# Compressive Sensing for Feedback Reduction in MIMO Broadcast Channels

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**Abstract**

We propose a generalized feedback model and compressive sensing based opportunistic feedback schemes for feedback resource reduction in MIMO Broadcast Channels under the assumption that both uplink and downlink channels undergo block Rayleigh fading. Feedback resources are shared and are opportunistically accessed by users who are *strong*, i.e. users whose channel quality information is above a certain fixed threshold. Strong users send the same feedback information on all shared channels. They are identified by the base station via compressive sensing. Both analog and digital feedbacks are considered. The proposed analog & digital opportunistic feedback schemes are shown to achieve the same sum-rate throughput as that achieved by dedicated feedback schemes, but with feedback channels growing only logarithmically with number of users. Moreover, there is also a reduction in the feedback load. In the analog feedback case, we show that the proposed scheme reduces the feedback noise which eventually results in better throughput, whereas in the digital feedback case the proposed scheme in a noisy scenario achieves almost the throughput obtained in a noiseless dedicated feedback scenario. We also show that for a given fixed budget of feedback bits, there exists a trade-off between the number of shared channels and thresholds accuracy of the fed back SNR.

**Index Terms**

Compressed sensing, feedback, lasso, multiple-input multiple-output (MIMO) systems, opportunistic, protocols, scheduling.

## I. INTRODUCTION

Recently, it has been shown that dirty paper coding (DPC) achieves the sum-rate throughput of the multiple-input multiple-output (MIMO) broadcast channel [1], [2]. However, it requires a great deal of

feedback as the transmitter needs perfect channel state information for all users and is computationally expensive [3]. Since then, many works have attempted to achieve the same sum-rate throughput with imperfect channel state information (reduced feedback load). This was done by applying opportunistic communication in the forward link [4]- [10]. By reviewing the feedback protocols suggested in literature, one can note that generally the following three components are fed back and user selection is based on either one or a combination of these components [4]- [9]:

1) Channel Direction Information (CDI), e.g., beam index(BI), quantized channel index (QCI),..., etc.

2) Channel Quality Information (CQI), e.g., SNR, SINR, channel norm, ..., etc.

3) User identity (ID)

Another way to differentiate Feedback schemes according to whether their feed back data is analog or digital [11]- [14]. While the forward link is opportunistic in nature, almost all feedback schemes mentioned above are non-opportunistic, and so, each user has a *dedicated* feedback channel. To see this, consider the random beamforming (RBF) scheme proposed by Sharif and Hassibi [4]. Here, the users are differentiated according to the channel direction (i.e. what beam direction is the channel mostly aligned with) and as a result, users feedback to Base Station (BS) the SINR corresponding to that direction only. So, each user feeds back one integer and one real number. In order to reduce the feedback load further Diaz et. al. in [7] propose a threshold based RBF. Here, instead of feeding back the SINR for the best beam for each receive antenna (one real plus one integer numbers), the user only transmits one bit to the BS, indicating whether or not the SINR on a pre-selected beam for any receive antenna is above a given threshold. The scheme is repeated for each beam. Since interaction or cooperation among the competing users is not allowed, hence defying opportunism, there is a linear increase in the feedback resources (or channels) with the number of users [5], [6]. Even if thresholding is applied, there is no reduction in the number of feedback channels. This is because the channels are reserved even when users are not sending any feedback information.

Recently, some works have started to consider opportunistic feedback schemes where feedback resources are shared and are opportunistically accessed by strong users, i.e. users whose CQI is above the given thresholds. Thus, in [8], Tang et. al. propose a feedback scheme with fixed number of feedback slots (channels) that are randomly accessed by strong users. In every slot, each strong user independently attempts to send back to the BS a data package containing its user identity (ID) with a probability. If two or more users feedback in the same slot, collision occurs and the feedback in that slot is discarded. When multiple users successfully feed back, the BS randomly selects one of the successful users. Although the

scheme requires only an integer feedback per slot, it is suboptimal as the user is selected randomly. The scheme was only proposed for the single-input single-output (SISO) case. In [9], Rajiv et. al. propose a similar feedback scheme based on random access slots for the MIMO case that requires only user identity feedback (i.e. an integer feedback per slot). Both of the two schemes above require accurate timing-synchronization to avoid collisions, which is difficult to achieve in practice. Moreover, feedback to the BS is successful if there are no collisions, i.e., only one user is attempting to feed back in a slot. In addition, the two schemes only work for digital feedback but not for analog. In all the feedback schemes discussed above, the feedback links were assumed ideal whereas the forward links were subject to both fading and noise. This asymmetry in the way the two links are treated is unrealistic[1].

In this paper, we consider a broadcast scenario where the forward and the feedback links are symmetric in that they are both i) non-ideal and ii) opportunistic or shared. Thus, both links undergo Rayleigh fading and are subject to additive Gaussian noise. Moreover, the channels in both links are shared and are opportunistic in the sense that feedback channels are dominated by strong users. Finally, the feedback links can be used for both analog and digital feedback. The paper proposes a generalized feedback model and compressive sensing (CS) [20] - [25] based opportunistic feedback protocols for feedback resource reduction.

### A. Our Feedback Approach

To setup the stage, we summarize in what follows our feedback technique. This also helps in laying out our contributions and the organization of the paper. Just as in all existing feedback techniques, a number of channel directions or beams is first determined. For each direction, the number of feedback channels is fixed and strong users feedback their CQI information on *all* channels. In the analog feedback case, each strong user feeds back its CQI value whereas in the digital feedback case, each strong user feeds back "1" if its CQI is above a particular threshold and remains silent otherwise. This creates an undetermined system of equations in a sparse vector of users. We use the emerging compressive sensing technique to identify users who have fed back and to estimate the feedback CQI. Users with higher value CQI have a stronger chance of being recovered. The results obtained via compressive sensing are refined using least-squares and the strongest user is selected. As the feedback links are noisy, the BS backs off on the noisy CQI based on the variance of the noise. We obtain the optimum back off on the noisy CQI that maximizes the throughput. The scheme is repeated for each channel direction. Although we have

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[1]A notable exception is [35] and [36] which some how account for this by considering the effect of feedback error.

used SINR feedback, the proposed schemes can work with any kind of CQI (e.g. SNR). It is important to note that our scheme is less sensitive to timing-synchronization errors, as the scheme will be affected only if out of synchronization user is selected for a particular channel direction (the probability of which is low).

The remainder of the paper is organized as follows. In Section II, a generalized feedback model is introduced. In Section III we discuss the proposed feedback strategy. In Section IV, we present the sum-rate throughput obtained by the proposed schemes in the RBF case. In Section V, performance evaluation of the proposed feedback schemes is presented. Feedback channel training is discussed in Section VI followed by numerical results and conclusions in Sections VII and VIII respectively.

## II. SYSTEM MODEL

### A. Downlink Transmission Model

We consider a single cell multi-antenna broadcast channel with $p$ antennas at the base station (transmitter) and $n$ users (receivers) each having one antenna[2]. The channel is described by a propagation matrix which is constant during the coherence interval and is known completely at the receiver. Let $\mathbf{u} \in \mathbb{C}^{p \times 1}$ be the transmit symbol vector then the signal $x_i$ received by the $i$-th user is given by

$$x_i = \sqrt{\rho_i} \mathbf{h}_i \mathbf{u} + z_i, \qquad i = 1, \ldots, n \qquad (1)$$

where $\mathbf{h}_i \in \mathbb{C}^{1 \times p}$ is the channel gain vector between the transmitter and the user, and $z_i$ is the additive noise. The entries of $\mathbf{h}_i$ and $z_i$ are i.i.d. complex Gaussian with zero mean and unit variance, $\mathcal{CN}(0, 1)$. Moreover, $\mathbf{u}$ satisfies unity average transmit power constraint $\mathbb{E}\{\mathbf{u}^* \mathbf{u}\} = 1$ and $\rho_i$ is the SNR of the $i$-th user. We consider a homogeneous network in which all users have the same SNR, i.e. $\rho_i = \rho = P/p$, where $P$ is the total power available at the transmitter. We also assume that the number of mobiles is greater than or equal to the number of transmit antennas, i.e., $n \geq p$, and that the BS selects $p$ out of $n$ users to transmit to.

