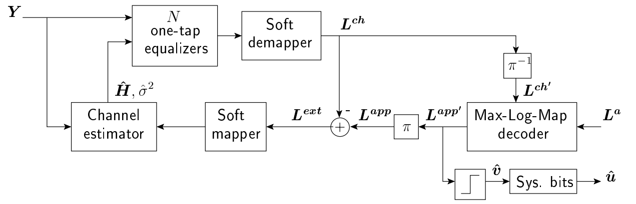

Fig. 2. The core iterative decoding and channel estimation/tracking algorithm.

of $\boldsymbol{X}_i$ as an input. The soft demapper and soft mapper modules convert extrinsic symbol probabilities to equivalent extrinsic bit probabilities and vice versa, given the specific mapping of bits to constellation points. The steps of the algorithm and the function of each module are described in more details below.

Since the noise variance does not usually vary too fast, for simplicity we will assume that σ^2 is known to the receiver, i.e., $\hat{\sigma}^2 = \sigma^2$. In practice, once the channel and data have been estimated for the jth OFDM symbol, $\hat{\sigma}^2$ can be obtained as

$$\hat{\sigma}_j^2 = \alpha \hat{\sigma}_{j-1}^2 + (1-\alpha)\frac{1}{N}\sum_{i=0}^{N-1}\left|Y_i^j - \hat{H}_i^j \hat{X}_i^j\right|^2, \tag{14}$$

where α is an exponential smoothing factor. We can then use $\hat{\sigma}_j^2$ as an estimate of σ^2 for the next symbol.

- **Step 1. Initialization:**

With a maximum of L active channel taps, an equal number of pilots is needed to uniquely identify the channel. Therefore, for the first OFDM symbol in the packet, L pilots uniformly spaced across the N subchannels are used to initially acquire the channel. In that case, the pilots induce the following input output relationship:

$$\boldsymbol{Y}_{i_{I_p}} = (\mathrm{diag}(\boldsymbol{X}_i))_{I_p}\boldsymbol{Q}_L \boldsymbol{h}_i + \boldsymbol{N}_{i_{I_p}}. \tag{15}$$

where I_p denotes the index set of the pilot subchannels, and $\boldsymbol{Q}_L$ is an $L \times L$ DFT matrix scaled by $\sqrt{L}$. Therefore, for the first symbol in the packet ($i = 0$), the iterative algorithm is initialized with

$$\hat{\boldsymbol{h}}_i^{(it=0)} = \boldsymbol{\Pi}_0 \boldsymbol{B}_i^* \left(\sigma^2 \boldsymbol{I}_L + \boldsymbol{B}_i \boldsymbol{\Pi}_0 \boldsymbol{B}_i^*\right)^{-1}, \tag{16}$$

where $\boldsymbol{B}_i = (\mathrm{diag}(\boldsymbol{X}_i))_{I_p}\boldsymbol{Q}_L$, and $\boldsymbol{\Pi}_0$ is the initial covariance matrix of the channel.

For the ith OFDM symbol of the subsequent $p-1$ symbols ($i > 0$), where no pilots are used, the iterative algorithm is initialized with

$$\hat{\boldsymbol{h}}_i^{(it=0)} = \boldsymbol{F}\hat{\boldsymbol{h}}_{i-1}. \tag{17}$$

Steps 2 through 4 deal solely with quantities related to the ith OFDM symbol. Thus, for convenience, we will drop the subscript i at these steps since it is understood from context, and later reintroduce it in Step 5.

- **Step 2. Equalization and soft demapping:**

Given the channel estimate $\hat{\boldsymbol{H}}^{(it)} = \boldsymbol{V}\hat{\boldsymbol{h}}^{(it)}$, we equalize the received vector $\boldsymbol{Y}$ using N parallel single-tap equalizers. We can then obtain the vector $\boldsymbol{L^{ch}}^{(it+1)}$ of the *extrinsic* channel log-likelihood ratios (LLRs) for the $b \cdot (N - N_p)$ coded bits as

