Low-Complexity MAP Based Channel Support Estimation for Impulse Radio Ultra-Wideband (IR-UWB) Communications

S. F. Ahmed, T. Y. Al-Naffouri and A. H. Muqaibel
Department of Electrical Engineering
King Fahd University of Petroleum and Minerals
Dhahran, Saudi Arabia.

sfaraz, naffouri, muqaibel@kfupm.edu.sa

***Abstract*—The paper addresses the problem of channel estimation in Impluse-Radio Ultra-Wideband (IR-UWB) communication system. The IEEE 802.15.4a channel model is used where the channel is assumed to be Linear Time Invariant (LTI) and thus the problem of channel estimation becomes the estimation of the sparse channel taps and their delays. Since, the bandwidth of the signal is very large, Nyquist rate sampling is impractical, therefore, we propose to estimate the channel taps from the sub-sampled versions of the received signal profile. We adopt the Bayesian framework to estimate the channel support by incorporating the a priori multipath arrival time statistics. In the first approach, we adopt a two-step method by employing Compressive Sensing to obtain coarse estimates and then refine them by applying Maximum A Posteriori (MAP) criterion. In the second approach, we develop a Low-Complexity MAP (LC-MAP) estimator. The computational complexity is reduced by identifying nearly orthogonal clusters in the received profile and by leveraging the structure of the sensing matrix.**

I. Introduction

Ultra-wideband (UWB) radio is a fast emerging technology with uniquely attractive features in wireless communications, networking, radar, imaging and positioning systems [1]. Conceptually, UWB is characterized by a transmission with an instantaneous spectrum in excess of 500 MHz, or, a fractional bandwidth of more than 20%. The Impulse-Radio Ultra-wideband (IR-UWB) communications utilize low duty cycle pulses to transmit data over the wireless channel. The small pulse duration which is on the order of nanoseconds implies large bandwidth.

UWB communication systems offer several advantages which include high data rates, high multipath resolution, low transmission power and simple transmitters. These systems are primarily envisioned for very high data rates indoor applications. There are several challenges that accompany the advantages of UWB communications. The Nyquist sampling frequency for UWB signals is prohibitively high. In addition, the transmitted energy is distributed over a large number of multipath components (MPCs). At the receiver, these MPCs need to be estimated accurately to capture sufficient energy for successful communications. Therefore, the channel estimation problem in IR-UWB becomes an important and challenging task and several approaches have been considered for this problem.

The work of S. F Ahmed and T. Y. Al-Naffouri was supported by SABIC through an internally funded project from DSR, KFUPM (Project No. SB101006).

In [2], the Maximum Likelihood (ML) criterion is applied to estimate the MPCs of a UWB transmitted pulse with Nyquist rate sampling. Symbol-long samples are used for both data-aided and non-data-aided ML estimation of the UWB channel in [3]. In [4] and [5], the authors use a pair of successive symbol long segments at the receiver and perform correlation to estimate the timing. In [6], the repetition pattern of the pulses is exploited by working with the second order statistics of the received signal to estimate the timing and perform synchronization at the receiver. In [7] the authors estimate the channel's Fourier co-efficients and use them to estimate the MPCs delays. *Finite Rate of Innovation* is employed in [8] and [9] for developing a low-complexity UWB channel estimator in the frequency domain, but there are no guarantees to avoid the potential ill-conditioning. The methods in [4]-[9] use sub-Nyquist symbol-rate sampling but are not optimal for parameter estimation and do not utilize any statistical information of the channel.

The received UWB signal profile contains a number of finley resolvable multipath components which gives it multipath sparsity and this makes the application of *Compressive Sensing* (CS) attractive. Thus, [10] applied compressive sensing for the first time to the UWB channel estimation problem. It basically reconstructs the received signal from the random samples but does not estimate the channel parameters.

This paper proposes a channel estimation algorithm that differentiates itself from the algorithms mentioned above as it is (1) low-complexity, (2) operates at sub-Nyquist rates and (3) makes optimal use of the channel statistics. As our simulations show, it beats other approaches both in performance and complexity. Specifically, we decompose the channel estimation problem into two parts: (i) estimation of the channel support (i.e. MPCs delays), followed by, (ii) estimation of the support co-efficients (i.e. MPCs amplitudes). First, we propose *Compressive Sensing MAP* (CS-MAP) estimator where we perform coarse estimation of the UWB channel in a compressive sensing framework and then refine the estimates using the MAP criterion. Second, we develop a *Low-Complexity MAP* (LC-MAP) estimator based on the sparsity of the received signal profile and the structure of the

978-1-4577-1764-2/11/$26.00 ©2011 IEEE

sensing matrix.

