NARROW BAND INTERFERENCE CANCELATION IN OFDM: A STRUCTURED MAXIMUM LIKELIHOOD APPROACH

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ABSTRACT

This paper presents a maximum likelihood (ML) approach to mitigate the effect of narrow band interference (NBI) in a zero padded orthogonal frequency division multiplexing (ZP-OFDM) system. The NBI is assumed to be time variant and asynchronous with the frequency grid of the ZP-OFDM system. The proposed structure based technique uses the fact that the NBI signal is sparse as compared to the ZP-OFDM signal in the frequency domain. The structure is also useful in reducing the computational complexity of the proposed method. The paper also presents a data aided approach for improved NBI estimation. The suitability of the proposed method is demonstrated through simulations.

1. INTRODUCTION

The vast deployment of wireless communication systems coupled with the congestion of available spectrum has resulted in various systems operating in the same or overlapping frequency bands. For instance, the unlicensed ISM band, used by several devices like cordless phones, garage door openers, baby monitors etc., also falls within the operating range of WiFi, WiMAX, Bluetooth, and Zigbee. As a result, today's wireless systems are very likely to experience interference from other devices/systems. This interference results in severe degradation in performance of the affected communication system.

Several works in literature deal with NBI cancelation, for instance [1]-[5]. The approach of [1] is based on finding a linear minimum mean square error solution and requires placement of a few unmodulated tones near the center frequency of the NBI; a requirement that might not be fulfilled in most scenarios. The approach presented in [2] is based on compressed sensing. Both [3] and [4] aim to notch out the tones where the NBI is present, meaning those tones will not be used for data transmission by the communication system. The approach of [5] is based on successive interference cancelation and hence suffers from error propagation.

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In this paper, we present a low complexity receiver design for ZP-OFDM system based on a structured maximum likelihood approach that mitigates the effect of narrow band interference. Our method estimates the NBI and cancels it from the received signal instead of notching out the interference tones. We consider a sophisticated model for the interferer with grid offset and allow the NBI to change from one symbol to the next. We use the guard interval of the ZP-OFDM symbol to obtain an ML estimate of the NBI. We make use of the rich structure inherent to the OFDM system and show how the computational complexity of the ML approach can be reduced significantly. Although the method of [2] is also based on the sparsity of NBI, it is quite different from the approach presented here as it casts the NBI estimation problem as a compressed sensing problem that requires the solution of a second order cone program (SCOP) for each estimate of NBI. It also does not utilize the structure of the measurement matrix.

The rest of the paper is organized as follows. Section 2 presents the system model. Section 3 formulates the NBI cancelation problem as a sparse signal estimation problem. Section 4 presents the proposed method. Section 5 presents the simulation results and Section 6 gives the conclusion.

2. SYSTEM MODEL

We consider a wireless ZP-OFDM system operating in the presence of a narrow band interferer. Let N be the number of carriers available for data transmission and M be the length of guard interval consisting of trailing zeros with $N + M = P$. Let $\mathbf{F}_N^{\mathrm{H}}$ be the $N \times N$ inverse discrete Fourier transform (IDFT) matrix where $(.)^{\mathrm{H}}$ indicates the Hermitian operation. Mathematically, the NBI affected system is given as

$$\mathbf{y} = \mathbf{H}\mathbf{F}_{\mathrm{zp}}\boldsymbol{\mathcal{S}} + \mathbf{x}_{\mathrm{eq}} + \mathbf{n} \tag{1}$$

where $\mathbf{H}$ is the time domain channel convolutional matrix of size $P \times P$, $\mathbf{F}_{\text{zp}}$ is zero padding transform matrix of size $P \times N$ and defined as $\mathbf{F}_{\text{zp}} := [\mathbf{F}_N \;\; \mathbf{0}_{N\times M}]^{\text{H}}$, $\boldsymbol{\mathcal{S}}$ is the $N \times 1$ frequency domain[1] data vector while $\mathbf{y}$, $\mathbf{x}_{\text{eq}}$ and $\mathbf{n}$ are time domain received signal, NBI signal and additive white Gaussian noise (AWGN) vectors of size $P \times 1$ each. The AWGN noise vector is modeled as $\mathbf{n} \sim \mathcal{CN}(0, \sigma_n^2\mathbf{I})$ while the time domain interference signal, $\mathbf{x}_{\text{eq}}$, is modeled as