### B. Generic Multi-Channel Feedback Model

We present here a general model for the multiuser feedback channel with $r$ feedback channels (possibly shared) among $n$ users, in which users report channel quality information (CQI) to the base station in

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[2]The broadcast model (1) is MISO. However, the feedback strategy in the paper applies to the general MIMO case. For in both cases, the users are feeding back a few number of coefficients, something that can be achieved by our feedback strategy.

order to exploit multiuser diversity. Let $\mathbf{v} \in \mathbb{C}^{n \times 1}$ be transmit feedback vector and let $y_i$ be the signal received via the $i$-th feedback channel described by

$$
\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{r1} & a_{r2} & \cdots & a_{rn} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_r \end{bmatrix}
$$

or equivalently

$$
\mathbf{y} = \mathbf{A}\mathbf{v} + \mathbf{w} \tag{2}
$$

where $r \leq n$ and $a_{ij}$ represents the gain of the $i$-th channel for the $j$-th user. The feedback propagation matrix $\mathbf{A}$ is independent from the downlink channel and is assumed constant during the coherence interval and perfectly known at the BS[3]. The entries of $\mathbf{A}$ are assumed to be zero mean unit variance complex i.i.d. complex Gaussian. The entries of the additive noise $\mathbf{w}$ are also complex i.i.d. Gaussian with zero mean and variance $\sigma^2$.

We summarize in Table I, how our feedback model (2) applies to the feedback models (opportunistic or not) suggested in literature [4]- [9]. Thus, in the non-opportunistic feedback model, each user is allocated its own feedback channel and the uplink channel matrix $\mathbf{A}$ becomes diagonal and of size $n$ (equal to the number of users). For the opportunistic models proposed in [8] by Tang et. al., the feedback channel matrix $\mathbf{A}$ becomes diagonal of size $r \times r$, where $r$ is the number of feedback slots and is less than $n$. The vector $\mathbf{v}$ represents feedback data in each slot, and when a collision in a particular slot takes place, the corresponding entry of $\mathbf{v}$ is not valid. The same model holds for [9] except that in this scheme $r$ is not fixed but varies randomly. Also, $r$ may not necessarily be less than $n$. In most of the proposed schemes, the additive noise $\mathbf{w}$ is set to zero.

In this paper, we focus on the case when $\mathbf{A}$ is fading, the links are noisy, and $r \leq n$. Unlike the approach taken in [15], [16], and [32], which also consider noisy feedback, we consider a contention-based feedback protocol, which assigns independent multi-access contention channels for CQI reporting in which the feedback process should itself be a filter that selects strongest users. We assume that each single-antenna user is going to feedback the same information over $r$ frequency bands shared with the other users. Thus, $a_{ij}$ represents the gain of the $j$-th user in the $i$-th band (frequency feedback channels).

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[3]We discuss in Section IV how this is done through feedback channel training.

## III. PROPOSED FEEDBACK STRATEGY

Before we discuss the proposed feedback strategy, we present an important compressive sensing result that we will refer in our work.

### A. Compressive Sensing using LASSO

We discuss an important result from a recent paper by Candes and Yaniv [25] for recovering $\mathbf{v}$ in a noisy setting (cf. (2)) using the following LASSO estimate

$$\hat{\mathbf{v}} = \arg \min_{\mathbf{v} \in \mathbb{R}^n} \|\mathbf{y} - \mathbf{A}\mathbf{v}\|_{l_2}^2 + \alpha\sigma\|\mathbf{v}\|_{l_1} \tag{3}$$

Assumptions:

1) $\mathbf{A}$ has unit normed columns and obeys the coherence property.

2) The noise is i.i.d. real Gaussian with variance $\sigma^2$, and $\mathbf{v}$ is sparse with support $S$.

3) $s \leq c_0 n/[\|\mathbf{A}\|^2 \log n]$ for some positive numerical constant $c_0$ (where $\|\mathbf{A}\|$ is the operator norm of $\mathbf{A}$). $\min_{i \in S} |\mathbf{v}_i| > 8\sigma\sqrt{2 \log n}$, and $\alpha$ is set to $\alpha = 2\sqrt{2 \log n}$.

Under these assumptions, the LASSO estimate $\hat{\mathbf{v}}$ obeys $\text{supp}(\hat{\mathbf{v}}) = \text{supp}(\mathbf{v})$ and $\text{sgn}(\hat{\mathbf{v}}) = \text{sgn}(\mathbf{v})$ with probability at least $1 - 2n^{-1}\left(\frac{1}{\sqrt{2\pi \log n}} + \frac{s}{n}\right) - O\left(\frac{1}{n^{2 \log 2}}\right)$. Moreover, once the support $S$ is known, least squares can be used to estimate or refine $\mathbf{v}_S$ (non-zero entries of $\mathbf{v}$) as $\mathbf{v}_S^{ls} = (\mathbf{A}_S^* \mathbf{A}_S)^{-1} \mathbf{A}_S^* \mathbf{y}$, where $\mathbf{A}_S$ denotes the sub-matrix formed by the columns $\{\mathbf{A}_j | j \in S\}$, indexed by the support $S$ [26].

### B. General Strategy of Using Compressive Sensing for Feedback

Any feedback scheme has two components, a direction component and a magnitude component. The transmitter usually has certain pre-determined directions for which it seeks user feedback. Thus, the BS announces that it is seeking feedback for a particular direction. The users whose channels lie at or are close to this direction, feedback their CQI (SNR, SINR, channel strength etc.). Now a limited number of users will feedback on the set of shared feedback channels according to equation (2).

This makes the vector $\mathbf{v}$ in (2) sparse with sparsity level determined by the number of users who feedback. CS can now be used to recover the sparsity pattern of $\mathbf{v}$. Moreover, the larger the value of particular CQI, the higher the chances of its recovery. Note that we need at least one strong user (i.e. $s \geq 1$) for each beam or direction in order to achieve full multiplexing gain which implies that small values of $s$ are sufficient. To reduce the number of users who feedback $s$, we pursue a thresholding strategy where the user will feedback if its CQI is greater than a threshold $\zeta$ to be determined.

Now consider a particular beam (CDI) (all beams will behave in an identical manner as the users are i.i.d. and the beams are equi-powered). Noting that the users' CQI are i.i.d., we can choose $\zeta$ to produce a sparsity level $s$. This happen by requiring that

$$\bar{F}(\zeta) = \arg \max_{u \in (0,1)} \binom{n}{s} u^s (1-u)^{n-s} \tag{4}$$

where $u = \bar{F}(\zeta)$ or $u$ is the complementary cumulative distribution function (CCDF) of CQI (SINR) defined as[4] $\bar{F}(\zeta) = \mathbb{P}[\text{SINR} > \zeta] = \frac{\exp(-\zeta/\rho)}{(1+\zeta)^{p-1}}, \zeta \geq 0$. By taking the logarithm of both sides of (4), it is easy to show that the level needed to produce a sparsity level $s$ is given by $\zeta = \bar{F}^{-1}\left(\frac{s}{n}\right)$.

*C. Applying Candes and Yaniv result to our problem*

To apply Candes and Yaniv result to our problem, note first that the vector $\mathbf{v}$ in our case is real. So we increase the number of measurements by decomposing the observation $\mathbf{y}$ into its real and imaginary parts

$$\begin{bmatrix} \Re(\mathbf{y}) \\ \Im(\mathbf{y}) \end{bmatrix} = \begin{bmatrix} \Re(\mathbf{A}) \\ \Im(\mathbf{A}) \end{bmatrix} \begin{bmatrix} \mathbf{v} \end{bmatrix} + \begin{bmatrix} \Re(\mathbf{w}) \\ \Im(\mathbf{w}) \end{bmatrix},$$

where $\Re(\mathbf{A})$ & $\Im(\mathbf{A})$ represents real and imaginary part of $\mathbf{A}$, or,

$$\underline{\mathbf{y}} = \underline{\mathbf{A}}\mathbf{v} + \underline{\mathbf{w}} \tag{5}$$

The above model gives us the $2r \times n$ real measurement matrix, and $2r \times 1$ real noise vector. To apply Candes and Yaniv result we still have to normalize the columns, thus

$$\underline{\mathbf{y}} = \hat{\mathbf{A}}\hat{\mathbf{v}} + \underline{\mathbf{w}} \tag{6}$$

where $\hat{\mathbf{A}} = \frac{A}{\sqrt{r}}$ and $\hat{\mathbf{v}} = \sqrt{r}v$. The outcome of the rearrangement gives us the following

1) The columns of $\hat{\mathbf{A}}$ are i.i.d. Gaussian and normalized with $\|\hat{\mathbf{A}}\| = \sqrt{n/2r}$. This implies that $s \leq c_0 2r/\log n$ or $2r \geq (1/c_0)s \log n$ or $2r \geq cs \log n$.