$$\begin{aligned} L_l^{ch(it+1)} &= \log \frac{p(\boldsymbol{Y}|\hat{\boldsymbol{H}}^{(it)}, v_l = 1)}{p(\boldsymbol{Y}|\hat{\boldsymbol{H}}^{(it)}, v_l = 0)}, & (18)\\ &= \log \frac{\sum_{A_m:v_l=1} p(\boldsymbol{Y}|\hat{\boldsymbol{H}}^{(it)}, X_n^s = A_m)}{\sum_{A_m:v_l=0} p(\boldsymbol{Y}|\hat{\boldsymbol{H}}^{(it)}, X_n^s = A_m)}, & (19)\\ &= \log \frac{\sum_{A_m:v_l=1} p(Y_n^s|\hat{H}_n^s, X_n^s = A_m)}{\sum_{A_m:v_l=0} p(Y_n^s|\hat{H}_n^s, X_n^s = A_m)}, & (20) \end{aligned}$$

where $l = 0, 1, \ldots, b \cdot (N - N_p) - 1$, $n = \lfloor \frac{l}{b} \rfloor$, and the notation $A_m : v_l = 0(1)$ represents the set of constellation points that correspond to $v_l = 0(1)$. For the first $b/2$ bits of the n'th b-tuple, the expression in (20) can be evaluated as

$$L_l^{ch(it+1)} = \log \frac{\sum_{Re\{A_m\}:v_l=1} e^{-\frac{|\hat{H}_n^s|^2}{2\sigma^2}\left(Re\left\{\frac{Y_n^s}{\hat{H}_n^s}\right\} - Re\{A_m\}\right)^2}}{\sum_{Re\{A_m\}:v_l=0} e^{-\frac{|\hat{H}_n^s|^2}{2\sigma^2}\left(Re\left\{\frac{Y_n^s}{\hat{H}_n^s}\right\} - Re\{A_m\}\right)^2}}. \tag{21}$$

Similarly, for the second $b/2$ bits of the n'th b-tuple, the expression in (20) can be evaluated as

$$L_l^{ch(it+1)} = \log \frac{\sum_{Im\{A_m\}:v_l=1} e^{-\frac{|\hat{H}_n^s|^2}{2\sigma^2}\left(Im\left\{\frac{Y_n^s}{\hat{H}_n^s}\right\} - Im\{A_m\}\right)^2}}{\sum_{Im\{A_m\}:v_l=0} e^{-\frac{|\hat{H}_n^s|^2}{2\sigma^2}\left(Im\left\{\frac{Y_n^s}{\hat{H}_n^s}\right\} - Im\{A_m\}\right)^2}}. \tag{22}$$

- **Step 3. Soft decoding:**

Given the channel soft extrinsic information $\boldsymbol{L^{ch}}^{(it+1)}$, soft MAP sequence estimation is performed using the Max-Log-Map algorithm [9], [10]. We obtain the *extrinsic* log-likelihood ratios for the coded bits $\boldsymbol{L^{ext}}$ as

$$\boldsymbol{L^{ext}}^{(it+1)} = \boldsymbol{L^{app}}^{(it+1)} - \boldsymbol{L^{ch}}^{(it+1)}, \tag{23}$$

where $\boldsymbol{L^{app}}$ is the interleaved version of the a posteriori LLRs vector for the coded bits $\boldsymbol{L^{app'}}$ provided by the Max-Log-Map algorithm.

The extrinsic probabilities of the coded bits are then obtained as

$$P^{ext}(v_l = 1) = \frac{e^{L_l^{ext}}}{1 + e^{L_l^{ext}}}, \quad P^{ext}(v_l = 0) = \frac{1}{1 + e^{L_l^{ext}}}, \tag{24}$$

where $l = 0, 1, \ldots, b \cdot (N - N_p) - 1$.

- **Step 4. Soft mapping:**
The first and second moments of X_n^s are given by

$$E[X_n^s] = \sum_{m=0}^{M-1} A_m \cdot P^{ext}(X_n^s = A_m), \quad (25)$$

$$E[|X_n^s|^2] = \sum_{m=0}^{M-1} |A_m|^2 \cdot P^{ext}(X_n^s = A_m), \quad (26)$$

where $n = 0, 1, \ldots, N - N_p$. The extrinsic probability of the nth signal QAM symbol X_n^s is simply the product of the probabilities of coded bits mapped to it, which are assumed to be independent because of the random interleaving and according to the iterative processing paradigm. Thus,

$$P^{ext}(X_n^s = A_m) = P^{ext}\left(\begin{bmatrix} v_{nb} & \cdots & v_{(n+1)b-1} \end{bmatrix}^T = \boldsymbol{bA_m}\right), \quad (27)$$

$$= \frac{1}{c} \prod_{j=0}^{b-1} P^{ext}(v_{nb+j} = \boldsymbol{bA_m}(j)), \quad (28)$$

where $\boldsymbol{bA_m}$ for $m = 0, 1, \ldots, M-1$ represents the vector of binary bits mapped to the constellation point A_m, and c is a normalization coefficient.