$$\mathbf{x}_{\text{eq}} \;\; = \;\; \boldsymbol{\Lambda}_{\text{fo}}\mathbf{H}_x\mathbf{F}_P^{\text{H}}\bar{\boldsymbol{\mathcal{X}}} \tag{2}$$

where $\mathbf{H}_x$ is the time domain channel convolution matrix of the interferer, $\boldsymbol{\Lambda}_{\text{fo}}$ is a diagonal matrix that models the frequency offset between the ZP-OFDM system and the NBI signal. Each individual diagonal entry of $\boldsymbol{\Lambda}_{\text{fo}}$ is defined as $\exp(i\frac{2\pi\alpha n}{P})$ for $n = 0, 1, \cdots, P-1$ where α is a random number uniformly distributed over the interval $[-\frac{1}{2}, \frac{1}{2}]$. $\bar{\boldsymbol{\mathcal{X}}}$, is the r-sparse (contains only r non zero elements) NBI signal. The sparsity assumption is justified for a narrow band interferer.

As $\mathbf{H}_x$ is semi definite positive Hermitian Toeplitz matrix, it can be approximated to be nearly circulant for a large enough P [6] and thus (2) can be written as

$$\begin{aligned}\mathbf{x}_{\text{eq}} \;\; &= \;\; \boldsymbol{\Lambda}_{\text{fo}}\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}_x\bar{\boldsymbol{\mathcal{X}}} \\ &= \;\; \boldsymbol{\Lambda}_{\text{fo}}\mathbf{F}_P^{\text{H}}\boldsymbol{\mathcal{X}}\end{aligned} \tag{3}$$

where we observe that $\boldsymbol{\Lambda}_x = \mathbf{F}_P\mathbf{H}_x\mathbf{F}_P^{\text{H}}$ is a diagonal matrix representing the frequency domain channel of the interferer. We define effective NBI as $\boldsymbol{\mathcal{X}} := \boldsymbol{\Lambda}_x\bar{\boldsymbol{\mathcal{X}}}$ where both $\bar{\boldsymbol{\mathcal{X}}}$ and $\boldsymbol{\mathcal{X}}$ are r-sparse. For a given support (indices of the non zero entries), the effective interference signal $\boldsymbol{\mathcal{X}}$ is complex Gaussian with zero mean and variance σ_x^2. Our aim is thus to estimate the combined effect of the interferer channel and NBI signal, i.e. $\boldsymbol{\mathcal{X}}$, as observed by the ZP-OFDM receiver.

3. FORMULATING SPARSE SIGNAL ESTIMATION PROBLEM

Observe that $\mathbf{H}$ in (1) is a circulant matrix owing to the structure of ZP OFDM system [7] and therefore, can be decomposed as $\boldsymbol{\Lambda} = \mathbf{F}_P\mathbf{H}\mathbf{F}_P^{\text{H}}$ where $\boldsymbol{\Lambda}$ is a diagonal matrix representing the frequency domain channel of the user. Transforming the received signal to frequency domain yields

$$\begin{aligned}\mathbf{F}_P\mathbf{y} \;\; &= \;\; \mathbf{F}_P\mathbf{H}\mathbf{F}_{\text{zp}}\boldsymbol{\mathcal{S}} + \mathbf{F}_P\mathbf{x}_{\text{eq}} + \mathbf{F}_P\mathbf{n} \\ \boldsymbol{\mathcal{Y}} \;\; &= \;\; \boldsymbol{\Lambda}\mathbf{F}_P\mathbf{F}_{\text{zp}}\boldsymbol{\mathcal{S}} + \mathbf{F}_P\mathbf{x}_{\text{eq}} + \boldsymbol{\mathcal{N}}\end{aligned} \tag{4}$$

where $\boldsymbol{\mathcal{Y}}, \boldsymbol{\mathcal{N}}$ are frequency domain received signal and AWGN noise vectors, respectively.