2) $\underline{\mathbf{w}}$ are i.i.d. $\mathcal{N}\left(0, \sigma^2/2\right)$.

3) $\min_{i \in S} |\mathbf{v}_i|$ that can be recovered using LASSO is given by $\min_{i \in S} |\mathbf{v}_i| > 8\sigma\sqrt{\frac{\log n}{r}}$.

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[4]It was shown in [4] that the asymptotic behavior of the maximum of $n$ *i.i.d.* random variables when $n$ is sufficiently large converges in distribution to a random variable whose distribution function is exponential. The class of distribution functions we encounter in this paper are of the same type that were encountered in [4].

*D. Feedback Protocol for the Analog Feedback Case*

In the analog feedback scenario, users above threshold feed back their analog CQI value. For any given $\sigma$, $n$ and $r$, the threshold should be set such that $\min_{i \in S} |\mathbf{v}_i| > 8\sigma\sqrt{\frac{\log n}{r}}$ which implies that $\bar{F}^{-1}(s/n) > 8\sigma\sqrt{\frac{\log n}{r}}$, or $\zeta > 8\sigma\sqrt{\frac{\log n}{r}}$. Thus, only those value of $s$ that satisfies the above equation are allowed. The other way of setting the threshold is to fix $s$, $n$ and $r$ and vary $\sigma$ (i.e. vary the SNR in the feedback channel (2)) such that the above equation is satisfied. Note that $s$ and $r$ are also related by $2r \geq cs \log n$.

The CS strategy then allows the BS to recover all users who transmitted their CQI with very high probability. This off course will be true provided that the number of users who feedback is less than or equal to $s$. Here, we assume that the probability a user is strongest for more than one beam is negligible as the number of users are relatively much larger than the number of beams. It has been shown in [4] that this is a valid assumption under these conditions. The the proposed compressive sensing based opportunistic feedback protocol is shown in Table II.

*E. Feedback Protocol for the Digital Feedback Case*

The digital feedback is similar to analog feedback except that user feeds back "1" if its CQI for a particular beam is above a particular threshold. Otherwise, the user remains silent. Here, for a given $n$ and $r$, $\sigma$ must be selected such that $\min_{i \in S} |\mathbf{v}_i| > 8\sigma\sqrt{\frac{\log n}{r}}$ is satisfied which implies that $\sigma < \frac{1}{8}\sqrt{\frac{r}{\log n}}$ From this inequality, it may first appear that $s$ has no role to play here. However, note that $s$ and $r$ are related by $2r \geq cs \log n$. To increase the feedback granularity, we let the users compare their CQI to a set of thresholds, not just one. Thus, suppose that we want to set $k$ thresholds $\zeta_1 < \zeta_2 < ......, < \zeta_k$ such that the number of users whose CQI lie between the two consecutive thresholds $Q_i = [\zeta_i, \zeta_{i+1})$ is equal to $s$. Note that the last interval is $[\zeta_k, \infty)$ as $\zeta_{k+1} = \infty$. Following our discussion in subsection III-B, we can set the lowermost threshold as $\bar{F}(\zeta_1)n = sk$, or, $\zeta_1 = \bar{F}^{-1}\left(\frac{sk}{n}\right)$. Continuing in the same way, we get $\zeta_2 = \bar{F}^{-1}\left(\frac{s(k-1)}{n}\right), \cdots\cdots, \zeta_k = \bar{F}^{-1}\left(\frac{s}{n}\right)$. The feedback procedure is as detailed in Table III.

## IV. THROUGHPUT IN THE RBF CASE

In this section, we present the sum-rate throughput achieved by the proposed schemes. Although we focus on RBF, the proposed scheme can be applied to other beamforming methods (e.g. ZFBF).

## A. Throughput in the Analog Feedback Case

The sum-rate throughput achieved in the RBF case with dedicated ideal feedback links is given by [4]

$$\mathcal{R} \approx \mathbb{E}\left[\sum_{m=1}^{p} \log_2(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,m})\right] = p\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,m})\right] \tag{7}$$

where "$\approx$" is used instead of "$=$" since there is a small probability that user $i$ may be the strongest user for more than one beam. Using extreme value theory, it is shown in [4] that (7) can be written as

$$\mathcal{R} = p\log_2(1 + \rho \log(n) - \rho(p-2)\log\log(n)) \tag{8}$$

For the proposed scheme, the throughput in the analog case $\mathcal{R}_a$ is given by

$$\mathcal{R}_a = p\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,m}) \mid \mathcal{A}, \mathcal{B}\right] \mathbb{P}(\mathcal{A} \cap \mathcal{B}) \tag{9}$$

where $\mathcal{A}$ is the event that CS is successful, and $\mathcal{B}$ is the event that for $1 \leq i \leq n$, $\mathbf{v}_i > \zeta$ is not a null set. As the events $\mathcal{A}$ and $\mathcal{B}$ are independent, we can write (9) as

$$\mathcal{R}_a = p\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,m}) \mid \mathcal{A}, \mathcal{B}\right] \mathbb{P}(\mathcal{A})\mathbb{P}(\mathcal{B}) \tag{10}$$

As mentioned earlier, $\mathbb{P}(\mathcal{A}) = 1 - \frac{2}{n}\left(\frac{1}{\sqrt{2\pi \log n}} + \frac{s}{n}\right)$. To calculate $\mathbb{P}(\mathcal{B})$, note that the complement set $\mathcal{B}^c$ is the event that the CQIs of all users are below the threshold $\zeta$. This happens with probability

$$\mathbb{P}(\mathcal{B}^{\mathcal{C}}) = \mathbb{P}(\max_{1 \leq i \leq n} \mathbf{v}_i \leq \zeta) = [1 - \bar{F}(\zeta)]^n = \left(1 - \frac{s}{n}\right)^n$$

Thus,

$$\mathcal{R}_a = p\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,m})\right]\left(1 - \frac{2}{n}\left(\frac{1}{\sqrt{2\pi \log n}}\right)\right)\left(1 - \left(1 - \frac{s}{n}\right)^n\right) \tag{11}$$

Due to noise, the fed back SINR might be higher than the actual one. This is due to the feedback channel noise, i.e. the feedback noise could add/subtract the original SINR value by a small value. As such, we need to estimate $A$ correctly and subtract the effect of noise on the SINR. As the SINRs fed back by the users are transmitted *as is* and the feedback links are noisy, there is a need to back off the noisy received SINRs based on the noise variance. Specifically, the least square (LS) refined estimate is given by

$$\mathbf{v}_S^{LS} = (\mathbf{A}_S^* \mathbf{A}_S)^{-1} \mathbf{A}_S^* \mathbf{y} \tag{12}$$

$$= (\mathbf{A}_S^* \mathbf{A}_S)^{-1} \mathbf{A}_S^* (\mathbf{A}\mathbf{v} + \mathbf{w}) \tag{13}$$

$$= \mathbf{v}_S + \mathbf{e}' \tag{14}$$

Here, the noise represented by $\mathbf{e}'$ is Gaussian as linear operations preserve the gaussian noise distribution[5]. A representative scalar equation of (14) takes the form

$$\text{SINR}' = \text{SINR} + e' \tag{15}$$

where SINR and SINR$'$ stand for the actual and noisy SINR's respectively and where $e'$ represents Gaussian noise. Now, if we decide to back off the received SINR by an amount $\triangle$, then the back-off efficiency ($\eta$) i.e. the probability that this backed off SINR is less than or equal to the actual SINR is given by $\eta = \mathbb{P}[\text{SINR}' - \triangle \leq \text{SINR}] = \mathbb{P}[e' \leq \triangle] = 1 - Q\left(\frac{\triangle}{\sigma_{e'}}\right)$. Thus, the effective throughput (with back-off on noisy SINR) can be written as

$$\mathcal{R}_{eff} = \left(1 - Q\left(\frac{\triangle}{\sigma_{e'}}\right)\right)\mathcal{R}_a \tag{16}$$

$$= p\left(1 - \frac{2}{n}\left(\frac{1}{\sqrt{2\pi \log n}}\right)\right)\left(1 - \left(1 - \frac{s}{n}\right)^n\right)\left(1 - Q\left(\frac{\triangle}{\sigma_{e'}}\right)\right)\log_2(\beta - \triangle) \tag{17}$$

where $\beta = 1 + \rho \log(n) - \rho(p-2)\log\log(n)$. We can now choose $\triangle$ to maximize $R_{eff}$. Differentiating $\mathcal{R}_{eff}$ w.r.t. $\triangle$ and setting it equal to 0 yields

$$Q\left(\frac{\triangle}{\sigma_{e'}}\right) + \left(\frac{\beta - \triangle}{\sqrt{2\pi}\sigma_{e'}}\right)\exp\left(-\frac{\triangle^2}{2\sigma_{e'}^2}\right)\log(\beta - \triangle) = 1 \tag{18}$$

The value of $\triangle$ (which is a function of $\sigma_{e'}$) that satisfies the above equation maximizes the effective throughput. It remains of course to calculate $\sigma_{e'}$ which we undertake in subsection V-B further ahead.

