Impulsive Noise Estimation and Cancellation in DSL Using Compressive Sampling

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***Abstract*—Impulsive noise is the bottle neck that determines the maximum length of the DSL. Impulsive noise seldom occurs in DSL but when it occurs, it is very destructive and results in dropping the affected DSL symbols at the receiver as they cannot be recovered. By considering impulsive noise a sparse vector, recently developed sparse reconstruction algorithms can be utilized to combat it. We propose an algorithm that utilizes the null carriers for the impulsive noise estimation and cancellation. Specifically, we use compressive sampling for a coarse estimate of the impulse position, an a priori information based MAP metric for its refinement, followed by MMSE estimation for estimating the impulse amplitudes. We also present a comparison of the achievable rate in DSL using our algorithm and recently developed algorithms for sparse signal reconstruction.**

***Index Terms*—Impulsive noise, DSL, Sparse signal reconstruction, and Compressive sampling.**

I. Introduction

One of the most severe problems that is encountered in Digital Subscriber Line (DSL) design is impulsive noise. As DSL technology works at extremely high SNR, additive white Gaussian noise (AWGN) is generally not a problem. Impulsive noise, as its name suggests, is a phenomenon that happens rarely. However, when it occurs it almost destroys the DSL signal. It is generally attributed to switching electronic equipment in the telephone network or nearby disturbances (such as starting of an automobile or vacuum cleaner). As its name suggests, impulsive noise is an impulse or a group of large individual impulses that take place in the time domain and then spread out in the frequency domain. It is difficult to design DSL systems to combat impulsive noise because it happens rarely (and thus it is not economical to design a system for a worst case scenario) but it cannot be ignored completely because when it takes place, it could devastate transmission and force the receiver to drop a few DSL symbols as they cannot be recovered.

Impulsive noise estimation and cancellation in OFDM systems is an area of active research. A Gaussian erasure matrix is used in [1] that assumes the impulses to be i.i.d. and occurring with a certain probability p. The receiver is given information of the exact position of the impulses by a genie, thus enabling it to eliminate the impulsive noise. In [2] and [3], precoding and frequency algebraic interpolation techniques using Reed-Solomon coding and decoding are proposed. Specifically, the presence of impulsive noise with few samples creates certain syndromes on a sequence of pilots or null frequencies which can be used to detect the location of impulsive noise, estimate it, and cancel it. The drawback of these techniques is that they require a certain structure of the null frequencies or pilots and they can be very sensitive to background noise. In [4], impulsive noise is considered a sparse vector (as it occurs as spikes in time domain) and compressive sampling (CS) [6], [7] is used to recover it by projections onto a small-dimensional space in background noise. This method allows much more flexible system design and much better robustness to the background noise.

In this paper, we present an extended version of the algorithm proposed in [4]. Instead of directly using the estimate obtained by using CS [4], we consider it a coarse estimate of the support (impulse positions) and utilize the a priori information that impulsive noise is Bernoulli-Gaussian to refine it. MMSE estimation is then employed for estimating the impulse amplitudes. We also compare the performance of our algorithm with the recently developed sparse reconstruction algorithms over a practical DSL channel. Since the advent of CS theory [5], [6], [7], there has been an increased interest in the area of sparse signal recovery. The main idea of CS is that a sparse signal can be reconstructed with high probability from an under-determined system of linear equations by l_1 optimization using linear programming techniques. The drawback of linear programming is its high complexity. Thus, recently many low complex alternatives have been proposed in literature for sparse signal recovery that include algorithms based on belief propagation [8], subspace based algorithms like subspace pursuit (SP) [9], and iterative greedy approaches including orthogonal matching pursuit (OMP) [10], gradient pursuit (GP), and nearly orthogonal matching pursuit (NOMP) [11].

II. Transmission Model

The time-domain complex baseband equivalent DSL channel is given by

$$y_k = \sum_{\ell=0}^{L} h_\ell x_{k-\ell} + z_k + e_k \tag{1}$$

where x_k is the channel input, y_k is the channel output, $\mathbf{h} = (h_0, \ldots, h_L)$ is the impulse response of the DSL channel, z_k is circular complex Gaussian noise $\sim \mathcal{CN}(0, N_0)$, and e_k is impulsive noise. We assume impulsive noise process to be Bernoulli-Gaussian, i.e. $e_k = \lambda_k g_k$, where λ_k are i.i.d. Bernoulli random variables, with $P(\lambda_k = 1) = p$, and g_k are i.i.d. Gaussian random variables $\sim \mathcal{CN}(0, I_0)$. We define the channel SNR as $\mathcal{E}_x/N_0$ and the impulse to noise ratio (INR) as I_0/N_0.