The zero padded portion of the received ZP-OFDM symbol (last M samples) is corrupted by ISI and NBI. In absence of ISI, the zero padding can be used for support estimation of the NBI source. This is achieved by equalizing the received signal in the frequency domain, transforming it to time domain and selecting the last M samples. Mathematically,

$$\begin{aligned}\mathbf{S}_T\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}^{-1}\boldsymbol{\mathcal{Y}} &= \mathbf{S}_T\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}^{-1}\boldsymbol{\Lambda}\mathbf{F}_P\mathbf{F}_{\text{zp}}\boldsymbol{\mathcal{S}} + \\ &\quad \mathbf{S}_T\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}^{-1}\mathbf{F}_P\mathbf{x}_{\text{eq}} + \mathbf{S}_T\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}^{-1}\boldsymbol{\mathcal{N}} \\ &= \mathbf{0} + \mathbf{S}_T\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}^{-1}\mathbf{F}_P\boldsymbol{\Lambda}_{\text{fo}}\mathbf{F}_P^{\text{H}}\boldsymbol{\mathcal{X}} + \mathbf{n}' \\ \mathbf{y}' &= \Psi\boldsymbol{\mathcal{X}} + \mathbf{n}'\end{aligned} \tag{5}$$

where $\mathbf{S}_T = [\mathbf{0}_{M\times N} \quad \boldsymbol{I}_M]$ is a $M \times P$ selection matrix, $\mathbf{y}'$ is the $M \times 1$ time domain observation vector free of ISI, $\Psi := \mathbf{S}_T\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}^{-1}\mathbf{F}_P\boldsymbol{\Lambda}_{\text{fo}}\mathbf{F}_P^{\text{H}}$ is the $M \times P$ measurement matrix and $\mathbf{n}'$ is the $M \times 1$ time domain colored noise vector. Note that we can also do minimum mean square error equalization in (5) instead of least squares (LS) equalization to avoid the noise enhancement drawback of the LS equalizer. In that case the measurement matrix would be defined as

$$\Psi := \mathbf{S}_T\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}^{\text{H}}\left[\boldsymbol{\Lambda}\boldsymbol{\Lambda}^{\text{H}} + \sigma_n^2\mathbf{I}\right]^{-1}\mathbf{F}_P\boldsymbol{\Lambda}_{\text{fo}}\mathbf{F}_P^{\text{H}} \tag{6}$$

4. MAP BASED NBI CANCELATION

We start by assuming $\alpha = 0$, corresponding to no offset between the grids of the OFDM system and the NBI source and then extend the approach for any arbitrary α. Note that $\alpha = 0$ yields $\boldsymbol{\Lambda}_{\text{fo}} = \boldsymbol{I}$ which simplifies the measurement matrix to

$$\Psi = \mathbf{S}_T\mathbf{F}_P^{\text{H}}\boldsymbol{\Lambda}^{-1} \tag{7}$$

Next we find the maximum a posteriori probability (MAP) estimate of the support and then for the chosen support, calculate the linear minimum mean square error estimate of the NBI.

Let Γ denote the set of all possible supports and $\mathfrak{X}$ denote a particular support of NBI, then the MAP estimate of the support is the one that maximizes the objective function

$$\arg\max_{\mathfrak{X}} p(\mathfrak{X}|\mathbf{y}') = \arg\max_{\mathfrak{X}} p(\mathbf{y}'|\mathfrak{X})p(\mathfrak{X}) \tag{8}$$

where $p(\mathfrak{X})$ is the prior, and hence known at the receiver while $p(\mathbf{y}'|\mathfrak{X})$ is the likelihood that needs to be evaluated. For an NBI signal of unknown sparsity r, we have[2] $|\Gamma| = \sum_{r=0}^{r_{\max}}\binom{P}{r}$ where $|\Gamma|$ denotes the cardinality of Γ. Its evident that the complexity of the naive exhaustive search quickly becomes prohibitive.