The first and second moments of the known pilots symbols (if any) are then inserted to obtain $E[X_n]$ and $E[|X_n|^2]$ for $n = 0, 1, \ldots, N-1$. This directly gives $E[\boldsymbol{X}]$, and given the random interleaving and the independence of the N subchannels, $E[\boldsymbol{X}\boldsymbol{X}^*]$ is given by

$$E[\boldsymbol{X}\boldsymbol{X}^*] = E[\boldsymbol{X}]E[\boldsymbol{X}]^* + E[\mathrm{diag}(\boldsymbol{X}^* \odot \boldsymbol{X})] - \mathrm{diag}(\mathrm{E}[\boldsymbol{X}]^* \odot \mathrm{E}[\boldsymbol{X}]), \quad (29)$$

where $\odot$ represents a point-wise product operation.

- **Step 5. Channel estimation:**
The channel estimation is performed using an EM-based Kalman filter that exploits all the system and data constraints and statistical information available to the receiver. Data constraints include: coding, finite alphabet and cyclic-prefix. System constraints include: finite delay spread, frequency correlation, and time correlation. The updated channel estimate is obtained as

$$\hat{\boldsymbol{h}}_i^{(it+1)} = (\boldsymbol{I}_L - \boldsymbol{K}_{f,i}\, \boldsymbol{C}_i)\, \hat{\boldsymbol{h}}_{i|i-1} + \boldsymbol{K}_{f,i}\, \boldsymbol{Z}_i, \quad (30)$$

where

$$\boldsymbol{K}_{f,i} = \boldsymbol{P}_{i|i-1}\boldsymbol{C}_i^*\boldsymbol{R}_{e,i}^{-1}, \quad (31)$$

$$\boldsymbol{R}_{e,i} = \boldsymbol{I}_{N+2\nu} + \boldsymbol{C}_i\boldsymbol{P}_{i|i-1}\boldsymbol{C}_i^*, \quad (32)$$

$$\boldsymbol{P}_i = \boldsymbol{P}_{i|i-1} - \boldsymbol{K}_{f,i}\boldsymbol{R}_{e,i}\boldsymbol{K}_{f,i}^*, \quad (33)$$

$$\boldsymbol{C}_i = \begin{bmatrix} E[\boldsymbol{A}_i] \\ \mathrm{Cov}[\boldsymbol{A}_{\mathrm{i}}^*]^{1/2} \end{bmatrix}, \quad (34)$$

$$\boldsymbol{Z}_i = \begin{bmatrix} \boldsymbol{\mathcal{Y}}_i \\ \boldsymbol{0}_{L\times 1} \end{bmatrix}, \quad (35)$$

$$\boldsymbol{P}_{i|i-1} = \begin{cases} \boldsymbol{\Pi}_0 & \text{for } i = 0 \\ \boldsymbol{F}\boldsymbol{P}_{i-1}\boldsymbol{F}^* + \boldsymbol{G}\boldsymbol{G}^* & \text{for } i > 0 \end{cases} \quad (36)$$

$$\hat{\boldsymbol{h}}_{i|i-1} = \begin{cases} \boldsymbol{0}_{L\times 1} & \text{for } i = 0 \\ \boldsymbol{F}\hat{\boldsymbol{h}}_{i-1} & \text{for } i > 0 \end{cases} \quad (37)$$

It is straightforward to express $E[\boldsymbol{A}_i]$ in terms of $E[\boldsymbol{X}_i]$ by noticing that $E[\boldsymbol{x}_i] = \boldsymbol{Q}^* E[\boldsymbol{X}_i]$. Similarly, $\mathrm{Cov}[\boldsymbol{A}_{\mathrm{i}}^*]$ can be expressed in terms of $E[\boldsymbol{X}_i]$ and $E[\boldsymbol{X}_i\boldsymbol{X}_i^*]$ obtained from Step 4 [11].