In matrix form, the channel model (1) is given by

$$\mathbf{y} = \mathbf{Hx} + \mathbf{z} + \mathbf{e} \tag{2}$$

where $\mathbf{y} \in \mathbb{C}^n$ and $\mathbf{x} \in \mathbb{C}^n$ are the time-domain OFDM receive and transmit signal blocks (after cyclic prefix removal) and $\mathbf{z} \sim$

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$\mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$. The vector $\mathbf{e}$ is an impulse noise process as specified above. Due to the presence of the cyclic prefix (which is inserted to avoid inter-block interference), $\mathbf{H}$ is a circulant matrix describing the cyclic convolution of the channel impulse response with the block $\mathbf{x}$. Let $\mathbf{F}$ denote a unitary DFT matrix with (k,ℓ) element $[\mathbf{F}]_{k,\ell} = \frac{1}{\sqrt{n}} e^{-j2\pi k\ell/n}$ with $k,\ell \in \{0,\ldots,n-1\}$. The time domain signal is related to the frequency domain signal by $\mathbf{x} = \frac{1}{\sqrt{n}}\mathbf{F}^{\mathsf{H}}\check{\mathbf{x}}$. The received signal block (in frequency domain) is given by

$$\check{\mathbf{y}} = \mathbf{D}\check{\mathbf{x}} + \check{\mathbf{z}} + \mathbf{F}\mathbf{e} \tag{3}$$

where $\mathbf{D} = \mathrm{diag}(\check{\mathbf{h}})$, $\check{\mathbf{h}} = \sqrt{n}\mathbf{F}\mathbf{h}$ is the DFT of the channel impulse response (whose coefficients are found, by construction, on the first column of $\mathbf{H}$), and $\check{\mathbf{z}}$ has the same statistics of $\mathbf{z}$, since $\mathbf{F}$ is unitary. Without impulsive noise, the transmitted data can be easily recovered using the element-by-element relationship,

$$\hat{\check{\mathbf{x}}} = \mathbf{D}^{-1}\check{\mathbf{y}} \tag{4}$$

In the presence of the impulsive noise, the performance of an OFDM receiver may suffer severe deterioration. Although impulsive noise attacks the DSL signal in time domain, it affects the symbols in the whole OFDM block in frequency domain, thus making the recovery of the OFDM block very difficult.

III. Impulsive Noise Elimination using Compressive Sampling

Consider the OFDM frequency domain channel model (3). Now trusting the fact that $\mathbf{e}$ is a sparse signal, we shall estimate it using a compressive sampling algorithm, and then remove it from the received signal. Let $\widehat{\mathbf{e}}$ denote the resulting estimate of $\mathbf{e}$ produced by the compressive sampling algorithm. Then, the signal actually fed to the receiver is given by

$$\widehat{\mathbf{y}} = \mathbf{D}\mathbf{S}_x\check{\mathbf{d}} + \mathbf{F}(\mathbf{e} - \widehat{\mathbf{e}}) + \check{\mathbf{z}} \tag{5}$$

The OFDM receiver will then treat this signal as if it was the output of a standard OFDM system without impulsive noise. Notice that a naive OFDM receiver that simply ignores the presence of the impulsive noise, would treat (5) as the output of an OFDM system with Gaussian noise. It is apparent that the gain of the proposed scheme is significant if the variance per component of the residual noise

$$\mathbf{v} = \mathbf{F}(\mathbf{e} - \widehat{\mathbf{e}}) + \check{\mathbf{z}} \tag{6}$$

is significantly less than the variance per component of the resulting frequency-domain Gaussian plus impulsive noise

$$\mathbf{w} = \mathbf{F}\mathbf{e} + \check{\mathbf{z}} \tag{7}$$

Let $\Omega \subset \mathbb{Z}_n$ denote the set of frequencies that are not used to send modulation symbols. As in [2], [3], we shall exploit these frequencies to estimate the impulsive noise vector $\mathbf{e}$ at the receiver. Our approach relies on using the null carriers that are available on the transmission spectrum to detect, estimate, and cancel impulsive noise.