### B. Throughput in the Digital Feedback Case

The sum-rate throughput achieved by $p$ beams in the multiple thresholds ($k$ in number) based digital feedback case for RBF is given by

$$\mathcal{R}_d \approx p\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq k} \zeta_i) \mid \mathcal{A}, \mathcal{B}\right]\mathbb{P}(\mathcal{A})\mathbb{P}(\mathcal{B})$$

where $\max_{1 \leq i \leq k} \zeta_i$ is the lower limit of the CQI of the highest active threshold interval. Note that $\mathcal{A}$ and $\mathcal{B}$ are independent events, where $\mathcal{A}$ is the event that CS is successful and $\mathcal{B}$ is the event that $\max_{1 \leq i \leq k} \zeta_i$ is not a null set which basically means $\max_{1 \leq i \leq n} \mathbf{v}_i > \zeta_1$ is not a null set. Thus, $\mathbb{P}(\mathcal{A}) = 1 - \frac{2}{n}\left(\frac{1}{\sqrt{2\pi \log n}} + \frac{s}{n}\right)$ and $\mathbb{P}(\mathcal{B})\left(1 - \left(1 - \frac{sk}{n}\right)^n\right)$ so that

$$\mathcal{R}_d = p\left(1 - \left(1 - \frac{sk}{n}\right)^n\right)\left(1 - \frac{2}{n}\left(\frac{1}{\sqrt{2\pi \log n}} + \frac{s}{n}\right)\right)\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq k} \zeta_i)\right] \tag{19}$$

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[5]While the elements of $\mathbf{e}'$ are not necessarily independent, we will not use this fact in our backoff calculations. After all, these calculations are based on a worst case scenario. This will result in a larger backoff and a less optimistic achievable throughput.

We are left to evaluate $\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq k} \zeta_i)\right]$ which can be derived analytically as (also see [27])

$$\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq k} \zeta_i)\right] = \sum_{i=1}^{k} \log_2(1 + \zeta_i)\mathbb{P}(\text{selected user is in } Q_i)\mathbb{P}(Q_i) \tag{20}$$

The probability of the threshold interval $Q_i$ is given by $\mathbb{P}(Q_i) = [F(\zeta_{i+1}) - F(\zeta_i)]$, where $F(\zeta)$ is the cumulative distribution function (CDF) of CQI (SINR) defined as $F(\zeta) = 1 - \frac{\exp(-\zeta/\rho)}{(1+\zeta)^{p-1}}, \zeta \geq 0$ [4]. Note that the probability that the selected user is in the threshold interval $Q_i$ is given as where $\mathcal{P}_1 = \mathbb{P}(j \text{ users other than the selected user are in } Q_i) = [F(\zeta_{i+1}) - F(\zeta_i)]^j$, and $\mathcal{P}_2 = \mathbb{P}((n - j - 1) \text{ users lies below the interval } Q_i) = [F(\zeta_i)]^{(n-j-1)}$. Substituting the values of $\mathcal{P}_1$ and $\mathcal{P}_2$ in (20), we find after straight forward manipulations

$$\mathbb{P}(\text{selected user is in } Q_i) = \frac{[F(\zeta_{i+1})]^n - [F(\zeta_i)]^n}{n[F(\zeta_{i+1}) - F(\zeta_i)]}$$

so that

$$\mathbb{E}\left[\log_2(1 + \max_{1 \leq i \leq k} \zeta_i)\right] = \frac{1}{n}\sum_{i=1}^{k} \log_2(1 + \zeta_i)([F(\zeta_{i+1})]^n - [F(\zeta_i)]^n)\text{and}$$

$$\mathcal{R}_d = p\left(1 - \left(1 - \frac{sk}{n}\right)^n\right)\left(1 - \frac{2}{n}\left(\frac{1}{\sqrt{2\pi \log n}} + \frac{s}{n}\right)\right)\frac{1}{n}\sum_{i=1}^{k} \log_2(1 + \zeta_i)([F(\zeta_{i+1})]^n - [F(\zeta_i)]^n).$$

## V. PERFORMANCE EVALUATION

We consider the following metrics for the performance evaluation of the proposed feedback schemes.

### A. Feedback Resources Reduction

There is a significant reduction in number of feedback channels required for carrying feedback information. The proposed schemes requires only $O(\log(n))$ feedback channels as opposed to $n$ feedback channels required in the dedicated feedback case[6]. More precisely, we have shown in subsection III-C that the number of multiple access feedback channels should be at least $r = \frac{c}{2}(s \log(n))$.

---

[6]We would like to stress here that we are reducing the resources dedicated to the user's feedback. In the independent feedback scenario that is usually pursued in the literature, the number of feedback channels is always $n$ regardless of how small the feedback load is.

## B. Feedback Noise Reduction in the Analog Feedback Case

The other important benefit of this scheme is the feedback noise reduction (which eventually results in better throughput) in the analog feedback case. This is because the feedback data of each user is carried over all shared channels. This allows the base station to make use of the extra measurements available to reduce the effect of additive noise. This comes in contrast to dedicated channel feedback case where each user's feedback is carried over one channel only. In what follows, we study the error covariance that results from the LS refinement of the CS output. This will also allow us to determine the estimation error variance $\sigma_{\hat{\mathbf{w}}}$ required in Subsection IV-A for SNR backoff in the analog feedback.

*1) Shared Feedback Channels:* The error covariance matrix that results from the post-CS *LS* refinement of (12) is given by

$$\mathbf{R}_e = \mathbb{E}_{\mathbf{A}_s} \left[ \mathbf{R}_v^{-1} + \mathbf{A}_s^* \mathbf{R}_w^{-1} \mathbf{A}_s \right]^{-1} \tag{21}$$

where $\mathbf{R}_{\mathbf{v}} = \mathbb{E}[\mathbf{v}_s \mathbf{v}_s^*] = \sigma_{\mathbf{v}}^2 \mathbf{I}$, and $\mathbf{R}_{\mathbf{w}} = \mathbb{E}[\mathbf{w}_s \mathbf{w}_s^*] = \sigma_{\mathbf{w}}^2 \mathbf{I}$ and where $S$ refers to the support indicating the users who feedback (obtained using CS). Thus, the $S \times S$ covariance matrix reads

$$\mathbf{R}_e = \sigma_{\mathbf{v}}^2 \mathbb{E}_{\mathbf{A}_s} \left[ (I + \rho \mathbf{A}_s^* \mathbf{A}_s)^{-1} \right] \tag{22}$$

Here $\rho = \frac{\sigma_{\mathbf{v}}^2}{\sigma_{\mathbf{w}}^2}$ is the signal to noise ratio. It is difficult to calculate this expectation in closed form. To go around this, note that what really matters to us are the diagonal values of $\mathbf{R}_e$ which correspond to the effective estimated noise variance of the fed back SNR (see (19)). It is not difficult to see that the maximum diagonal value of $\mathbf{R}e$ can be upper bounded by

$$\max_{1 \leq i \leq s} \sigma_{\mathbf{e}}(i) \leq \sigma_{\mathbf{v}}^2 E_{\mathbf{A}_s} \left[ \frac{1}{1 + \rho \lambda_{min}(\mathbf{A}_s^* \mathbf{A}_s)} \right] \tag{23}$$

To calculate this expectation, we need to have the pdf of the minimum eigenvalue of the matrix $\mathbf{A}_s^* \mathbf{A}_s$. Now $\mathbf{A}_s$ is a $2r \times s$ matrix *i.i.d.* Gaussian matrix making $\mathbf{A}_s^* \mathbf{A}_s$ a real central Wishart matrix $W(s, 2r)$. It is difficult to obtain the *pdf* of $\lambda_{min}$ and evaluate the expectation above for general $s$[7], so we concentrate on the cases $s = 1$ and $s = 2$, and to understand the general trend we also consider the case of $s/2r = \beta$, where $r \to \infty$. Considering these cases makes sense given the fact that only a very low number of users are expected to feed back. We can show the following

1) For $s = 1$, we have (Figure 7)
$$E \left[ \frac{1}{1 + \rho \lambda_{min}} \right] = \frac{\rho^{-r} e^{1/2\rho}}{2^r} \Gamma(1 - r, 1/2a)$$

---

[7]Obtaining the pdf of $\lambda_{min}$ for even the simple case of $s = 2$ proves to be very difficult to evaluate.