- **Step 6. Repeating:**
Return to Step 2, and repeat until a stopping criterion, such as a maximum number of iterations, is reached.

IV. Optional Serial Concatenation

For multi-amplitude constellations and at a relatively high Doppler frequency, the proposed iterative algorithm may occasionally fail to converge within the allowed number of iterations, which causes channel tracking to be lost and results in a burst of errors for the reminder of the packet. This problem can result in an error flooring behavior that may not be acceptable in some applications. Therefore, we propose the use of an optional rate-1/2 outer LDPC code in serial concatenation with the inner convolutional code. This outer code also allows exploiting the time diversity of the channel, and through iterative serial decoding leads to an increased coding gain. The decoding of the outer LDPC is only invoked after the core iterative channel-estimation-and-decoding algorithm described in Section III has stopped iterating. Therefore, the proposed serial concatenation does not affect the complexity of the core iterative algorithm. LDPC codes and their iterative decoding are described in detail in [5].

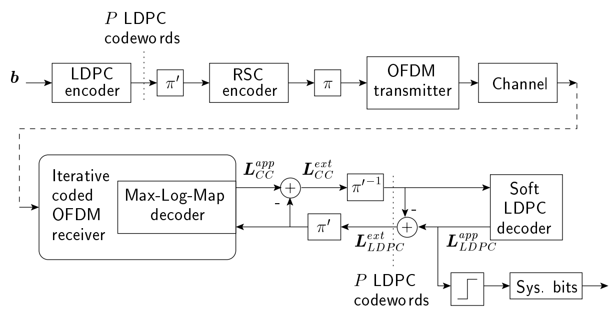

Fig. 3. Optional serial concatenation.

Fig. 3 illustrates the proposed serial-concatenation scheme. The information bits of a single OFDM packet (p OFDM symbols) are coded using a single LDPC codeword. Then the bits of P LDPC codewords are interleaved using the outer interleaver π' before passing them through the RSC encoder. The outer interleaving is needed to disperse the burst of bit errors in an OFDM packet caused by lost channel tracking and distribute it among P LDPC codewords. Although the loss of channel tracking is a relatively rare event for reasonable Doppler rate and constellation size, when it occurs, it leads to a large number of bit errors in an OFDM packet. Therefore, a powerful enough LDPC code, like a rate-1/2 code, is needed to correct these bursts of bit errors. Alternatively, the length of the outer interleaver, i.e., P should be increased, but that results in increased latency which may not be acceptable in certain applications. Thus, the proposed serial concatenation offers a trade-off between latency and performance that can be adjusted as needed for a given application.

When the core iterative algorithm stops, iterative decoding of the serially concatenated codes is performed by exchanging soft extrinsic information for the LDPC coded bits between the Max-Log-Map decoder and the iterative LDPC decoder as shown in Fig. 3. One of the attractive features of LDPC codes is that they have a natural stopping criterion. Thus, the iterative decoding of an LDPC codeword $\boldsymbol{c}$ can be stopped once $\boldsymbol{H}\hat{\boldsymbol{c}} = \boldsymbol{0}$, where $\boldsymbol{H}$ is the parity-check matrix of the code. Similarly, serial iterations can be stopped once the parity-check condition is satisfied for all the P LDPC codewords.

V. Simulation Results and Discussion

The 4-state rate $r = 1/2$ convolutional code with $G(D) = [1 \ \frac{1+D^2}{1+D+D^2}]$ is used, and its trellis is terminated on each OFDM symbol with $m = 2$ bits. The OFDM system is assumed to have a total bandwidth of 2 MHz divided into $N = 64$ subchannels with a carrier frequency of 5 GHz. The guard interval is assumed to be 8 μsec; hence, the system has a symbol period of 40 μsec. Gray coding on both dimensions of the square M-QAM constellation is used to map bits to constellation points. The channel delay spread is assumed to be equal to the guard interval with $L = \nu + 1 = 16$ active taps. The channel is assumed to follow the WSSUS model with an exponentially decaying power profile and to have a Rayleigh fading. The Doppler rate is assumed to be the same for all of the L channel taps. In our simulations, we consider three Doppler frequency values: 250 Hz, corresponding to a mobile speed of 33.7 miles/hr and a normalized Doppler rate of 1%; 500 Hz, corresponding to a mobile speed of 67.5 miles/hr and a normalized Doppler rate of 2%; and 1000 Hz, corresponding to a mobile speed of 135 miles/hr and a normalized Doppler rate of 4%. The first OFDM symbol in the packet, which is assumed to consist of $p = 10$ symbols, has $L = 16$ uniformly spaced pilots across the $N = 64$ subchannels, while the next $p - 1$ symbols have no pilots.