We construct the time domain transmit signal as

$$\mathbf{x} = \mathbf{F}^{\mathsf{H}}\mathbf{S}_x\check{\mathbf{d}} \tag{8}$$

where $\check{\mathbf{d}}$ is frequency-domain data symbol vector of dimension $k \leq n$ and where $\mathbf{S}_x$ is an $n \times k$ "selection matrix" containing only one element equal to 1 per column, and with $m = n - k$ zero rows. The columns of $\mathbf{S}_x$ index the subcarriers that are used for data transmission in the OFDM system. The remaining subcarriers are either not used, or used for transmitting known pilot symbols in the frequency domain, which are not shown here since we do not deal with channel estimation, and these can be ideally subtracted from the received signal at the receiver. Therefore, to all extents, the subcarriers not indexed by columns of $\mathbf{S}_x$ are not used. We shall denote by $\mathbf{S}$ the matrix with a single element equal to 1 per column, that span the orthogonal complement of the columns of $\mathbf{S}_x$.

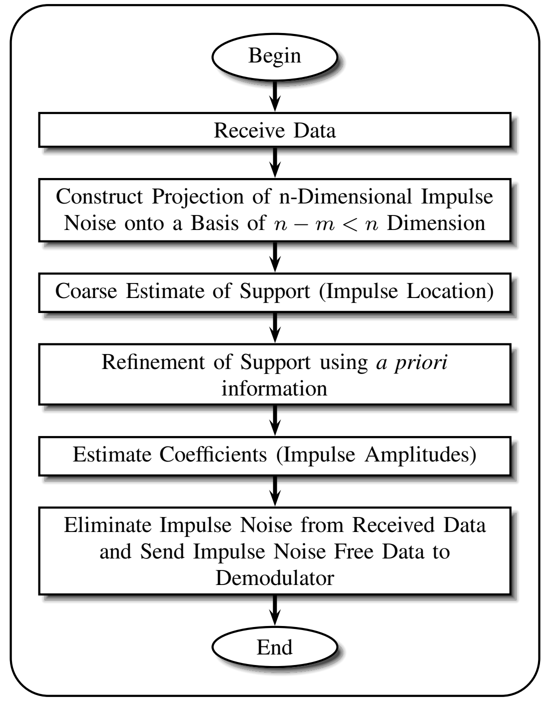

Fig. 1. Flowchart of the proposed method

The frequency domain vector is thus given by

$$\check{\mathbf{y}} = \mathbf{F}\mathbf{y} = \mathbf{D}\mathbf{S}_x\check{\mathbf{d}} + \mathbf{F}\mathbf{e} + \check{\mathbf{z}} \tag{9}$$

We shall estimate $\mathbf{e}$ from the projection into the orthogonal complement of the signal subspace. This is given by

$$\mathbf{y}' = \mathbf{S}^{\mathsf{T}}\check{\mathbf{y}} = \mathbf{S}^{\mathsf{T}}\mathbf{F}\mathbf{e} + \mathbf{z}' \tag{10}$$

where the observation vector $\mathbf{y}'$ is a projection of the n-dimensional impulsive noise onto a basis of dimension $n - m < n$ corrupted by the AWGN $\mathbf{z}'$ which is an i.i.d. Gaussian vector with variance N_0 per component, of length m. For later use, we shall denote the $m \times n$ projection matrix obtained by a row selection of $\mathbf{F}$ (according to $\mathbf{S}$) by $\mathbf{\Psi} = \mathbf{S}^{\mathsf{T}}\mathbf{F}$.

In (10), the impulsive noise $\mathbf{e} \in \mathbb{C}^n$ and the observation vector $\mathbf{y}' \in \mathbb{C}^m$ where $m < n$. Thus we have an under-determined system of linear equations which is an ill-posed problem. Similar to [4], we use the CS algorithm [5], [6], [7] to identify the support. Our work is different from [4] in that we refine the estimate obtained by CS algorithm using a MAP metric and then estimate the amplitudes of non-zero components of $\mathbf{e}$ using MMSE estimation. This clearly decomposes the algorithm into the coarse estimation of the support (impulse location), its refinement, and the estimation of the coefficients (impulse amplitudes) as shown in Figure 1.