2) For $s = 2$, we have (Figure 7)

$$E\left[\frac{1}{1+\rho\lambda_{min}}\right] = \frac{1}{(2r-1)!(2r-2)!}\left(\int_0^\infty \frac{\lambda^{2r-2}e^\lambda}{1+\rho\lambda}\Gamma(2r+1)d\lambda - 2\int_0^\infty \frac{\lambda^{2r-1}e^{-\lambda}}{1+\rho\lambda}\Gamma(2r,\lambda)d\lambda + \int_0^\infty \frac{\lambda^{2r}e^{-\lambda}}{1+\rho\lambda}\Gamma(2r-1,\lambda)d\lambda\right).$$

3) For $s/2r = \beta$, with $m \to \infty$, we have,(Figure 8)

$$E\left[\frac{1}{1+\rho\lambda_{min}}\right] = \frac{1}{1+2\rho r(1-\sqrt{\beta})^2}$$

*2) Dedicated Feedback Channels:* For the dedicated feedback case, the error covariance matrix is given by,

$$\text{ECM} = \left[\left(\frac{1}{\sigma_\mathbf{v}^2}\mathbf{I} + \frac{1}{\sigma_\mathbf{w}^2}\mathbf{I}\right)^{-1}\right] = \frac{\sigma_\mathbf{v}^2}{1+\rho}\mathbf{I} \tag{24}$$

Thus, from (24), we conclude that the back off on the SINR is $O(\frac{\sigma_\mathbf{w}}{\sqrt{r}})$ in the shared feedback channels case as opposed to $O(\sigma_\mathbf{w})$ in the dedicated feedback channel case.

### C. Feedback Load Reduction

In addition to the feedback resources reduction, there is a reduction in the amount of feedback. In RBF scheme with dedicated feedback channel [4], $n$ real values and $n$ integer values ($n\log_2 p$ bits) are fed back, as there are $n$ users in the system.

*1) Analog Feedback Case:* The proposed CS based analog feedback scheme requires only $pr$ real values to be fedback. This is because there are $r$ shared channels and the scheme is repeated for each beam. Note that the feedback load reduction is more dominant in systems with large number of users, as $r \sim O(\log(n))$ and $p$ is small.

*2) Digital Feedback Case:* The proposed CS based digital feedback scheme requires only $pkr$ bits to be fedback. This is because there are $r$ shared channels and the scheme is repeated for each beam & threshold. Note that the feedback load reduction is more dominant in systems with large number of users, as $r \sim O(\log(n))$ and $p$ & $k$ are small.

### D. Trade-off in the Digital Feedback Case

Given a budget of bits that can be fed back, using intuition, it was shown in [32] that trade-off exists between the multi-user diversity and feedback accuracy. In our context, multi-user diversity is related to the number of shared channels $r$ whereas feedback accuracy is related to the number of thresholds $k$, and so a similar trade-off may exist. The number of shared channels and thresholds must be chosen such that the throughput is maximized. This is explored using simulation in section VII.

## VI. FEEDBACK CHANNEL TRAINING

In the previous sections, we assumed that the channel $\mathbf{A}$ estimation is given to the system with the aid of a "genie" at no cost. In this section, we present how the feedback channel training can be accomplished and explore ways to reduce it. Here, we assume that (2) represents frequency feedback channels i.e., the entries of $\mathbf{A}$, $a_{ij}$ represents the gain of the $j$-th user in the $i$-th frequency band.

### A. Channel Matrix is Full

The optimal number of symbols required for channel training is at least equal to the number of transmit antennas [33]. So we need at least $p$ training symbols for the downlink channel and at least $n$ training symbol for the uplink channel (as there are $n$ users each having one transmit antenna). In this section, we will discuss the uplink training for determining the entries of $\mathbf{A}$ and give an insight on the effect of error in $\mathbf{A}$ on compressive sensing. Note that training for each user in the uplink is performed one by one. So without loss of generality, we will discuss the training for user 1.

Let us assume that training for user 1 is carried out using $\tau$ symbols for the $i$-th frequency channel. Same procedure is repeated for all frequency channel. Mathematically we can write it as

$$y_i = \sqrt{\rho} a_{i1} \mathbf{s}_\tau + w_i, \qquad i = 1, \ldots, r \qquad (25)$$

where $\mathbf{s}_\tau \in \mathbb{C}^{\tau \times 1}$ is the training data (known at the BS), $y_i$ is the received signal at the BS, $w_i$ is the gaussian noise and $a_{i1}$ is the feedback channel gain to be determined.

The channel gains can be computed using the linear minimum-mean-square-error approach given in [33]. Thus,

$$\hat{a_{i1}} = \sqrt{\frac{1}{\rho}} \left( \frac{1}{\rho} + \mathbf{s}_\tau{}^* \mathbf{s}_\tau \right)^{-1} \mathbf{s}_\tau{}^* y_i, \qquad i = 1, \ldots, r \qquad (26)$$

Thus, we can write

$$a_{i1} = \hat{a_{i1}} + \tilde{a_{i1}}, \qquad i = 1, \ldots, r \qquad (27)$$

where $\tilde{a_{i1}}$ is the zero-mean channel estimation error.

Note that training for all frequency channels can be done simultaneously. Thus, continuing in the same way for all users, we need $n\tau$ symbols for the training. Also, it is important to note that we need one symbol for sending feedback data, so almost all of the uplink coherence time can be used for feedback training. Coherence time is typically of the order of few thousand symbols, so training would not be an issue for systems with moderate number of users.

In [34], Tune et. al. showed that when compressive sensing is done using perfect **A** (i.e. with no estimation error), the number of channels satisfy the following

$$r_{perfect} \geq \frac{\log(n - k + 1)}{\log(1 + \rho \hat{a}^2)} \tag{28}$$

where the denominator is the capacity of the channel with perfect channel gain $\hat{a}$ of a user.

With estimation error, denominator in the above equation modifies to $\log(1 + \frac{\rho \hat{a}^2}{\rho \tilde{a}^2 + 1})$. Note that this is obtained assuming $\tilde{a}$ to be gaussian which is the worst case. Thus,

$$r \geq \frac{\log(n - k + 1)}{\log(1 + \frac{\rho \hat{a}^2}{\rho \tilde{a}^2 + 1})}. \tag{29}$$

From the above equation, we can infer that estimation error results in the increase in the number of channels (for a fixed $\rho$) by a factor

$$\frac{r}{r_{perfect}} = \frac{\log(1 + \rho a^2)}{\log(1 + \frac{\rho \hat{a}^2}{\rho \tilde{a}^2 + 1})}. \tag{30}$$

The other way to mitigate the effect of estimation error is by increasing the SNR. Let us assume that $\rho_{perfect}$ and $\rho$ be the SNR in (28) and (29) respectively. Then for a fixed number of channels, following equation must be satisfied

$$\rho_{perfect} = \frac{1}{a^2} \left( \frac{\rho \hat{a}^2}{\rho \tilde{a}^2 + 1} \right) \tag{31}$$

A method for reducing the amount of feedback channel training time is discussed in the next subsection.

## B. Channel Matrix is Block Diagonal

In order to reduce the feedback training time, we divide the users into groups with each group being allowed to feedback only on a set of feedback channels, thereby reducing the full channel matrix to a block diagonal one

$$\mathbf{A}_{BD} = \begin{bmatrix} \mathbf{A}_1 & & \\ & \ddots & \\ & & \mathbf{A}_k \end{bmatrix}.$$

Compressive sensing is applied in the same way as discussed in Section III, the only difference being that it is now applied on each block. Strong users in each block (or group) are found and the user corresponding to the maximum SINR among the strong users from all blocks is selected. As the users are i.i.d., so we divide the feedback resource equally among the $k$ groups. Thus, training can now be performed for each block simultaneously. This approach reduces the feedback training time considerably,

e.g. if we divide the total number of users into two groups, then the training will require $n/2$ symbol time as opposed to $n$ symbol time required for the case when the channel matrix is full.