The system performance using the proposed iterative algorithm is compared to the case with no iterations and to the ideal case with perfect channel state information (CSI) at the receiver. To account for the rate loss due to pilots, coding, trellis termination and cyclic-prefix, the performance curves are plotted against the average E_b/N_0 instead of the average SNR, which are related as

$$SNR_{ave} = 2\bar{b}R_{eff}\left(\frac{E_b}{N_0}\right)_{ave}, \qquad (38)$$

where $\bar{b} = b/2$ is the normalized number of bits per QAM symbol, and

$$R_{eff} = \frac{[(N - N_{p_0}) + (p-1)N]\, r - p \cdot m/b}{p(N + \nu)}, \qquad (39)$$

where N_{p_0} is the number of pilots in the first symbol, which is L for the unknown CSI cases and 0 for the ideal case with perfect CSI at the receiver. If an outer code is used, its rate loss is incorporated by multiplying R_{eff} in (39) by the rate of that code.

Figure 4 shows the bit error rate (BER) curves for the system using the proposed iterative algorithm with 4-QAM and 16-QAM modulations. As can be seen in this figure, compared to the non-iterative case, significant improvement in performance, with as little as 3 iterations, is achieved in the 4-QAM case, which shows no significant error flooring effect even at a Doppler rate of 4%. As for the 16-QAM case, significant improvement is achieved in the high SNR range, but the 2% Doppler rate case still shows a slight error-flooring effect. The next section will show how such error flooring can be completely eliminated by using an outer code.

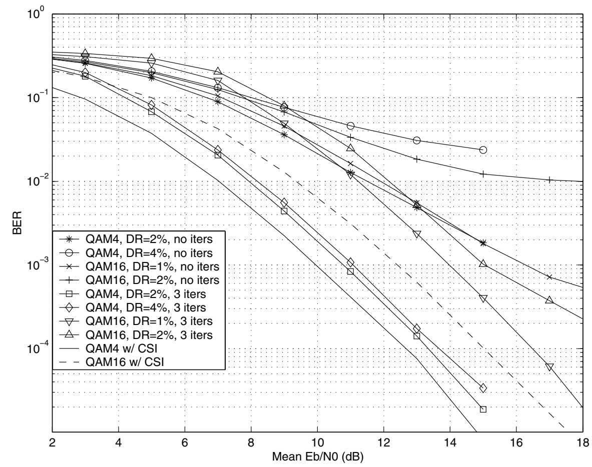

Fig. 4. BER versus average E_b/N_0 for the various cases without outer coding.

For the 16-QAM case, we observe that at low SNR the iterative algorithm actually has a degraded performance compared to the non-iterative case. This behavior is caused by the extra sensitivity of multi-amplitude modulation to the bad initial channel estimate. This, in turn, causes the iterative algorithm to diverge, resulting in an even worse channel estimate that is then passed on to the next symbol in the packet, thereby causing error propagation across symbols. This divergence behavior in the low SNR range can be more clearly seen in Figure 5, which shows the average mean square error $MSE = ||\boldsymbol{h} - \hat{\boldsymbol{h}}||^2$ of the channel estimate versus E_b/N_0.

For the optional serial concatenation, we used a regular rate-1/2 LDPC code with a randomly generated parity-check matrix that has a column weight of 3. The binary data of each packet of $p = 10$ OFDM symbols is coded using a single LDPC codeword; then, $P = 10$ LDPC codewords are interleaved before passing them through the RSC encoder. Figure 6 shows the BER curves for 4-QAM and 16-QAM modulations with 3 iterations of the core iterative algorithm, followed by up to a maximum of 5 serial decoding iterations between the inner and outer codes. The iterative LDPC decoder itself uses a maximum of 20 iterations. As can be seen in this figure, the error floor behavior at the high constellation sizes is completely eliminated. This performance improvement is achieved at the expense of an increased latency of $P \cdot p = 100$ OFDM symbols, as opposed to the latency of a single OFDM symbol of the core iterative algorithm without serial concatenation.