A. Coarse Estimation of the Support

We specifically use the convex optimization algorithm proposed by Candes, Randall, and Tao [6], [7] to recover $\mathbf{e}$. In our notation, it is given by

$$\begin{aligned} \text{minimize} \quad & \|\widetilde{\mathbf{e}}\|_1, \\ \text{subject to} \quad & \|\mathbf{y}' - \boldsymbol{\Psi}\widetilde{\mathbf{e}}\|_2 \le \epsilon \end{aligned} \tag{11}$$

for some small enough ϵ.

B. Refining the Support Estimate

We wish to estimate optimally the support of $\mathbf{e}$ denoted by $\mathcal{I}_e$, from the observation $\mathbf{y}'$, where $\mathbf{e}$ is Bernoulli-Gaussian with parameters p and I_0, as specified in Section II. The a priori probability of $\mathcal{I}_e$ depends only on its size $J = |\mathcal{I}_e|$ (number of non-zero components). For a given binary vector $\mathbf{b}$ of Hamming weight r, we have $P(\mathcal{I}_e = \mathbf{b}) = P(|\mathcal{I}_e| = r) = p^r(1-p)^{n-r}$. The Maximum A-posteriori Probability rule (optimal Bayesian estimation rule) is given by

$$\widehat{\mathcal{I}} = \arg\max_{\mathcal{I}} \ P(\mathcal{I}|\mathbf{y}')$$

Up to an irrelevant proportionality factor, we can maximize the joint probability (density) $P(\mathcal{I}, \mathbf{y})$, i.e., the MAP metric

$$p(\mathbf{y}'|\mathcal{I})P(\mathcal{I}) = p^{|\mathcal{I}|}(1-p)^{n-|\mathcal{I}|}\frac{\exp\left(-\frac{1}{N_0}(\mathbf{y}')^{\mathsf{H}}\boldsymbol{\Sigma}(\mathcal{I})^{-1}\mathbf{y}'\right)}{\det\left(\boldsymbol{\Sigma}(\mathcal{I})\right)} \tag{12}$$

where we define the covariance matrix of $\mathbf{y}'$ given $\mathcal{I}$, normalized by the noise variance, as

$$\boldsymbol{\Sigma}(\mathcal{I}) = \frac{1}{N_0}\mathbb{E}[\mathbf{y}'(\mathbf{y}')^{\mathsf{H}}|\mathcal{I}] = \mathbf{I} + \frac{I_0}{N_0}\boldsymbol{\Psi}(\mathcal{I})\boldsymbol{\Psi}(\mathcal{I})^{\mathsf{H}} \tag{13}$$

where $\boldsymbol{\Psi} = \mathbf{S}^{\mathsf{T}}\mathbf{F} = [\boldsymbol{\psi}_1, \ldots, \boldsymbol{\psi}_n]$, and where $\boldsymbol{\Psi}(\mathcal{I})$ denotes the submatrix formed by the columns $\{\boldsymbol{\psi}_j : j \in \mathcal{I}\}$, indexed by the support $\mathcal{I}$. Here, we have used the fact that, under the support hypothesis $\mathcal{I}_e = \mathcal{I}$, the observation $\mathbf{y}'$ is conditionally Gaussian with covariance $N_0\boldsymbol{\Sigma}(\mathcal{I})$.

An optimal MAP support detector would test each hypothesis and find the one that maximizes the MAP metric above. Even by limiting to a subset of most probable supports, i.e., of weight at most $r_{\max}$ for some reasonable value of $r_{\max} > np$, this scheme would be prohibitively complex. Nevertheless, we can use the following augmented CS scheme: we use a CS algorithm in order to find a set of candidate positions. Let $\widehat{\mathbf{e}}$ denote the estimated impulse vector from the CS algorithm. Sort its components in decreasing order of magnitude, and consider the candidate supports

1) $\mathcal{I}_0 = \mathbf{0}$ (no impulses);
2) $\mathcal{I}_1$ containing a single 1 in the position of the largest element of $\widehat{\mathbf{e}}$;
3) $\mathcal{I}_2$ containing two 1's in the position of the two largest elements of $\widehat{\mathbf{e}}$;
4) ... so on, till a maximum number $r_{\max} > np$ of ones.

We eventually select the support as the one that maximizes the above MAP metric among the above set of candidates.