The flip side of this approach is that compressive sensing is now applied on the group of users instead of all users as one block. Thus, for same sparsity level $s$ (overall), with block diagonalization, the number of feedback channels required is given below

$$f_{\mathbf{A}_{BD}} = f_{\mathbf{A}_1} + \cdots + f_{\mathbf{A}_k}$$
$$= k f_{\mathbf{A}_1}$$
$$= k \left[ \frac{1}{2} \left( c' \left( \frac{s}{k} \right) \log \left( \frac{n}{k} \right) \right) \right].$$

Note that from the above equation it may first appear that the number of channels have reduced as the quantity inside the logarithm is reduced by a factor of $k$, however, it is the other way round. This is because now $c'$ has increased as the problem dimension ($n$) is reduced by a factor of $k$ [23]. Thus, there is a trade-off between the reduction in the amount of feedback training and the number of feedback channels. Also, note that there is now an additional constraint requiring $s/k$ to be an integer.

*C. Non-fading Channels*

When the channels are non-fading i.e., the channel gains are constant (or 1), then each strong users multiples its CQI with a unique binary chip sequence (consisting of $\pm 1$ each with probability 0.5) of length equal to the number of shared feedback channels $r$ and send it over the multiple access shared channels. There are two ways of assigning chip sequences to the users: pre-programmed in users' device or sending it over the air. If it is send over the air, then the training time (used in the case of fading channels) can be used to send unique binary chip sequences to all $n$ users. Thus, $\mathbf{A}$ is $r \times n$ Bernoulli matrix and so CS can be applied as Bernoulli matrices are shown to satisfy the RIP [20].

## VII. NUMERICAL RESULTS

In this section, we present numerical results for CS-based feedback schemes by applying it in RBF context. We use $p = 4$ base station antennas, and $n = 100$ users. We set the threshold according to the sparsity level $s$, and use the maximum correlation technique (unless mentioned otherwise) for compressive sensing as this is much more computationally efficient than LASSO. Each point in the figures represents the sum-rate throughput achieved for shared number of channels determined by $c$ & $s$. We use SNR = 10 dB for both downlink and feedback link (unless stated otherwise) for calculating the sum-rate throughput.

*A. Analog Feedback Case*

In Fig. 2, we present the sum-rate throughput with shared channel feedback in the analog feedback case. We use optimum back off on noisy SINRs in the analog feedback case. From this figure, we note that for small values of $s$ the throughput is low. This is because the threshold works well for systems with large number of users but for systems with moderate number of users, we may have more or less number of users above the threshold than desired. So, if we set $s$ low, then the probability that a beam has no strong user is relatively higher (resulting in a multiplexing loss) to the case when $s$ is large. However, large values of $s$ requires more feedback channels. Also, we see that the number of shared channels required to achieve the maximum possible throughput obtained in a noisy dedicated feedback scenario is 11 (corresponds to $c/2 = 0.4$ and $s = 6$). Also, it worth mentioning that the proposed scheme comes close to achieving the throughput obtained in a noiseless dedicated feedback scenario (dedicated feedback with ideal feedback links) due to feedback noise reduction. Note that $90\%$ of throughput in noiseless dedicated feedback case is achieved by 19 shared channels (corresponds to $c/2 = 0.8$ and $s = 5$).

In Fig. 3, we present results on block digitalization method proposed for reducing the feedback training time (section VI-B). Here, we divide 100 users into two groups of 50 users and compressive sensing is applied on each group. It is clear from the figure that this method requires few more feedback channels. Also in this figure, we present result based on LASSO which shows that LASSO method performs marginally better than maximum correlation method.

In Fig. 4, we present the sum-rate throughput achieved by two-stage RBF in the analog feedback case when the feedback channels are noiseless. In the second-stage of the two-stage RBF, additional feedback information (beam gain information (BGI) [6]) is requested from the selected users only and Iterative Beam Power Control (IBPC) algorithm proposed in [6] is used for the re-distribution of the total power among the active beams (beams for which there are strong users) in an optimized manner. From the figure, we see that there is hardly any gain for two stage RBF with dedicated channel feedback, however, it is evident that for compressive sensing based opportunistic feedback protocol, two-stage RBF is effective even for moderate to large number of users. This is because if no user is strong for some beams, the system still suffers from the multiplexing loss but the power of those beam are distributed among the active beams in an optimized way. Also, note that there is no back-off required here as the feedback links are noiseless. With two-stage RBF, the number of shared feedback channels required is 15 (corresponds to $c/2 = 0.8$ and $s = 4$).

## B. *Digital Feedback Case*

For all digital feedback cases, we chose $s = 1$ (the minimum possible value) and set multiple thresholds as discussed in section III-E. This is because for the proposed scheme, $s = 1$ will allow us to set the highest possible uppermost threshold thereby ensuring a higher throughput.

In Fig. 5, we present the sum-rate throughput achieved with shared channel feedback in the digital feedback case. Is is evident form the figure that the proposed scheme in a noisy scenario achieves the throughput obtained in a noiseless dedicated feedback scenario (dedicated feedback with ideal feedback links). Also, we see that the throughput increases with the increase in the number of shared channels & thresholds. Taking the pessimistic view, we need only 10 feedback channels (corresponds to $c/2 = 2$ and $s = 1$). However, it is important to note that beyond a certain number of shared channels or thresholds, the throughput either becomes stagnant or increases marginally.

In Fig. 6, we consider fixed budgets of $p \times kr$ bits that can be fed back. From the figure, we note that such a trade-off exists and for a given fixed budget there is an optimum number of thresholds and shared feedback channels that maximizes the throughput.

## VIII. CONCLUSIONS

In this paper, a generic feedback channel model and compressive sensing based opportunistic feedback schemes are proposed. The proposed generic feedback channel model is shown to encompass all existing feedback channel models proposed in the literature. We have shown that the proposed analog & digital opportunistic feedback schemes achieves the same sum-rate throughput as that achieved by dedicated feedback schemes, but with feedback channels growing only logarithmically with number of users. Also, we derived an expression for the sum-rate throughput in the digital feedback case with multiple thresholds.

In the analog feedback case (noisy scenario), it has also been shown that due to feedback noise reduction, the proposed scheme comes close to achieving the throughput obtained in the case of noiseless dedicated feedback. In the digital feedback case, it has also been shown that beyond a certain number of shared channels or thresholds, the throughput either becomes stagnant or increases marginally. Also, given a budget on the amount of bits that can be fedback, we have shown that there exist a trade-off between the number of shared channels and thresholds and therefore they must be chosen such that the throughput is maximized.

Although the results presented here only show the performance of the the proposed schemes in the RBF context, the schemes can easily work with other beamforming methods.

# REFERENCES

[1] X. Qin and R. Berry "Distributed Approach for Exploiting Multiuser Diversity in Wireless Networks," *IEEE Trans. Inf. Theory*, vol.52, no. 2, pp. 392406, Feb.2006.

[2] W. Yu and J. Cioffi, "Sum Capacity of Gaussian Vector Broadcast Channels," *IEEE Trans. Inf. Theory*, vol. 50, pp. 1875–1892, Sept. 2004.

[3] Q. H. Spencer, C. B. Peel, A. L. Swindlehurst, and M. Haardt, "An Introduction to the Multi-User MIMO Downlink," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 60-67, Oct. 2004.

[4] M. Sharif and B. Hassibi, "On the Capacity of MIMO Broadcast Channels with Partial Side Information," *IEEE Trans. Inf. Theory*, vol. 51, pp. 506–522, Feb. 2005.

[5] T. Yoo, N. Jindal, and A. Goldsmith, "Multi-Antenna Downlink Channels with Limited Feedback and User Selection," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sept. 2007.

[6] M. Kountouris, D. Gesbert, and T. Salzer "Enhanced Multiuser Random Beamforming: Dealing with the Not So Large Number of Users Case," *IEEE J. Sel. Areas Commun., Sp. Issue on Limited Feedback Wir. Comm. Networks*, vol. 26, no. 8, pp. 1536–1545, Oct. 2008.

[7] J. Diaz, O. Simeone, and Y. Bar-Ness, "How Many Bits of Feedback is Multiuser Diversity Worth in MIMO Downlink?," *IEEE ISSST*, Manaus, Brazil, Aug. 2006, pp. 505–509.