As mentioned earlier, LDPC codes have a natural parity-

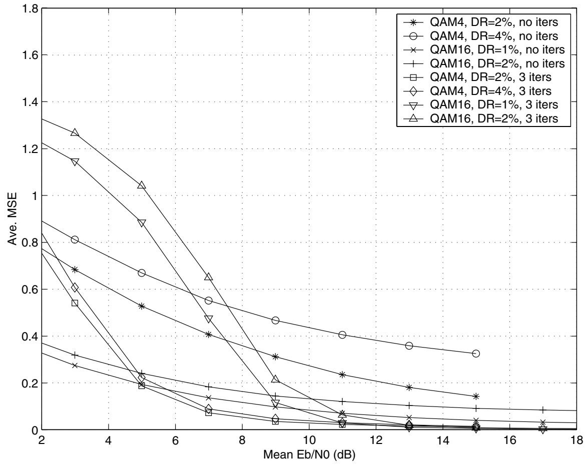

Fig. 5. Average MSE versus average E_b/N_0 for the iterative and non-iterative cases.

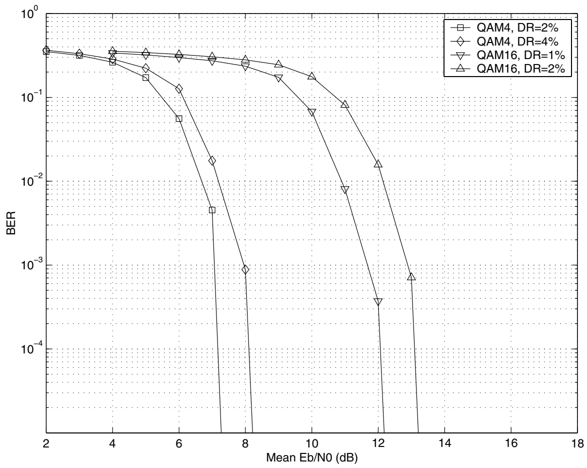

Fig. 6. BER versus average E_b/N_0 for the various cases with outer coding.

check stopping criterion that allows stopping the iterative LDPC decoding as well as the iterative serial decoding, which can save significant power consumption in practice. Figure 7 shows the average number of serial decoding iterations and the average number of LDPC decoding iterations at the last LDPC decoding step before reaching the stopping criterion versus E_b/N_0. Clearly, the iterative decoding can be stopped at a much smaller number of iterations than the maximum at the SNR range in which the LDPC code actually shows most of its benefit in terms of performance improvement. This observation indicates that the maximum numbers of serial decoding iterations and LDPC decoding iterations can be lowered significantly without noticeably affecting performance.

VI. Conclusion

We presented an EM-based iterative algorithm for channel estimation and tracking that collectively exploits all of the data and system constraints that may be available to the receiver of a coded OFDM system. The algorithm uses

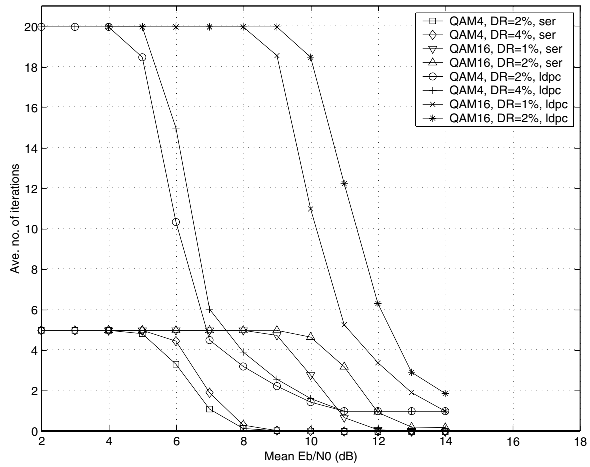

Fig. 7. Average number of iterations of serial decoding and last LDPC decoding versus average E_b/N_0.

a simple 4-state convolutional code, and with as few as 3 iterations results in significant improvement relative to the non-iterative approach. If the Doppler rate is too high, it can still causes an unacceptable error floor for multi-amplitude modulation cases. We proposed using an outer LDPC code with serial concatenation to alleviate this problem at the cost of increased latency, but without significantly increasing the complexity of the receiver.

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