As $\mathbf{y}'$ is a linear function of $\boldsymbol{\mathcal{X}}$, so for a given support $\mathfrak{X}$ and observation vector $\mathbf{y}'$, the NBI estimate $\hat{\boldsymbol{\mathcal{X}}}$ is given as

$$\begin{aligned}\hat{\boldsymbol{\mathcal{X}}} = \mathbb{E}[\boldsymbol{\mathcal{X}}|\mathbf{y}', \mathfrak{X}] &= \sigma_x^2\Psi_{\mathfrak{X}}^{\text{H}}\left[\sigma_x^2\Psi_{\mathfrak{X}}\Psi_{\mathfrak{X}}^{\text{H}} + \sigma_n^2\Psi\Psi^{\text{H}}\right]^{-1}\mathbf{y}' \\ &= \sigma_x^2\Psi_{\mathfrak{X}}^{\text{H}}\boldsymbol{\Sigma}(\mathfrak{X})^{-1}\mathbf{y}'\end{aligned} \tag{9}$$

where $\boldsymbol{\Sigma}(\mathfrak{X}) := \left[\sigma_x^2\Psi_{\mathfrak{X}}\Psi_{\mathfrak{X}}^{\text{H}} + \sigma_n^2\Psi\Psi^{\text{H}}\right]$ and $\Psi_{\mathfrak{X}}$ is a matrix consisting of only those columns of Ψ that correspond to the indices of $\mathfrak{X}$.

[1] We use calligraphic notation to denote vectors in frequency domain.

[2] $r_{\max}$ is chosen such that the sparsity assumption is valid, typically $r_{\max} < M/4$.

When NBI given the support $\boldsymbol{\mathcal{X}}|\mathfrak{X}$ is Gaussian, $\mathbf{y}'|\mathfrak{X}$ is also Gaussian [8]. Assuming the receiver has knowledge of second order statistics of the NBI source, the likelihood $p(\mathbf{y}'|\mathfrak{X})$ is evaluated as

$$p(\mathbf{y}'|\mathfrak{X}) = \frac{1}{(2\pi)^{r/2}\det(\mathbf{\Sigma}(\mathfrak{X}))}\exp\Big(- \frac{1}{2}\mathbf{y}'^{\mathrm{H}}[\mathbf{\Sigma}(\mathfrak{X})]^{-1}\mathbf{y}'\Big) \tag{10}$$

The problems in calculating the optimum MAP solution are i) the search space is huge ($|\Gamma| = \sum_{r=0}^{r_{\max}}\binom{P}{r}$) ii) it involves inverting a matrix of size $M \times M$ for each support combination and iii) its computationally expansive to repeatedly calculate the likelihood (10).

Next we show how structure can be used to efficiently calculate an approximate MAP estimate.

4.1. Exploiting structure

The measurement matrix Ψ has a very rich structure owing to the fact that its a partial DFT matrix. To this end, let $\beta := \frac{P}{M}$. As Ψ is a fat matrix ($M < P$), its columns are linearly dependent. However, a subset of M columns are independent such that by collecting these M columns together, we can form a matrix Ψ_M of full column rank. Particularly, if we choose $M = \frac{N}{l}$ for some integer l then it turns out that every kth column is *highly correlated* to its neighboring $\pm(\beta - 1)$ columns (i.e. with its $2\beta - 2$ neighboring columns) and (semi) orthogonal to the remaining distant columns. The correlation beyond $\pm k'\beta$ is small and can be neglected [9]. Based on this, we can make a cluster of size $2\beta - 1$ centered around the kth column.

This rich structure of Ψ is very helpful to limit the search space of Γ. Specifically, if we project the transpose of observation vector of (5) onto Ψ, the magnitude of the individual entries of the resulting vector

$$\mathbf{c} = \mathbf{y}'^{\mathrm{T}}\Psi \tag{11}$$

correspond to the likelihood of the NBI support being located in a cluster centered around it (where $(.)^{\mathrm{T}}$ denotes the transpose). Thus if the mth entry of $\mathbf{c}$ is the largest, it means the cluster centered on the mth column of Ψ is a candidate for the true support of $\boldsymbol{\mathcal{X}}$. If two clusters overlap or are close by, they are lumped together to form a single cluster. Selecting the largest T entries of $\mathbf{c}$ reduces the search space to only T non overlapping clusters within Ψ. Thus, the matrix $\Psi_{\mathfrak{X}}$ can be written as a combination of T clusters as