This paper is organized as follows: We present the UWB channel model and its parameters in Section II-A and our formulation of the IR-UWB channel estimation problem in Section II-B. In Section III we present the CS-MAP and LC-MAP estimation algorithms. Section IV discusses the simulation results and finally Section V concludes the paper.

II. UWB Communication Model

A. IR-UWB Channel

In an IR-UWB communications system, data is transmitted by sending very short duration low duty cycle pulses. These pulses have periods on the order of nanoseconds. When the transmitted pulse travels through the channel it gets delayed and attenuated. Since the bandwidth is very large, many MPCs are resolvable at the receiver. In order to estimate the channel, the receiver needs to estimate the delays and the attenuations of the MPCs. The channel is modeled as the finite impulse response (FIR) filter and so the MPCs' delays and attenuations correspond to the locations of the filter taps and their coefficients, respectively. Mathematically speaking, the received signal profile is given by the following,

$$r(t) = g(t) * h(t) + \omega(t) \tag{1}$$

$$= \sum_{l=0}^{L-1} \alpha_l g(t - \tau_l) + \omega(t) \tag{2}$$

where L is the number of MPCs, $g(t)$ is the transmitted pulse which is shaped as second derivative of the Gaussian pulse,[1] $h(t)$ is the channel impulse response and $*$ denotes linear convolution. In Eq. (2), α_l denotes the gain and τ_l denotes the delay of the l^{th} path while $\omega(t)$ is the additive noise at the receiver which is assumed to be white Gaussian (AWGN).

Due to the large bandwidth of the signal, the small-scale fading co-efficients, α_l's, are modelled as Nakagami distributed in the IEEE 802.15.4a model [11]. Furthermore, the UWB channel impulse response (CIR) has been found empirically to exhibit a clustered structure [12]. The number of clusters is an important parameter and is modelled as Poisson-distributed in [11] whereas the arrival of the clusters and the paths within a cluster are modelled as Poisson processes. This implies that if L MPCs occur in a given duration of time, they are uniformly distributed. The rate of arrival of the MPCs within a cluster, λ, is given in [11] for the different Line-of-Sight (LOS) and non Line-of-Sight (NLOS) environments. Therefore, the probability of a path occuring in a time bin of small duration, δ_t, is given by $\lambda\delta_t$. Thus the occurance of a single MPC in a bin can be assumed to be a Bernoulli trial, with probability of success $\mathrm{p}_b = \lambda\delta_t$ [13]. This assumption is valid if the bin duration, δ_t, is small enough such that it either contains exactly one or no MPC.

[1] Second and fifth derivatives of the Gaussian pulse are generally used in IR-UWB systems since their spectrum follows the FCC specifications.

B. Matrix Model Formulation

Consider the signal profile in Eq. (2), which we would like to express in matrix form. We can represent $r(t)$ using its Nyquist rate (F_N) samples. Thus, the samples are taken at every $\delta_t = \frac{1}{F_N}$ seconds which is much less than the pulse duration T_g and we can write,

$$r(n\delta_t) = \sum_{l=0}^{L-1} \alpha_l g(n\delta_t - l\Delta\delta_t) + \omega(n\delta_t) \tag{3}$$

$$r(n) = \sum_{l=0}^{L-1} \alpha_l g(n - l\Delta) + \omega(n) \tag{4}$$

where we assume that the delays τ_l's can be represented as integral multiples of δ_t, i.e. $\tau_l = l\Delta\delta_t$ (Δ is the amount of the basic shift of the pulse as shown in Eq. (6) further ahead) and where we have dropped δ_t from the argument (4) for notational convenience. The number of multipath components L is generally large but only the L_{max} strongest MPCs capture the significant portion of the transmitted signal energy [14]. This leads to a practical Selective Rake Receiver implementation where estimates of only L_{max} τ's and the corresponding α's are required.

Now, while we can represent $r(t)$ using its Nyquist rate samples, we sub-sample it at a lower rate $\mu F_N = \frac{M}{N} F_N$ where $M < N$. We represent this in the matrix form as,

$$\mathbf{y} = \mathbf{\Psi}\boldsymbol{\alpha} + \boldsymbol{\omega} \tag{5}$$

where

$$\mathbf{\Psi} = \begin{bmatrix} g(n-\Delta) & \dots & g(n-N\Delta) \\ g(n+1-\Delta) & \dots & g(n+1-N\Delta) \\ . & \dots & . \\ . & \dots & . \\ g\left(n+(M-1)-\Delta\right) & \dots & g\left(n+(M-1)-N\Delta\right) \end{bmatrix} \tag{6}$$