C. Estimating the Impulse Amplitudes

Assuming that $\widehat{\mathcal{I}}$ is correct, we apply MMSE estimation for the non-zero components $\mathbf{u}_e$ of $\mathbf{e}$, and eventually reconstruct $\widehat{\mathbf{e}} = \widehat{\mathbf{S}}_e\widehat{\mathbf{u}}_e$, where $\widehat{\mathbf{S}}_e$ is the selection matrix corresponding to $\widehat{\mathcal{I}}$. Mathematically, we can write

$$\mathbf{e} = \widehat{\mathbf{S}}_e\mathbf{u}_e$$

and proceed to estimate the impulse amplitudes from the system of equations

$$\begin{aligned} \mathbf{y}' &= \mathbf{S}^{\mathsf{T}}\mathbf{F}\widehat{\mathbf{S}}_e\mathbf{u}_e + \mathbf{z}' \\ &\triangleq \boldsymbol{\Phi}_e\mathbf{u}_e + \mathbf{z}' \end{aligned} \tag{14}$$

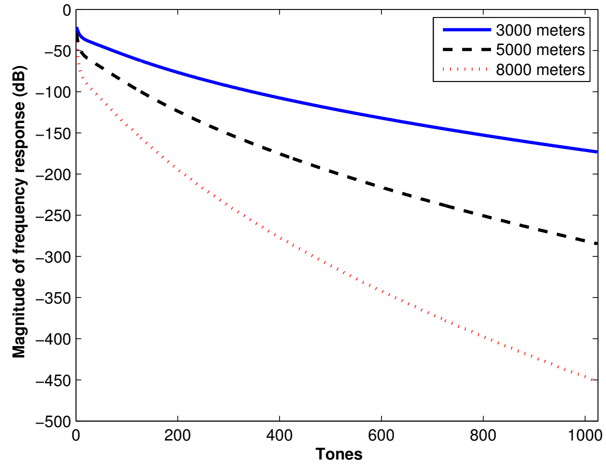

Fig. 2. Magnitude of the channel frequency response in dB for different distance between CO and CPE

This gives us the MMSE estimate

$$\widehat{\mathbf{u}}_e^{\text{mmse}} = \boldsymbol{\Phi}_e^{\mathsf{H}}(\frac{N_0}{I_0}\mathbf{I} + \boldsymbol{\Phi}_e\boldsymbol{\Phi}_e^{\mathsf{H}})^{-1}\mathbf{y}' \tag{15}$$

so that

$$\widehat{\mathbf{e}}^{\text{mmse}} = \widehat{\mathbf{S}}_e\boldsymbol{\Phi}_e^{\mathsf{H}}(\frac{N_0}{I_0}\mathbf{I} + \boldsymbol{\Phi}_e\boldsymbol{\Phi}_e^{\mathsf{H}})^{-1}\mathbf{y}' \tag{16}$$

IV. Performance Comparison over a DSL Channel

The characteristics of a DSL channel are determined by numerous factors that include wire length, wire diameter and gauge, bridged taps, load coils, different resistor-capacitor terminations and shunt resistances [12]. The DSL channel is modeled here based on the transmission line theory provided in [13]. Specifically, the "ABCD" model and the transmission line RLCG characterization are used to model the american wire gauge (AWG26) wire. The parameters used to model the AWG26 twisted pair [13] are given in Table I.

TABLE I
AWG26 Twisted Pair Parameters

Resistance	r_{0c}	r_{0s}	a_c	a_s
(value)	286.18 Ω/km	∞ Ω/km	0.15	0
Inductance	l_0	l_∞	b	f_m
(value)	675.37 μH/km	488.95 μ H/km	0.93	806.34 kHz
Capacitance	c_∞	c_0	c_e	
(value)	49 nF/km	0 nF/km	0	
Conductance	g_0	g_e		
(value)	43 nS/km	0.7		

The channel frequency response for the AWG26 cable with different distance between the central office (CO) and customer premises equipment (CPE) is presented in Figure 2. The number of tones is fixed at $n = 1024$ and a carrier spacing of 4.3125 KHz is used according to the ITU G.992.1 DSL standard. It can be observed that the channel gain reduces with the increase in frequency and longer length cables are attenuated more compared to the shorter ones. Due to this reason, most of the carriers at high frequencies are not utilized to transmit data and thus can be utilized for impulsive noise estimation and cancellation. The performance of the proposed algorithm is compared with [4] that uses CS to find the impulse support followed by MMSE estimation to recover the impulse amplitudes. We also compare the proposed algorithm with three recently developed sparse reconstruction algorithms namely, subspace pursuit (SP) [9], gradient