$$\Psi_{\mathfrak{X}} = [\Psi_{\mathfrak{X}_1}\Psi_{\mathfrak{X}_{T'}}] = [\Psi_{\mathfrak{X}_1}\Psi_{\mathfrak{X}_2}\cdots\Psi_{\mathfrak{X}_T}] \tag{12}$$

where $\mathfrak{X}_t$ is the support set corresponding to the tth cluster. This allows us to rewrite (5) as

$$\mathbf{y}' = [\Psi_{\mathfrak{X}_1}\Psi_{\mathfrak{X}_2}\cdots\Psi_{\mathfrak{X}_T}]\begin{bmatrix}\boldsymbol{\mathcal{X}}_1\\ \boldsymbol{\mathcal{X}}_2\\ \vdots\\ \boldsymbol{\mathcal{X}}_T\end{bmatrix} + \mathbf{n}' \tag{13}$$

The MAP estimate of the support within cluster t is obtained by maximizing the objective function

$$\arg\max_{\mathfrak{X}_t} p(\mathfrak{X}_t|\mathbf{y}') = \arg\max_{\mathfrak{X}_t} p(\mathbf{y}'|\mathfrak{X}_t)p(\mathfrak{X}_t) \tag{14}$$

where $|\Gamma_t| = \sum_{r=0}^{\acute{r}}\mathbf{C}_r^{(2\beta-1)}$, $\acute{r}$ is the maximum number of possible support within a cluster and is a design parameter. Note that $|\Gamma_t| \ll |\Gamma|$.

The semi-orthogonal structure of the measurement matrix allows us to reuse calculations from one cluster to another. Next we show how $\mathbf{\Sigma}(\mathfrak{X})$ and $\mathbf{\Sigma}(\mathfrak{X})^{-1}$ can be calculated recursively exploiting the structure. From (9), we have

$$\begin{aligned}\mathbf{\Sigma}(\mathfrak{X}) &= \sigma_x^2\Psi_{\mathfrak{X}}\Psi_{\mathfrak{X}}^{\mathrm{H}} + \sigma_n^2\Psi\Psi^{\mathrm{H}}\\ &= \sigma_x^2\Psi_{\mathfrak{X}_1}\Psi_{\mathfrak{X}_1}^{\mathrm{H}} + \sigma_x^2\Psi_{\mathfrak{X}_{T'}}\Psi_{\mathfrak{X}_{T'}}^{\mathrm{H}}\\ &\quad + \sigma_n^2\Psi\Psi^{\mathrm{H}}\\ &= \mathbf{\Sigma}(\mathfrak{X}_1) + \sigma_x^2\Psi_{\mathfrak{X}_{T'}}\Psi_{\mathfrak{X}_{T'}}^{\mathrm{H}}\end{aligned} \tag{15}$$

where we observe that

$$\mathbf{\Sigma}(\mathfrak{X}_1) = \sigma_x^2\Psi_{\mathfrak{X}_1}\Psi_{\mathfrak{X}_1}^{\mathrm{H}} + \sigma_n^2\Psi\Psi^{\mathrm{H}} \tag{16}$$

Using the matrix inversion lemma, the inverse can be written as in (17). Inserting (16) in (17) and observing that product of two semi-orthogonal clusters is approximately zero, $\Psi_{\mathfrak{X}_1}^{\mathrm{H}}\Psi_{\mathfrak{X}_{T'}} = \Psi_{\mathfrak{X}_{T'}}^{\mathrm{H}}\Psi_{\mathfrak{X}_1} \approx \mathbf{0}$, it can be shown that

$$\mathbf{\Sigma}(\mathfrak{X})^{-1} \approx \sum_{t=1}^{T}\left(\sigma_x^2\Psi_{\mathfrak{X}_t}\Psi_{\mathfrak{X}_t}^{\mathrm{H}} + \sigma_n^2\Psi\Psi^{\mathrm{H}}\right)^{-1} \tag{18}$$

