Data-aided Channel Estimation in OFDM Systems using Kalman Filter

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There has been increased interest in Orthogonal Frequency Division Multiplexing (OFDM) due to its high achievable data rates, multipath robustness and simple receiver implementation. This is reflected by the many standards that considered and adopted OFDM, e.g. ADSL, VDSL, power line communication, WiFi (IEEE 802.11a), WiMAX (IEEE 802.16), terrestrial broadcast (DVB-T), and ultrawideband personal area networks (IEEE 802.15.3a) [1]. In OFDM systems, channel must be estimated accurately for high speed communication. The channel estimation techniques present in the literature can be broadly divided into four categories:

1. *Training-based:* These techniques involve sending training data (pilots) along with the data symbols for estimating the channel [2], [3], [4]. Use of pilots decreases the bandwidth efficiency.

2. *Blind:* Blind techniques use the structure of the communication system created by such constraints as finite alphabet constraint [5], cyclic prefix [5], [6], and time and frequency correlation [7], [8]. Blind techniques usually require averaging over many symbols before they converge (this implicitly assumes that the channel remains invariant over these symbols).

3. *Semi-blind:* Semi-blind techniques are hybrid of training-based and blind methods. These methods use pilots to obtain an initial channel estimate and improve the estimate by using a variety of a priori information [7], [9].

4. *Data-aided:* The main and perhaps the only reason to perform channel estimation at the receiver is to use the estimate along with the channel output to recover the transmitted data. One can, in turn, use the detected data to enhance the channel estimate giving rise to an iterative technique for joint channel and data recovery [7], [8].

1 Signal model and problem formulation

Consider the iid sequence $\mathcal{X}_0^{(T+1)N} = (\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_{(T+1)N})$ to be transmited. In an OFDM system, the sequence is parsed into a sequence of $T+1$ data symbols $\boldsymbol{\mathcal{X}}_0^T = (\boldsymbol{\mathcal{X}}_0, \boldsymbol{\mathcal{X}}_1, \cdots, \boldsymbol{\mathcal{X}}_T)$ each of length N. Each symbol $\boldsymbol{\mathcal{X}}_i$ undergoes an IDFT operation to produce the time domain symbol $\boldsymbol{x}_i = \sqrt{N}\boldsymbol{Q}^*\boldsymbol{\mathcal{X}}_i$, where $\boldsymbol{Q}$ is the $N \times N$ DFT matrix for which $q_{l,m} = e^{-j\frac{2\pi(l-1)(m-1)}{N}}$. The transmitter then appends a cyclic prefix (CP) $\underline{\boldsymbol{x}}_i$ (of length P) to $\boldsymbol{x}_i$, resulting finally in a sequence of super-symbols $\overline{\boldsymbol{x}}_0^T$ each of length $N+P$.

We assume that the channel $\underline{\boldsymbol{h}}_i$ (of maximum length $P+1$) remains fixed over a given OFDM symbol (and its associated CP) but varies from one symbol to the next according to a state-space model

$$\underline{\boldsymbol{h}}_{i+1} = \boldsymbol{F}\underline{\boldsymbol{h}}_i + \boldsymbol{G}\underline{\boldsymbol{u}}_i, \quad \underline{\boldsymbol{h}}_0 \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Pi}_0), \quad \underline{\boldsymbol{u}}_i \sim \mathcal{N}(\boldsymbol{0}, \sigma_u^2\boldsymbol{I}) \tag{1.1}$$

The matrices $\boldsymbol{F}$ and $\boldsymbol{G}$ in (1.1) are square matrices of size $P+1$ and are assumed available to the receiver. The details of how we can construct such a model from the knowledge of the Doppler frequency (time-correlation) and the power-delay profile (frequency-correlation) is present in [7] and [8]. At the channel output, we obtain a sequence of time-domain super-symbols $\overline{\boldsymbol{y}}_0^T$, which after stripping the cyclic prefix $\underline{\boldsymbol{y}}_i$, produce a sequence of time-domain symbols $\boldsymbol{y}_0^T$. The input/output (I/O) relationship of the OFDM system is best described in the frequency domain

$$\boldsymbol{\mathcal{Y}}_i = \mathrm{diag}(\boldsymbol{\mathcal{X}}_i)\boldsymbol{\mathcal{H}}_i + \boldsymbol{\mathcal{N}}_i = \mathrm{diag}(\boldsymbol{\mathcal{X}}_i)\boldsymbol{Q}_{P+1}\underline{\boldsymbol{h}}_i + \boldsymbol{\mathcal{N}}_i \tag{1.2}$$

where $\boldsymbol{\mathcal{N}}_i \sim \mathcal{N}(\boldsymbol{0}, \sigma_n^2\boldsymbol{I})$ is the additive noise. The second equality in (1.2) follows from the DFT relationship $\boldsymbol{\mathcal{H}}_i \triangleq \boldsymbol{Q}\begin{bmatrix} \underline{\boldsymbol{h}}_i \\ \boldsymbol{0} \end{bmatrix} = \boldsymbol{Q}_{P+1}\underline{\boldsymbol{h}}_i$, where $\boldsymbol{Q}_{P+1}$ consists of the first $P+1$ columns of $\boldsymbol{Q}$. Alternatively, with $\boldsymbol{X}_i \triangleq \mathrm{diag}(\boldsymbol{\mathcal{X}}_i)\boldsymbol{Q}_{P+1}$, we can write

$$\boldsymbol{\mathcal{Y}}_i = \boldsymbol{X}_i\underline{\boldsymbol{h}}_i + \boldsymbol{\mathcal{N}}_i \tag{1.3}$$

2 Objective

The main objective of this project is to estimate the channel using an Expectation Maximization (EM) based forward backward Kalman filter [7], [8] for the system model described in the previous section. Specifically, the following cases should be studied:

1. Known input case (Training-based channel estimation).
2. Unknown input case (Data-aided channel estimation).
3. Performance comparison of using soft and hard estimate of the input.
4. Performance comparison of different implementations of Kalman filter including Forward-only Kalman filter, Cyclic FB Kalman filter, Helix FB Kalman filter [10].

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