Note that in (5) $\mathbf{y}$ is the $M \times 1$ received vector, and $\boldsymbol{\omega}$ is the $M \times 1$ AWGN vector with zero-mean and $M \times M$ covariance matrix $\mathbf{C}_\omega = N_0\mathbf{I}$. The vector $\boldsymbol{\alpha}$ is the $N \times 1$ sparse channel parameter vector with its active elements at the channel delays. We further decompose $\boldsymbol{\alpha}$ as, $\boldsymbol{\alpha} = \mathbf{a} \odot \underline{\boldsymbol{\alpha}}$ ($\odot$ denotes element by element multiplication) where $\underline{\boldsymbol{\alpha}}$ is an $N \times 1$ binary vector that represents the support of $\boldsymbol{\alpha}$ i.e., $\underline{\boldsymbol{\alpha}} = supp(\boldsymbol{\alpha})$[2], and $\mathbf{a}$ is an $N \times 1$ vector of the amplitudes of $\boldsymbol{\alpha}$ where the amplitudes are zero except at the active locations of $\underline{\boldsymbol{\alpha}}$; we collect these non-zero amplitudes in the vector $\mathbf{a}_{\underline{\boldsymbol{\alpha}}}$. Now, if we set $\mathbf{A} = diag(\boldsymbol{\alpha})$[3], then we can rewrite eq. (5) as

$$\mathbf{y} = \mathbf{\Psi}\mathbf{A}\underline{\boldsymbol{\alpha}} + \boldsymbol{\omega} \tag{7}$$

With this formulation, the estimation of the τ's and α's of Eq. (4) translate into the estimation of the vectors $\underline{\boldsymbol{\alpha}}$ and $\mathbf{a}_{\underline{\boldsymbol{\alpha}}}$ respectively, i.e., the estimation of the location of active elements of $\boldsymbol{\alpha}$ and the corresponding amplitudes.

[2] $supp(\boldsymbol{\alpha})$ is a binary vector having elements as 1's and 0's where a 1 indicates that there is a non-zero element at the same location in $\boldsymbol{\alpha}$.

[3] $diag(\boldsymbol{\alpha})$ is a diagonal matrix formed by placing the elements of $\boldsymbol{\alpha}$ along its diagonal.

III. Channel Estimation

The formulation of the last section allows us to decompose the channel estimation into two parts. Once the support vector $\underline{\boldsymbol{\alpha}}$ is estimated, the amplitudes vector $\mathbf{a}_{\underline{\boldsymbol{\alpha}}}$ corresponding to that support is estimated using Least-Squares (LS)[4],

$$\hat{\mathbf{a}}_{\underline{\boldsymbol{\alpha}}_{LS}} = (\boldsymbol{\Psi}^H_{\underline{\boldsymbol{\alpha}}}\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}})^{-1}\boldsymbol{\Psi}^H_{\underline{\boldsymbol{\alpha}}}\mathbf{y} \tag{8}$$

where $\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}$ represents the sub-matrix of $\boldsymbol{\Psi}$ consisting of the columns indicated by the active elements of $\underline{\boldsymbol{\alpha}}$.

Now, we present the estimation of the channel support in a Bayesian Framework, where $\underline{\boldsymbol{\alpha}}$ is the unknown random vector with a known prior probability distribution. Let $\aleph$ be the set consisting of all 2^N possible support vectors $\underline{\boldsymbol{\alpha}}$. We need to search for the best estimate of $\underline{\boldsymbol{\alpha}}$ over the entire set $\aleph$. In our first method we reduce this search space by using CS and in the second method we attempt to find the low-complexity MAP estimate by jointly exploiting the sparsity of $\mathbf{y}$ and the structure of the matrix $\boldsymbol{\Psi}$. In the following, we assume that the start of the received signal profile, i.e., the location of the first multipath component, is known. Therefore our task is to estimate the support of the remaining MPCs.

A. Maximum A Posteriori (MAP) Based Support Estimation

In order to find the MAP estimate of the support, we need the pdf $p(\boldsymbol{\alpha}|\mathbf{y})$ which is given using the Baye's Rule as,

$$p(\boldsymbol{\alpha}|\mathbf{y}) = \frac{p(\mathbf{y},\boldsymbol{\alpha})}{p(\mathbf{y})} \tag{9}$$

$$= \frac{p(\mathbf{y},\mathbf{a}_{\underline{\boldsymbol{\alpha}}}|\underline{\boldsymbol{\alpha}})p(\underline{\boldsymbol{\alpha}})}{p(\mathbf{y})} \tag{10}$$