Similarly it can be shown that the determinant of $\mathbf{\Sigma}(\mathfrak{X})$ is approximately given by

$$\det(\mathbf{\Sigma}(\mathfrak{X})) \approx \prod_{t=1}^{T}\det(\mathbf{\Sigma}(\mathfrak{X}_t) \tag{19}$$

The likelihood calculation can also be reduced in a similar fashion. Let the likelihood of cluster t for the Gaussian case be denoted by $\mathfrak{L}_t$ given as

$$\mathfrak{L}_t = \frac{1}{(2\pi)^{r/2}\det(\mathbf{\Sigma}(\mathfrak{X}_t))}\exp\Big(- \frac{1}{2}\mathbf{y}'^{\mathrm{H}}[\mathbf{\Sigma}(\mathfrak{X}_t)]^{-1}\mathbf{y}'\Big) \tag{20}$$

then the likelihood in (10) can be approximated as

$$\mathfrak{L} = p(\mathbf{y}'|\mathfrak{X}) \approx \prod_{t=1}^{T}\mathfrak{L}_t \tag{21}$$

This result has far reaching effects. It shows that we can evaluate the likelihood in parts over individual clusters. The semi-orthogonal structure, thus, allows us to reduces the complexity of the MAP search dramatically as instead of performing a joint search over the entire length P for the support of $\boldsymbol{\mathcal{X}}$, as in (10), we can search within each of the T clusters independent of the other $T - 1$ clusters, and still obtain almost the same MAP estimate.

$$\mathbf{\Sigma}(\mathfrak{X})^{-1} = \mathbf{\Sigma}(\mathfrak{X}_1)^{-1} - \sigma_x^2 \mathbf{\Sigma}(\mathfrak{X}_1)^{-1} \Psi_{\mathfrak{X}_{T'}} \left[\mathbf{I} + \sigma_x^2 \Psi_{\mathfrak{X}_{T'}}^{\mathrm{H}} \mathbf{\Sigma}(\mathfrak{X}_1)^{-1} \Psi_{\mathfrak{X}_{T'}} \right]^{-1} \Psi_{\mathfrak{X}_{T'}}^{\mathrm{H}} \mathbf{\Sigma}(\mathfrak{X}_1)^{-1} \tag{17}$$

4.2. Estimation of NBI in the presence of grid offset

So far we had assumed $\alpha = 0$, i.e., perfect grid alignment between the NBI source and OFDM system. Now we show how to solve the general case in the presence of grid offset. Assume the range of α is divided into $2\xi + 1$ discrete levels where ξ is some positive integer and that α can lie randomly on any of these levels. Let $\mathbf{q}_\alpha$ be a vector of length P formed by collecting the diagonal entries of $\mathbf{\Lambda}_{\mathrm{fo}}$, then

$$\mathbf{\Lambda}_{\mathrm{fo}} \mathbf{F}_{\mathbf{P}}^{\mathrm{H}} = [\mathbf{q}_\alpha \odot \mathbf{a_0} \quad \mathbf{q}_\alpha \odot \mathbf{a_1} \quad \cdots \quad \mathbf{q}_\alpha \odot \mathbf{a_{(P-1)}}] \tag{22}$$

where $\mathbf{a}_j$ is the $j\mathrm{th}$ column of the IDFT matrix and $\odot$ denotes the Hadamard product. Next we expand the vector $\boldsymbol{\mathcal{X}}$ from a vector of size $P \times 1$ to one of size $P(2\xi + 1) \times 1$ and search for the most probable support. We call the new vector $\underline{\boldsymbol{\mathcal{X}}}$. The sensing matrix is also expanded from size $M \times P$ to $M \times (P(2\xi + 1))$ as

$$\Psi = \mathbf{S}_T \mathbf{F}_P^{\mathrm{H}} \mathbf{\Lambda}^{-1} \mathbf{F}_P \mathbf{A} \tag{23}$$

where $\mathbf{A}$ is given in (24) and is a $P \times (P(2\xi + 1))$ matrix. Quasi-orthogonality continues to apply. Thus if $\mathbf{a}_i$ and $\mathbf{a}_j$ are semi-orthogonal, $\mathbf{q}_\alpha \odot \mathbf{a}_i$ and $\mathbf{q}_\alpha \odot \mathbf{a}_j$ are also semi-orthogonal.

