Time-varying DOA estimation for multiple moving targets
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1 Signal Model and problem formulation

Assume that an array of M sensors receives K targets impinging from unknown time-varying directions. Over the observation interval, the Rayleigh fading amplitude of the target is assumed to vary in time according to Jakes' or first order autoregressive (AR1) correlation models. The $M \times 1$ array snapshot complex vectors at the output can be expressed as

$$\mathbf{y}(n) = \sum_{k=1}^{K} \mathbf{a}(\theta_k(n))h_k(n) + \mathbf{n}(n) = \mathbf{A}(\boldsymbol{\theta}_n)\mathbf{h}(n) + \mathbf{n}(n),\ n = 0, \ldots, N-1 \tag{1.1}$$

where $\mathbf{A}(\boldsymbol{\theta}_n)$ is an $M \times K$ ($K < M$) matrix formed by the steering vector $\mathbf{a}(\theta_k(n))$. We suppose $\|\mathbf{a}(\theta_k(n))\|^2 = M$. In (1.1), $\boldsymbol{\theta}_n \stackrel{\text{def}}{=} (\theta_k(n), \ldots, \theta_K(n))^T$ is the unknown time-varying DOA vector of K targets, $\mathbf{h}(n) = (h_1(n), \ldots, h_k(n))^T$ is the time-varying vector of complex amplitudes of K target returns. The process $h_k(n)$ is the sample of the fading amplitude of the kth target assumed to be zero-mean circular complex Gaussian with unknown variance σ_k^2 and correlation function given by:

$$R_h^J(m) \stackrel{\text{def}}{=} \sigma_h^2 \mathrm{E}(h_k(n)h_k^*(n-m) = \sigma_k^2 J_0(2\pi f_d T m),$$

where $J_0(.)$ is the first kind 0th-order Bessel function, T is the symbol period and f_d denotes the maximum Doppler shift. This is frequently referred to as the Jakes' model [3].

AR1 model of fading Among various channel models, the information theoretic results in [4] show that the first-order AR model provides a sufficiently accurate model for time fading channels

$$h_k(n) = \gamma h_k(n-1) + e_k(n) \tag{1.2}$$

where $e_k(n) \sim \mathcal{N}(0, \sigma_k^2(1-\gamma^2))$ is the additive driving noise and where $\gamma \stackrel{\text{def}}{=} J_0(2\pi f_d T)$ is assumed to be unknown.

For simplicity, the time-varying $\boldsymbol{\theta}(n)$ is modeled as

$$\boldsymbol{\theta}(n) = \boldsymbol{\theta}_0 + n\boldsymbol{\theta}_1, \tag{1.3}$$

where $\boldsymbol{\theta}_0 = (\theta_{01}, \ldots, \theta_{0K})^T$ and $\boldsymbol{\theta}_1 = (\theta_{11}, \ldots, \theta_{1K})^T$. The linear polynomial (1.3) can be seen as a truncated Taylor expansion which gives a good description for the source motion in a small observation [6, 7].

Having the model for the variation of the channel (1.2) and the DOA (1.3), and from (1.1), we can obtain the following state space representation of the problem

$$\begin{cases} \mathbf{y}(n) = \mathbf{A}(\boldsymbol{\theta}(n))\mathbf{h}(n) + \mathbf{n}(n) \\ \mathbf{h}(n) = \gamma\mathbf{h}(n-1) + \mathbf{e}(n) \\ \boldsymbol{\theta}(n) = \boldsymbol{\theta}_0 + n\boldsymbol{\theta}_1 \end{cases} \tag{1.4}$$

where $\mathbf{e}_n \stackrel{\text{def}}{=} (e_1(n), \ldots, e_K(n))^T \sim \mathcal{N}(0, (1-\gamma^2)\mathbf{Q})$ and where $\mathbf{Q} \stackrel{\text{def}}{=} \mathrm{Diag}(\sigma_1^2, \ldots, \sigma_K^2)$.

The estimation problem can now be formulated as follows: Given the received signal $\mathbf{y} = (\mathbf{y}(0), \ldots, \mathbf{y}(N-1))^T$ an unknown parameter vector $\boldsymbol{\alpha} \stackrel{\text{def}}{=} (\mathbf{h}^T, \gamma, \boldsymbol{\sigma}, \boldsymbol{\theta}_0^T, \boldsymbol{\theta}_1^T)^T$, estimate $\boldsymbol{\theta} \stackrel{\text{def}}{=} (\boldsymbol{\theta}_0^T, \boldsymbol{\theta}_1^T)^T$. In this problem, $\boldsymbol{\theta}$ is the parameter of interest and the other parameters are nuisance parameters.

2 Objective

The main objective of this project is to extend the algorithm proposed in [1, 2] by combining the expectation-maximization (EM) algorithm with Kalman smoother algorithm to yield time-varying DOA estimation and ML estimate of channel parameters.

To evaluated the performance of this estimator it should be interesting to calculate the hybrid Cramér Rao-bound basing on the work presented in [5].

References

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