The prior $p(\underline{\boldsymbol{\alpha}})$ is to calculate since the entries of $\underline{\boldsymbol{\alpha}}$ are independent Bernoulli trials, with $P(\underline{\alpha}_i = 1) = \mathrm{p}_b$ and $P(\underline{\alpha}_i = 0) = (1-\mathrm{p}_b)$ for $i = 1,\ldots,N$ (see Sec. II-A). Therefore,

$$p(\underline{\boldsymbol{\alpha}}) = \mathrm{p}_b^{\|\underline{\boldsymbol{\alpha}}\|_0}(1-\mathrm{p}_b)^{N-\|\underline{\boldsymbol{\alpha}}\|_0} \tag{11}$$

The vector $\mathbf{a}_{\underline{\boldsymbol{\alpha}}}$ is non-Gaussian and therefore, the joint probability distribution given by $p(\mathbf{y},\mathbf{a}_{\underline{\boldsymbol{\alpha}}}|\underline{\boldsymbol{\alpha}})$ is difficult to obtain in a closed form. We assume the vectors $\mathbf{a}_{\underline{\boldsymbol{\alpha}}}$ and $\underline{\boldsymbol{\alpha}}$ to be independent, and since we are interested in estimating the support vector $\underline{\boldsymbol{\alpha}}$, we consider $\mathbf{a}_{\underline{\boldsymbol{\alpha}}}$ as the nuisance parameter and inegrate it out to obtain the likelihood as,

$$p(\mathbf{y},\mathbf{a}_{\underline{\boldsymbol{\alpha}}}|\underline{\boldsymbol{\alpha}}) = p(\mathbf{y}|\mathbf{a}_{\underline{\boldsymbol{\alpha}}},\underline{\boldsymbol{\alpha}})p(\mathbf{a}_{\underline{\boldsymbol{\alpha}}})$$

$$i.e., \int_{-\infty}^{+\infty} p(\mathbf{y},\mathbf{a}_{\underline{\boldsymbol{\alpha}}}|\underline{\boldsymbol{\alpha}})d\mathbf{a}_{\underline{\boldsymbol{\alpha}}} = \int_{-\infty}^{+\infty} p(\mathbf{y}|\mathbf{a}_{\underline{\boldsymbol{\alpha}}},\underline{\boldsymbol{\alpha}})p(\mathbf{a}_{\underline{\boldsymbol{\alpha}}})d\mathbf{a}_{\underline{\boldsymbol{\alpha}}}$$

$$= p(\mathbf{y}|\underline{\boldsymbol{\alpha}})$$

The MAP estimate of $\underline{\boldsymbol{\alpha}}$ is now given by the following,

$$\hat{\underline{\boldsymbol{\alpha}}}_{MAP} = \arg\max_{\underline{\boldsymbol{\alpha}}\in\aleph} p(\mathbf{y}|\underline{\boldsymbol{\alpha}})p(\underline{\boldsymbol{\alpha}}) \tag{12}$$

[4]In the abscence of any a priori statistics about $\mathbf{a}$, LS is the best we can do.

For a given support vector $\underline{\boldsymbol{\alpha}}$, we can say that the received vector $\mathbf{y}$ lies in the subspace spanned by the columns of $\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}$, plus an AWGN vector $\boldsymbol{\omega}$. Thus, the orthogonal projection of $\mathbf{y}$ onto the orthogonal complement of $\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}$ is Gaussian. Specifically, the vector $\Pi^{\perp}_{\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}}\mathbf{y}$ is Gaussian where $\Pi^{\perp}_{\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}}$ is the projection matrix onto the orthogonal complement of $\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}$, i.e.

$$\Pi^{\perp}_{\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}} = \mathbf{I} - \boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}\left[\boldsymbol{\Psi}^H_{\underline{\boldsymbol{\alpha}}}\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}\right]^{-1}\boldsymbol{\Psi}^H_{\underline{\boldsymbol{\alpha}}} \tag{13}$$

Therefore, we can approximate the likelihood $p(\mathbf{y}|\underline{\boldsymbol{\alpha}})$ as

$$p(\mathbf{y}|\underline{\boldsymbol{\alpha}}) \propto \exp\left(-\frac{1}{2N_0}\|\Pi^{\perp}_{\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}}\mathbf{y}\|_2^2\right) \tag{14}$$

Substituting (11) and (14) in (12), we see that we can find the MAP support estimate by maximizing the log-likelihood

$$\hat{\underline{\boldsymbol{\alpha}}}_{MAP} = \arg\max_{\underline{\boldsymbol{\alpha}}\in\aleph} \quad \ln\left(\frac{\mathrm{p}_b}{1-\mathrm{p}_b}\right)^{\|\underline{\boldsymbol{\alpha}}\|_0} - \frac{1}{2N_0}\|\Pi^{\perp}_{\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}}\mathbf{y}\|_2^2 \tag{15}$$

where from (13),

$$\|\Pi^{\perp}_{\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}}\mathbf{y}\|_2^2 = \|\mathbf{y}\|_2^2 + \mathbf{y}^H\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}\left[\boldsymbol{\Psi}^H_{\underline{\boldsymbol{\alpha}}}\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}\right]^{-1}\boldsymbol{\Psi}^H_{\underline{\boldsymbol{\alpha}}}\mathbf{y} \tag{16}$$