4.3. Data aided estimation of NBI

In data aided approach, we start by obtaining an initial estimate of the NBI based on the MAP approach. Next we remove the estimated NBI from the received signal and get an initial estimate of the data, $\hat{\boldsymbol{\mathcal{S}}}$. This is done by equalizing the resulting signal corrupted by residual NBI. If the actual data symbol $\boldsymbol{\mathcal{S}}$ was known at the receiver, we could simply remove the effect of the data symbol and estimate the NBI term. Let ω, of cardinality l, be the set of carrier indices (and hence the data symbols) most affected by NBI[3]. Knowledge of ω helps to identify the indices of data symbols most likely to be in error. Based on the data constellation size, d, there are d^l possibilities for the received symbol $\hat{\boldsymbol{\mathcal{S}}}_{\omega_j}$ such that

$$\hat{\mathcal{S}}_{\omega_j}(i) = \begin{cases} \mathcal{S}'(i) & i \in \omega \\ \hat{\mathcal{S}}(i) & \text{otherwise} \end{cases} \tag{25}$$

where $i = 1, 2, \cdots, N$ and $j = 1, 2, \cdots, d^l$. The correct combination completely removes the effect of data symbol from the received signal enabling us to detect the NBI signal. Mathematically,

$$\boldsymbol{\mathcal{Y}}_r = \boldsymbol{\mathcal{Y}} - \mathbf{\Lambda} \mathbf{F}_P \mathbf{F}_{\mathrm{zp}} \hat{\boldsymbol{\mathcal{S}}}_{\omega_j} \tag{26}$$

[3] ω is known based on initial NBI estimate.

Next we define

$$\Psi_2 := \mathbf{S}_2 \mathbf{F}_P^{\mathrm{H}} \tag{27}$$

where $\mathbf{S}_2 = [\mathbf{I}_N \quad \mathbf{0}_{N \times M}]$ is a $N \times P$ selection matrix. Let $\mathbf{y}'_r = \Psi_2 \boldsymbol{\mathcal{Y}}_r$ then (26) can be written as

$$\mathbf{y}'_r = \Psi_2 \mathbf{\Lambda} \mathbf{F}_P \mathbf{F}_{\mathrm{zp}} (\boldsymbol{\mathcal{S}} - \hat{\boldsymbol{\mathcal{S}}}_{\omega_j}) + \Psi_2 \mathbf{F}_P \mathbf{x}_{\mathrm{eq}} + \Psi_2 \boldsymbol{\mathcal{N}} \tag{28}$$

When $\hat{\boldsymbol{\mathcal{S}}}_{\omega_j} = \boldsymbol{\mathcal{S}}$, the first term on the right hand side would disappear leaving the NBI term. Thus the NBI is given by the optimal data combination $\hat{\boldsymbol{\mathcal{S}}}_{\omega_j}^{\mathrm{o}}$ that solves

$$\arg \max_{\hat{\boldsymbol{\mathcal{S}}}_{\omega_j}} p(\mathbf{y}'' | \hat{\boldsymbol{\mathcal{S}}}_{\omega_j}) \tag{29}$$

where $p(.)$ is defined in (10) and $\mathbf{y}''$ is obtained by concatenating (28) and (5), i.e.,

$$\mathbf{y}'' = \begin{bmatrix} \mathbf{y}'_r \\ \mathbf{y}' \end{bmatrix} \tag{30}$$

Note that an erroneous initial support estimate of the NBI can be tackled by expanding ω to include its neighboring tones.