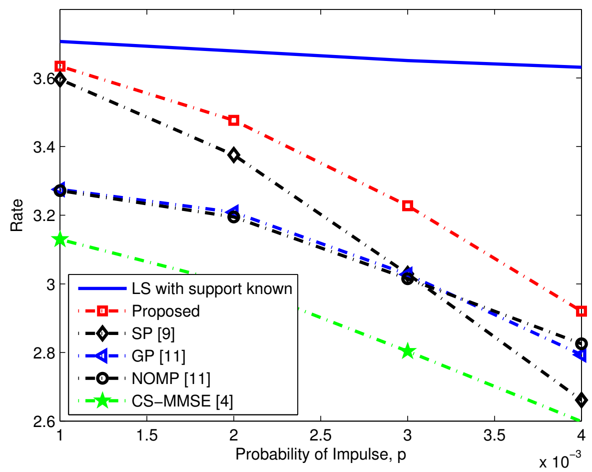

Fig. 3. Performance comparison for full band null carriers

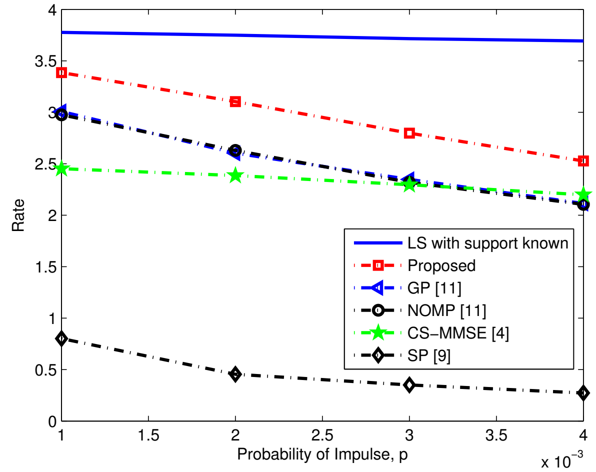

Fig. 4. Performance comparison for quarter band null carriers

pursuit (GP) [11], and nearly orthogonal matching pursuit (NOMP). The upper bound (benchmark) is given by the case when support is perfectly known and least squares (LS) is used for estimation of amplitudes. We consider an AWG26 wire of length 8000 meters, n = 1024 tones and m = 30 null carriers, with SNR = 20 dB and INR = 35 dB. The range of number of impulses (K) is $0 \leq K \leq 10$ and all the algorithms are run for 256 Monte Carlo iterations at each value of K. The probability of impulse p is assumed to be known a priori at the receiver. We evaluate the achievable rate (similar to [4]) for all the algorithms in the following two scenarios.

A. Full band

Full band implies that the m null carriers used for impulsive noise estimation and cancellation are chosen at random with uniform probability over all possible $\binom{n}{m}$ subsets. The performance of the algorithms in this case is presented in Figure 3. It can be seen that the proposed algorithm easily outperforms the CS-MMSE algorithm. This signifies the advantage of refining the CS output using a priori statistical information. The performance of the proposed algorithm is also superior to all the three recently developed algorithms.

B. Quarter band

Quarter band here means that the m null carriers are chosen at random with uniform probability over the last $\binom{n/4}{m}$ subsets i.e. null carriers can only be placed in the last 25% carriers. This is in accordance with DSL specifications as most of the carriers at spectrum edges are not considered for data transmission due to high attenuation. From Figure 4, it can be observed that the proposed algorithm performs better than all the other algorithms. The performance of greedy algorithms (GP and NOMP) is reasonable but the SP algorithm fails completely as the projection matrix $\mathbf{\Psi}$ is not rich enough in this case (as null carriers are not random enough). The proposed algorithm beats the CS-MMSE algorithm in this case also.

V. Conclusion

In this paper, we propose a CS based algorithm to detect, estimate and eliminate the effect of impulsive noise is DSL systems. The presented algorithm consists of three steps. First, the impulsive noise is projected onto a small dimensional space to obtain a coarse estimate of the impulse positions using CS. This is followed by its refinement using a MAP metric based on a priori information that the impulse noise is Bernoulli-Gaussian. Finally, the impulse amplitudes are obtained using MMSE estimation. Simulation results demonstrate that the proposed algorithm easily outperforms the CS-MMSE algorithm presented in [4] which signifies the gain achieved by refining the CS output. The proposed algorithm is also compared with three recently developed sparse reconstruction methods over a DSL channel and the simulations show the favorable performance of the proposed algorithm.

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