[8] T. Tang and R. Heath, "Opportunistic Feedback for Downlink Multiuser Diversity," *IEEE Commun. Lett.*, vol. 9, pp. 948-950, Oct. 2005.

[9] R. Agarwal, C.S. Hwang, and J.Cioffi, "Opportunistic Feedback Protocol for Achieving Sum-Capacity of the MIMO Broadcast Channel," *IEEE VTC*, Oct. 2007, pp. 606–610.

[10] S. Bhaskaran, L. Davis, A. Grant, S. Hanly, P. Tune, "Downlink Scheduling Using Compressed Sensing," *Information Theory Workshop (ITW)*, Apr 2009.

[11] A. Lau and F. Kschischang, "Optimal feedback quantization schemes for multiuser diversity systems," *IEEE Trans. Inf. Theory*, 2004.

[12] Y. Xue, T. Kaiser, "Exploiting multiuser diversity with imperfect one-bit channel state feedback". *IEEE Transactions on Vehicular Technology*, 56(1), 2007, 183193.

[13] T. Eriksson, T. Ottosson, "Compression of Feedback for Adaptive Transmission and Scheduling", *Proceedings of the IEEE*, vol.95, no.12, pp.2314-2321, Dec. 2007.

[14] D. Gesbert and M.-S. Alouini, "How much feedback is multi-user diversity really worth?" *in Proc. IEEE International Conference onbibitem Communications* 2004, pp. 234238.

[15] M. Agarwal, M. Honig, "Spectrum Sharing on a Wideband Fading Channel with Limited Feedback," *Cognitive Radio Oriented Wireless Networks and Communications*, 2007. CrownCom 2007. 2nd

[16] Giuseppe Caire, Nihar Jindal, Mari Kobayashi, Niranjay Ravindran, "Multiuser MIMO Achievable Rates with Downlink Training and Channel State Feedback," *http://arxiv.org/abs/0711.2642*

[17] Song Bongyong, R. Cruz, L. Milstein, "Multiuser Diversity with Imperfect Channel Quality Feedback in MIMO Broadcast Networks," *IEEE International Conference on Communications, 2007*, vol., no., pp.4652-4657, 24-28 June 2007

[18] Jin Zhang, James Lehnert, "Throughput-Optimal Precoding and Rate Allocation for MISO Systems With Noisy Feedback Channels," *IEEE Transactions on Information Theory,* vol.54, no.5, pp.2139-2155, May 2008

[19] Young-Han Nam, Jianzhong Zhang, H. El-Gamal, T. Reid, "Opportunistic Communications with Distorted CSIT," *3rd International Symposium on Wireless Communication Systems, 2006*, vol., no., pp.16-20, 6-8 Sept. 2006

[20] E. Cands, J. Romberg, and T. Tao, "Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489-509, Feb. 2006.

[21] E. Candes and T. Tao,"Decoding by Linear Programming" *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4203–4215, Dec. 2005.

[22] Richard Baraniuk, "Compressive Sensing", *IEEE Signal Process. Mag.*, vol. 24, no. 4, pp. 118–121, July 2007.

[23] M. J. Wainwright, "Sharp Thresholds for High-dimensional and Noisy Sparsity Recovery Using $l_1$-Constrained Quadratic Programming," Technical Report 709, Department of Statistics, UC Berkeley, 2006. [Available at: www.stat.berkeley.edu/tech-reports/709.pdf]

[24] A. K. Fletcher, S. Rangan, and V. K. Goyal, "Necessary and Sufficient Conditions on Sparsity Pattern Recovery," [Available at: http://arxiv.org/abs/0804.1839]

[25] E. Candes and Y. Plan, "Near-ideal model selection by $\ell_1$ Minimization," 2008.

[26] G. Caire, T. Y. Al-Naffouri, and A. Narayanan, "Impulse noise cancellation in OFDM: An application of compressed sensing," ISIT, Toronto, Canada, July 2008.

[27] F. Floren, O. Edfors, and B. Molin, "The Effect of Feedback Quantization on the Throughput of a Multiuser Diversity Scheme," *IEEE GLOBECOM*, Dec. 2003, pp. 497–501.

[28] A. Bayesteh and A. K. Khandani, "How Much Feedback is Required in MIMO Broadcast Channels?," [Available at: http://arxiv.org/pdf/cs/0703143]

[29] A. H. Sayed, *Adaptive Filters*, John Wiley & Sons, 2008.

[30] E. Telatar, "Capacity of Multi-Antenna Gaussian Channel," *European Trans. Telecommunications*, vol. 10, pp. 585-595, Nov. 1999.

[31] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed., Academic Press, 2007.

[32] N. Ravindran and N. Jindal, "Multi-User Diversity vs. Accurate Channel Feedback for MIMO Broadcast Channels," *IEEE International Conference on Communications, 2008. ICC '08*, pp.3684-3688, 19-23 May 2008.

[33] B. Hassibi and B. M. Hochwald, "How Much Training is Needed in Multiple-Antenna Wireless Links?," *IEEE Trans. Inf. Theory*, vol. 49, pp. 951–963, April 2003.

[34] P. Tune, S. Bhaskaran, and S. Hanly, "Number of Measurements in Sparse Signal Recovery?," *IEEE ISIT*, June 2009, pp. 16–20.

[35] A. K. Fletcher, S. Rangan and V. K. Goyal, "On-Off Random Access Channels: A Compressed Sensing Framework," submitted to IEEE Trans. Information Theory and arXived.

[36] K, Pedersen, G. Monghal, I. Kovacs, T. Kolding, A. Pokhariyal, F. Frederiksen, P. Mogensen, "Frequency Domain Scheduling for OFDMA with Limited and Noisy Channel Feedback," Vehicular Technology Conference, 2007. VTC-2007 Fall. 2007 IEEE 66th , vol., no., pp.1792-1796, Sept. 30 2007-Oct. 3 2007
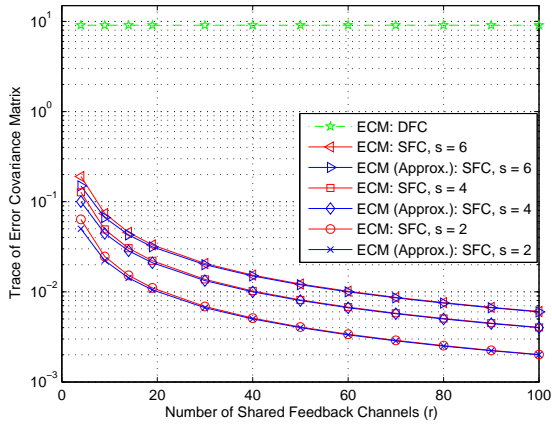
Figure 1. Trace of Error Covariance Matrix for shared feedback channel (SFC) and dedicated feedback channel (DFC), $n = 100$ and uplink SNR = 10 dB for different values of $s$.
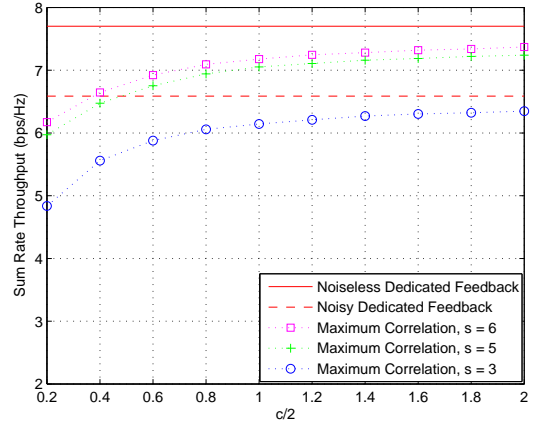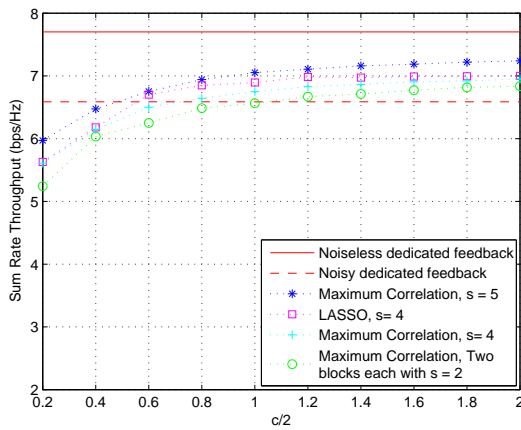


Figure 2. Analog Shared Channel Feedback: Throughput versus c/2 for RBF, $p = 4$, $n = 100$ and SNR = 10 dB (both downlink & feedback link) for different values of $s$.