5. SIMULATION RESULTS

We simulate an uncoded ZP-OFDM system with $N = 64$ data carriers modulated with 4-QAM data symbols. The length of zero padding is $M = 16$. The channel is assumed to be of $L = 13$ taps and changes from one OFDM symbol to the next. Both the location and offset of the NBI is assumed to vary from one symbol to the next. The width of the NBI, r, is assumed to vary from 0 to 3 according to a binomial distribution. For an NBI cancelation algorithm, it is also important to recognize those symbols which do not require NBI cancelation. Including the $r = 0$ case would test the proposed algorithm if it is able to make this distinction. Here we constraint the NBI to be confined to contiguous tones though the method will work for non contiguous interference as well. All algorithms are compared in an uncoded OFDM system so that the performance gain due to coding does not mask the performance of the individual algorithm. The signal to noise power ratio (SNR) is defined as SNR $= \sigma_s^2/\sigma_n^2$ and the signal to interference ratio (SIR) is defined as SIR $= \sigma_s^2/\sigma_x^2$ where σ_s^2, σ_x^2, σ_n^2 are OFDM signal power, interference power and noise power respectively.

Figure 1 shows the uncoded ZP-OFDM case where performance of the proposed MAP approach is compared to the CS method, data aided MAP approach and the exhaustive search in the presence of NBI of unknown width and SIR $= -10$ dB. Results are plotted for $\xi = 5$ (corresponding to 11

$$\mathbf{A} = [\mathbf{q}_{-\xi} \odot \mathbf{a}_0 \quad \mathbf{q}_{-\xi+1} \odot \mathbf{a}_0 \ \cdots \ \mathbf{q}_{\xi} \odot \mathbf{a}_0 \ \cdots \ \mathbf{q}_{-\xi} \odot \mathbf{a}_{(P-1)} \ \cdots \ \mathbf{q}_{\xi} \odot \mathbf{a}_{(P-1)}] \tag{24}$$

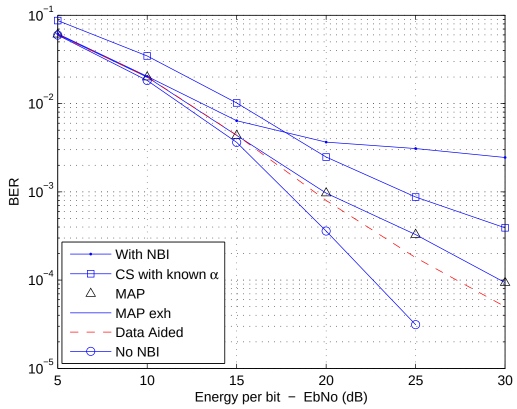

Fig. 1. Uncoded BER comparison for various NBI cancelation methods (SIR = -10 dB).

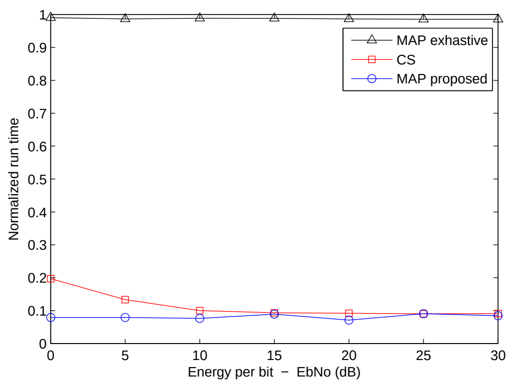

Fig. 2. Comparison of normalized run time.

levels of α). The CS method does not perform well in the uncoded case while the performance of the proposed algorithm is comparable to that of the exhaustive search method. Figure 2 shows the normalized running time of the CS based method, the exhaustive search and the proposed MAP method. The proposed method requires slightly less CPU time than the CS method at all SNR values. The runtime of the proposed method can be further reduced by using structure to reduce calculations for the expanded grid.

6. CONCLUSION

This paper presents a MAP approach for narrow band interference cancelation by formulating it as a sparse signal estimation problem. We consider a highly sophisticated model for the NBI signal by allowing the NBI to have a grid offset relative to the ZP-OFDM system and letting it change its location and amplitude on a symbol by symbol basis. The proposed technique exploits the inherent structure of the ZP-OFDM system to significantly reduce the search space. The computational complexity of the algorithm is low as many calculations can be reused owing to the rich structure of the sensing matrix. Further reduction in complexity is possible by exploiting the structure for within a cluster calculations. Simulations show that the proposed method is able to perfectly estimate and remove the interfering signal.

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