B. Compressive Sensing based MAP (CS-MAP) Estimation

The challenge in (15) is that we need to maximize the likelihood over the entire set $\aleph$ which is computationally very complex. Therefore, in the following we employ CS for coarse estimation to reduce the search space. From CS point of view [10], the uniformly sub-sampled matrix $\boldsymbol{\Psi}$ in the under-determined system of Eq. (5) is the dictionary matrix composed of atoms - the linear shifted versions of the discretized pulse $g(n)$. Since the received UWB signal $\mathbf{y}$ is made up of a linear superposition of only L atoms of the dictionary and $L < N$, therefore the vector $\boldsymbol{\alpha}$ is sparse and CS can be applied to reconstruct $\boldsymbol{\alpha}$ from $\mathbf{y}$ as follows:

$$\hat{\boldsymbol{\alpha}}_{cs} = \arg\min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_0 \quad s.t. \quad \mathbf{y} = \boldsymbol{\Psi}\boldsymbol{\alpha}$$

This is a combinatorial problem. Therefore, its convex relaxation where the l_0 norm is replaced by the l_1 norm is employed which lends itself to linear programming implementation [15]. Alternatively, greedy algorithms can also be used for reconstruction which are computationally more efficient but incur further approximations leading to larger reconstruction errors.

The CS based channel support estimate is $\hat{\underline{\boldsymbol{\alpha}}}_{cs} = supp(\hat{\boldsymbol{\alpha}}_{cs})$. In order to refine the CS estimates, we retain the support vector $\hat{\underline{\boldsymbol{\alpha}}}^{\star}_{cs}$ corresponding to the $L_{cs} > L_{max}$ largest entries of $\hat{\boldsymbol{\alpha}}_{cs}$ and apply the MAP criterion. Thus the CS-MAP support estimate is given as,

$$\hat{\underline{\boldsymbol{\alpha}}}_{CS-MAP} = \arg\max_{\underline{\boldsymbol{\alpha}}\in\aleph^{\star}} \quad \ln\left(\frac{p_b}{1-p_b}\right)^{\|\underline{\boldsymbol{\alpha}}\|_0} - \frac{1}{2N_0}\|\Pi^{\perp}_{\boldsymbol{\Psi}_{\underline{\boldsymbol{\alpha}}}}\mathbf{y}\|_2^2 \tag{17}$$

where $\aleph^{\star}$ represents the set consisting of all the $2^{L_{cs}}$ possible combinations of the columns of $\boldsymbol{\Psi}_{\hat{\underline{\boldsymbol{\alpha}}}^{\star}_{cs}}$. The computational complexity could still be high depending upon the value of L_{cs} and the reconstruction method.

We note that the CS reconstruction is performed with the objective of having the sparsest representation of the received signal in the dictionary but the sparsest estimate need not be a good estimate of the true channel support. The CS-MAP is only an approximation of the MAP estimator as it relies on the initial CS estimates and it can only perform well if these initial estiamtes are good. We tackle this drawback in the next sub-section where we show how the rich structure of the sensing matrix allows us to perform true MAP estimation (i.e. maximize (15)) with low complexity.

C. Low-Complexity MAP (LC-MAP) Estimation

The sensing matrix $\mathbf{\Psi}$ in Eqs. (5)-(7) is rich in structure. It is not only Toeplitz but is a banded diagonal matrix. Let $|g|$ denote the length of the basic pulse $g(n)$ at Nyquist rate, then for a given sub-sampling ratio μ the bandwidth of the matrix $\mathbf{\Psi}$ is given by $\beta = |g|\mu$. This implies that

$$\boldsymbol{\psi}_i^H \boldsymbol{\psi}_j = \begin{cases} 0, & |i-j| > \beta \\ f(|i-j|), & |i-j| \le \beta \end{cases} \tag{18}$$

where $\boldsymbol{\psi}_i$ is the i^{th} column of $\mathbf{\Psi}$ i.e., the columns that are distant enough are orthogonal. Therefore we can collect the columns of $\mathbf{\Psi}$ to form a number of mutually orthogonal clusters of width $s\beta$ where s is an integer. We correlate the received vector $\mathbf{y}$ with the columns of $\mathbf{\Psi}$ to identify the location and width of these clusters, as shown in Fig. 1. We select s for each of these nearly orthogonal clusters; this technique is refered to as *orhtogonal clustering (OC)*. A similar approach involving FFT matrix is adopted in [16] (see also [17] for more details). For ease of calculations, but without loss of generality, we assume the same s for every cluster and thus each cluster has a width of $s\beta$. We remark here that due to the multipath sparsity, only a few of these clusters are active (see Fig. 1).