Figure 3. Analog Shared Channel Feedback: Throughput versus c/2 for RBF, $p = 4$, $n = 100$ and SNR = 10 dB (both downlink & feedback link) for different methods viz. LASSO, maximum correlation, and maximum correlation with block diagonalization.
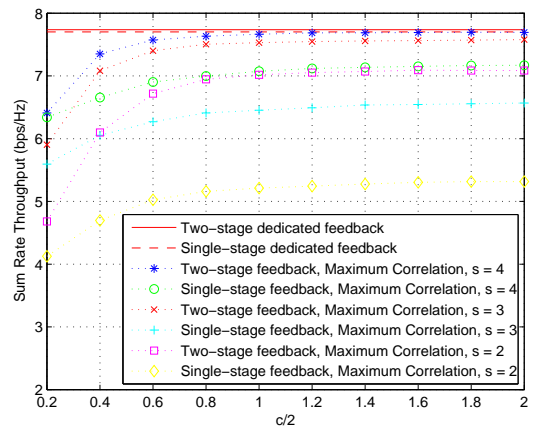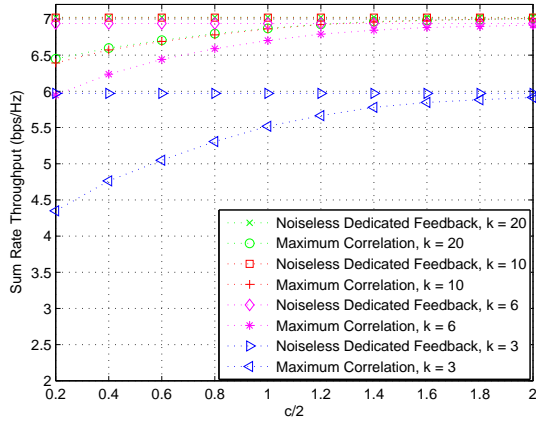


Figure 4. Analog Shared Channel Feedback: Throughput versus c/2 for Single stage and Two stage RBF, $p = 4$, $n = 100$ and SNR = 10 dB & $\infty$ for downlink and feedback link respectively.

Figure 5. Digital Shared Channel Feedback: Throughput versus c/2 for RBF, $p = 4$, $n = 100$ and SNR = 10 dB (both downlink & feedback link) for different values of $k$.
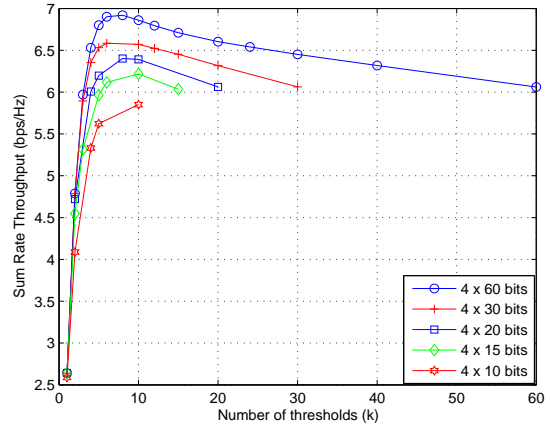


Figure 6. Digital Shared Channel Feedback: Throughput versus k for RBF, $p = 4$, $n = 100$ and SNR = 10 dB (both downlink & feedback link) for different budgets of bits that are to be fedback.
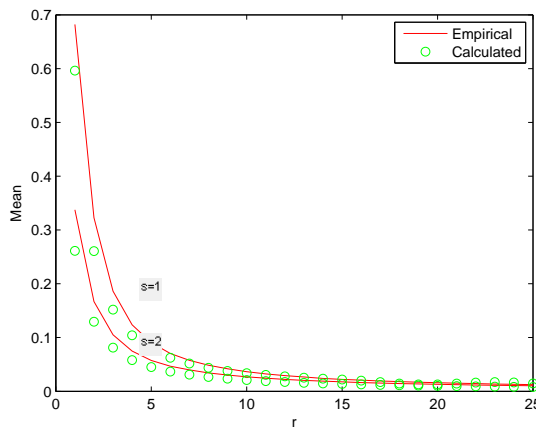


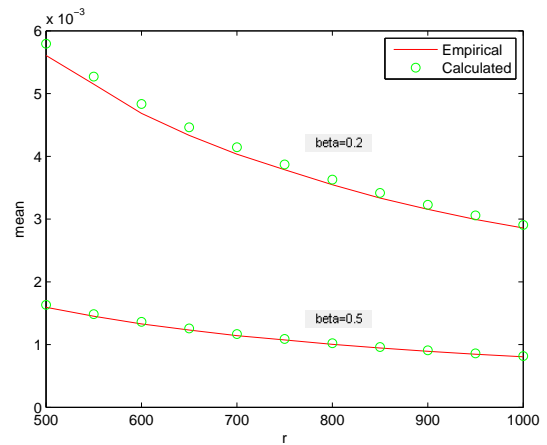Figure 7. Actual and simulated value of expected mean vs $r$ for $s = 1$, $s = 2$ and $\alpha = 2$



Figure 8. Empirical and simulated value of expected mean vs $r$ for $\beta = 0.2$, $\beta = 0.5$ and $\alpha = 2$

| Reference | Feedback Protocol | Entries of A | Noise variance ($\sigma^2$) | BF Type | Feedback Components |
|---|---|---|---|---|---|
| Sharif et. al. [4] | Dedicated | const. | 0 | RBF | BI & SINR |
| Yoo et. al. [5] | Dedicated | const. | 0 | ZFBF | QCI & SNR/SINR |
| Kountouris et al. [6] | Dedicated | const. | 0 | RBF (1st stage) | BI & SINR |
|  |  |  |  | RBF (2nd stage) | BGI |
| Kountouris et al. [6] | Dedicated | const. | 0 | ZFBF | QCI & SINR |
| Diaz et. al. [7] | Dedicated | const. | 0 | RBF | 1 bit |
| Tang et. al. [8] | Opportunistic | const. | 0 | SISO case | ID |
| Rajiv et. al. [9] | Opportunistic | const. | 0 | RBF | ID |
|  |  |  |  | ZFBF | ID & QCI |
| Proposed | Opportunistic | $\mathcal{CN}(0,1)$ | $> 0$ | RBF | CQI (Analog Case) |
|  |  |  |  | RBF | 1 bit (Digital Case) |

Table I

GENERIC FEEDBACK CHANNEL MODEL

Table II

PROPOSED COMPRESSIVE SENSING BASED OPPORTUNISTIC FEEDBACK PROTOCOL FOR ANALOG FEEDBACK.

1) **Threshold Determination:** BS decides on thresholding level $\zeta$ based on the sparsity level that can be recovered.

2) **User Feedback:** Repeat the following steps for each beam.
   - CQI Determination: Each user determines its best beam (corresponding to the highest CQI value).
   - CQI Feedback: Each user feeds back its CQI if it is higher than $\zeta$ on all shared channels. Otherwise, the user remains silent.
   - Compressive Sensing: BS finds the strong users using CS.
   - Least-squares estimation/refining: BS estimates or refines results obtained via CS using least-squares.
   - Optimum CQI Back off: BS backs off on the noisy CQI (SINR) based on the noise variance such that the throughput is maximized.

3) **User Selection:** Select users and schedule them to beams.

Table III

PROPOSED COMPRESSIVE SENSING BASED OPPORTUNISTIC FEEDBACK PROTOCOL FOR THE DIGITAL FEEDBACK.

1) **Threshold Determination:** BS decides on thresholding levels $\zeta_1, \zeta_2, ..., \zeta_k$ based on the sparsity level that can be recovered. For each threshold interval $[\zeta_i, \zeta_{i+1})$, repeat the *User Feedback* step.

2) **User Feedback:** Repeat the following steps for each beam.
   - CQI Determination: Each user determines its best beam (corresponding to the highest CQI value).
   - CQI Feedback: Each user feeds back its CQI if it lies in threshold interval $[\zeta_i, \zeta_{i+1})$ on all shared channels. Otherwise, the user remains silent.
   - Compressive Sensing: BS finds the strong users using Compressive Sensing.

3) **User Selection:** For each beam, BS randomly selects one of strong users of the highest active threshold interval, where active threshold interval here means that there is at least one user sending feedback data in the interval. Here, CQI is the lower limit of the highest active threshold interval.