Mean Convergence Analysis of NLMS and ϵ-NLMS

1 Problem Description

Adaptive filter have found applications in many diverse fields ranging from equalization, echo cancelation, linear prediction, noise cancelation, system identification, channel estimation and beam-forming. The most basic and most widely used adaptive algorithm is the Least Mean Squares (LMS) algorithm. Though it is simple and easy to implement, it suffers from slow convergence especially in case of highly correlated input data, e.g. speech. It turns out that introducing a normalization term in the update equation speeds up the convergence rate. This algorithm is termed the normalized LMS (NLMS) algorithm. The weight update equations of the NLMS and ϵ-NLMS algorithms have the following forms

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \frac{\mathbf{u}_i^*}{||\mathbf{u}_i||^2} e(i), \qquad i \geq 0 \tag{1.1}$$

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \frac{\mathbf{u}_i^*}{\epsilon + ||\mathbf{u}_i||^2} e(i), \quad i \geq 0 \tag{1.2}$$

respectively, where $\mathbf{w}_i$ is the estimate of $\mathbf{w}^o$, $\mathbf{u}_i$ is the input regression vector and $e(i)$ is the estimation error defined as

$$e(i) = d(i) - \mathbf{u}_i \mathbf{w}_{i-1} = \mathbf{u}_i \mathbf{w}^o - \mathbf{u}_i \mathbf{w}_i + v(i), \tag{1.3}$$

where $d(i)$ is the desired signal response, and $v(i)$ is zero mean i.i.d noise sequence with variance σ_v^2 that is independent of the input sequence.

The normalization in equations (1.1) and (1.2) however, complicates the performance analysis of the NLMS and ϵ-NLMS algorithm. It can be shown [1] that the mean behavior of the NLMS and ϵ-NLMS algorithm depends on the solution of expectation terms of the form $E[\frac{f(\mathbf{u}_i)}{g(\mathbf{u}_i)}]$. These expectations are difficult to evaluate and various approximations have been proposed in literature. Recently, a novel approach for solving such expectations has been proposed in [2]. This is done by first writing these expressions in an appropriate form, then using complex integration to evaluate the Cumulative Distribution Function (CDF) and then using the CDF to evaluate the exact multidimensional moments. Once the moments are obtained, the transient performance of these algorithms can be analyzed.

2 Objective

This project is a continuation of the work done in [2]. The project aims to reproduce some of the results of [2] and extend the work to evaluate the exact mean behavior of the NLMS and ϵ-NLMS algorithm for complex Gaussian data.

References

[1] A. H. Sayed, *Fundamentals of Adaptive Filtering*, New York: Wiley-Interscience, 2003.

[2] T. Y. Al-Naffouri and M. Moinuddin, "Exact Performance Analysis of the ϵ-NLMS Algorithm", *submitted to IEEE Transaction on Signal Processing.*