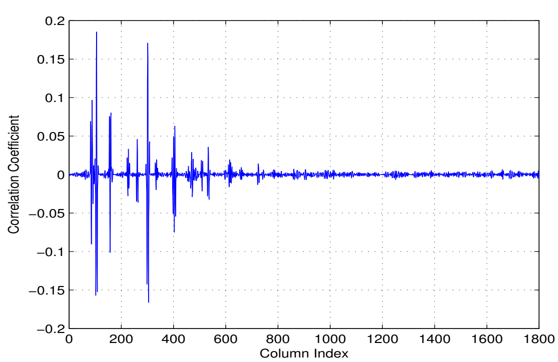

Figure 1. Correlation of the Received vector $\mathbf{y}$ with Columns of $\mathbf{\Psi}$ Showing Orhogonal Clustering, at SNR = 20 dB and $\mu = 1/4$

If for a certain received vector $\mathbf{y}$, C orthogonal clusters are identified after correlation, then, we can express $\mathbf{\Psi}_{\underline{\boldsymbol{\alpha}}}$ in a block matrix form as

$$\mathbf{\Psi}_{\underline{\boldsymbol{\alpha}}} = [\mathbf{\Theta}_1 \quad \mathbf{\Theta}_2 \ldots \mathbf{\Theta}_C] \tag{19}$$

where $\mathbf{\Theta}_i$ stands for the i^{th} cluster. Now we need to determine which columns of $\mathbf{\Theta}_i$ are active. Since, the $\mathbf{\Theta}_i$'s are orthogonal, the inverse term that appears in Eq. (16) becomes the inverse of the block diagonal matrix,

$$\begin{bmatrix} (\mathbf{\Theta}_1^H\mathbf{\Theta}_1)^{-1} & 0 & \ldots & 0 \\ 0 & (\mathbf{\Theta}_2^H\mathbf{\Theta}_2)^{-1} & \ldots & . \\ \vdots & 0 & \ddots & 0 \\ 0 & \vdots & \ldots & (\mathbf{\Theta}_C^H\mathbf{\Theta}_C)^{-1} \end{bmatrix} \tag{20}$$

Moreover, if we select the same set of active columns in $\mathbf{\Theta}_1$ and $\mathbf{\Theta}_i$, then it is easy to show that

$$(\mathbf{\Theta}_1^H\mathbf{\Theta}_1)^{-1} = (\mathbf{\Theta}_2^H\mathbf{\Theta}_2)^{-1} = \cdots = (\mathbf{\Theta}_C^H\mathbf{\Theta}_C)^{-1} \tag{21}$$

since in this case the $\mathbf{\Theta}_i$ is simply a shifted version of $\mathbf{\Theta}_1$ and where a maximum of only $(2\beta+1)$ computations are required for $(\mathbf{\Theta}_1^H\mathbf{\Theta}_1)$ as implied from (18). Furthermore, due to orthogonal clustering (16) is decomposed as follows:

$$\|\Pi^{\perp}_{\mathbf{\Psi}(\underline{\boldsymbol{\alpha}})}\mathbf{y}\|_2^2 = \|\mathbf{y}\|_2^2 + \mathbf{y}^H\Pi_{\mathbf{\Theta}_1}\mathbf{y} + \cdots + \mathbf{y}^H\Pi_{\mathbf{\Theta}_C}\mathbf{y} \tag{22}$$

where for $r = 1, 2, \ldots, C$,

$$\Pi_{\mathbf{\Theta}_r} = \mathbf{\Theta}_r[\mathbf{\Theta}_r^H\mathbf{\Theta}_r]^{-1}\mathbf{\Theta}_r^H \tag{23}$$

Moreover, from (21) we observe that only a single matrix inversion is required in Eq. (22). Now, for a certain received vector $\mathbf{y}$ the MAP metric in Eq. (15) can be evaluated for all $\underline{\boldsymbol{\alpha}} \in \aleph$ from only a few computations. Specifically, all we need to compute is the energy of $\mathbf{y}$ and the N correlations: $\mathbf{y}^H\boldsymbol{\psi}_i$ for $i = 1, \ldots, N$. In this way we determine the MAP estimate of the channel support where the objective is to find the most probable channel support for the observed signal profile. This estimator is presented in Eq. (15) which we reproduce below:

$$\hat{\underline{\boldsymbol{\alpha}}}_{MAP} = \arg\max_{\underline{\boldsymbol{\alpha}} \in \aleph} \quad \ln\left(\frac{\mathrm{p}_b}{1-\mathrm{p}_b}\right)^{\|\underline{\boldsymbol{\alpha}}\|_0} - \frac{1}{2N_0}\|\Pi^{\perp}_{\mathbf{\Psi}(\underline{\boldsymbol{\alpha}})}\mathbf{y}\|_2^2 \tag{24}$$

In evaluating the above we make use of the (20) and (22) to perform the maximization on each of the C clusters independently and obtain the *Low-Complexity Maximum A Posteriori* (LC-MAP) estimate of the true channel support.

For a cluster duration of $\delta_c = (s\beta \times \delta_t)$ seconds the expected number of paths is $\delta_c\lambda$ and the probability of having k paths in a single cluster is given by the Poisson distribution

$$P_c(k) = \frac{(\delta_c\lambda)^k \exp^{-(\delta_c\lambda)}}{k!} \tag{25}$$

We restrict the maximum number of paths in a single cluster, k_c, to be such that $P_c(k_c) < \epsilon$, where ϵ is arbitrarily small. For a certain cluster $\mathbf{\Theta}_r$ we compute the metric in eq. (24) for all $\underline{\boldsymbol{\alpha}}$ with $\|\underline{\boldsymbol{\alpha}}\|_0 = 1, 2, \ldots, k_c$. In doing so, and when moving from $\|\underline{\boldsymbol{\alpha}}\|_0 = l$ to $\|\underline{\boldsymbol{\alpha}}\|_0 = (l+1)$, we can make use of the Order Recursive approach as presented in [17] to further reduce the computations. Finally we note here that, the complexity of CS estimation based on convex relaxation using l_1 minimization is $O(M^2N^{3/2})$ [18] and that of our LC-MAP estimator is only $O(MNk_c/\beta)$.

IV. Simulations and Results

In our simulations the channel impulse response $h(t)$ was generated according to the IEEE 802.15.4a Indoor Residential LOS model. The second derivative of the Gaussian pulse with $T_g = 1$ nanosecond was generated as the IR-UWB signal and convolved with $h(t)$. This signal has a 3 dB bandwidth of 4 GHz and consequently the Nyquist rate sampling period of 0.125 nanosecond. The simulations were run for 1000 realizations of the channel in the prescence of AWGN and the normalized root mean square error (NRMSE), expressed as number of samples, in the estimation of $\underline{\alpha}$ was calculated for different values of SNR where the normalization was performed with respect to the Nyquist rate sampling period. The comparison of CS, CS-MAP and LC-MAP estimation methods for estimating 10 MPCs at $\mu = 1/2$ and $\mu = 1/4$ is shown in Fig. 2. CS-MAP performs little better than CS but LC-MAP performs better than both CS and CS-MAP over the whole SNR range. Furthermore, LC-MAP is computationally very efficient as shown in Fig. 3, where the mean computation times of the three methods are compared.

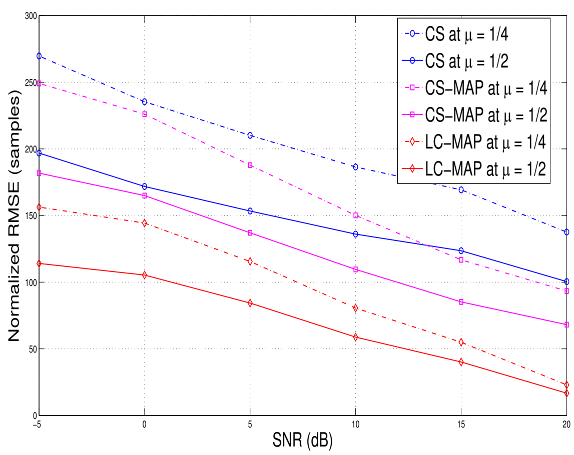

Figure 2. Performance Comparison of CS, CS-MAP and LC-MAP Methods at $\mu = 1/2$ and $\mu = 1/4$

V. Conclusion

The channel support estimation problem in IR-UWB is a challenging one. In this paper, we approached the problem of estimation using the sub-sampled versions of the received signal. The CS based estimates were improved using the MAP criterion and a novel Low-Complexity MAP (LC-MAP) estimator was developed by exploiting the rich structure of the sensing matrix. The simulations showed that LC-MAP improves the estimation with a significant reduction in computational complexity as well.

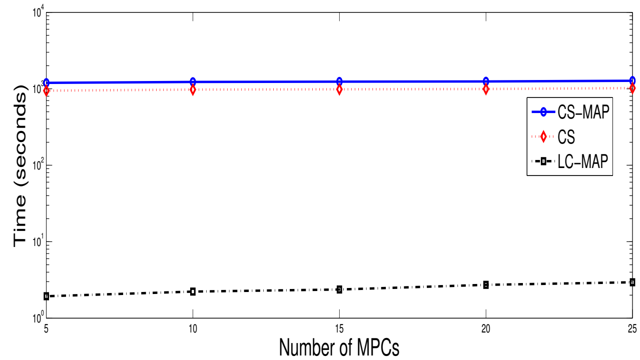

Figure 3. Average Run-Time for CS, CS-MAP and LC-MAP Methods for Different Number of Paths

References

[1] Liuqing Yang and Georgios B. Giannakis, "Ultra-Wideband: An Idea Whose Time Has Come," *IEEE Signal Processing Magazine*, vol. 21, no. 6, pp. 26–54, Nov. 2004.

[2] M. Win and R. Scholtz, "Characterization of ultra-wide bandwidth wireless indoor channels: a communication-theoretic view," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 9, pp. 1613–1627, Dec. 2002.

[3] V. Lottici, A. D'Andrea, and U. Mengali, "Channel estimation for ultra-wideband communications," *IEEE Journal on Selected Areas in Communications*, vol. 20, no. 9, pp. 1638–1645, Dec. 2002.

[4] Y. Li, A. Molisch, J. Zhang, and Others, "Channel estimation and signal detection for UWB," in *Proc. of WPMC*, 2003.

[5] L. Yang and G. Giannakis, "Timing Ultra-Wideband Signals With Dirty Templates," *IEEE Transactions on Communications*, vol. 53, no. 11, pp. 1952–1963, Nov. 2005.

[6] Z. Tian and L. Wu, "Timing Acquisition with Noisy Template for Ultra-Wideband Communications in Dense Multipath," *EURASIP Journal on Advances in Signal Processing*, vol. 2005, no. 3, pp. 439–454, 2005.

[7] G. Giannakis, "Blind timing and channel estimation for UWB multi-user ad hoc access," *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers, 2004.*, pp. 642–646, 2004.

[8] I. Maravic and M. Vetterli, "Low-complexity subspace methods for channel estimation and synchronization in ultra-wideband systems," in *Proc. International Workshop on UWB Systems*, no. 1, 2003.

[9] I. Maravic, M. Vetterli, and K. Ramchandran, "Channel estimation and synchronization with sub-Nyquist sampling and application to ultra-wideband systems," *2004 IEEE International Symposium on Circuits and Systems (IEEE Cat. No.04CH37512)*, no. 2, pp. V–381–V–384.

[10] J. L. Paredes, G. R. Arce, and Z. Wang, "Ultra-Wideband Compressed Sensing: Channel Estimation," *IEEE Journal of Selected Topics in Signal Processing*, vol. 1, no. 3, pp. 383–395, Oct. 2007.

[11] A. Molisch, K. Balakrishnan, C. Chong, S. Emami, A. Fort, J. Karedal, J. Kunisch, H. Schantz, U. Schuster, and K. Siwiak, "IEEE 802.15. 4a channel model-final report," *IEEE P*, vol. 15, pp. 802–1504, 2006.

[12] R.-M. Cramer, R. Scholtz, and M. Win, "Evaluation of an ultra-wide-band propagation channel," *IEEE Transactions on Antennas and Propagation*, vol. 50, no. 5, pp. 561–570, May 2002.

[13] T. Jia and D. I. Kim, "Analysis of Channel-Averaged SINR for Indoor UWB Rake and Transmitted Reference Systems," *IEEE Transactions on Communications*, vol. 55, no. 9, pp. 1822–1822, Sep. 2007.

[14] A. H. Muqaibel, "Directional modelling of ultra wideband communication channels," *IET Communications*, vol. 4, no. 1, pp. 51–62, 2010.

[15] H. Rauhut, K. Schnass, and P. Vandergheynst, "Compressed Sensing and Redundant Dictionaries," *IEEE Transactions on Information Theory*, vol. 54, no. 5, pp. 2210–2219, May 2008.

[16] T. Y. Al-Naffouri, A. Abdul Quadeer, and G. Caire, "Impulsive Noise Estimation and Cancellation in DSL using Orthogonal Clustering," in *submitted to IEEE International Symposium on Information Theory (ISIT), Russia*, 2011.

[17] T. Y. Al-Naffouri and A. A. Quadeer, "Structure-Based Bayesian Sparse Reconstruction," *to be submitted to IEEE Transactions on Signal Processing*, 2011.

[18] Y. Nesterov and A. Nemirovskii, *Interior-Point Polynomial Algorithms in Convex Programming*. SIAM, 1994.