

Solve the problem

$$\min_w \alpha_1 \|y - Hw\|_{W_1}^2 + \alpha_2 \|y - Hw\|_{W_2}^2$$

where  $\alpha_1, \alpha_2 > 0$  &  $W_1$  and  $W_2$  are positive definite

Solve the problem by articulating it as a LS problem.

Note that

$$\begin{aligned} \alpha \|x\|_{W_1}^2 + \alpha \|x\|_{W_2}^2 &= \alpha_1 x^* W_1 x + \alpha_2 x^* W_2 x \\ &= x^* \alpha_1 W_1 x + x^* \alpha_2 W_2 x \\ &= x^* (\alpha_1 W_1 + \alpha_2 W_2) x \end{aligned}$$

$$\therefore \alpha \|x\|_{W_1}^2 + \alpha \|x\|_{W_2}^2 = \|x\|_{\alpha_1 W_1 + \alpha_2 W_2}^2$$

(Diagonality property)

$$\text{So } \min_w \alpha_1 \|y - Hw\|_{W_1}^2 + \alpha_2 \|y - Hw\|_{W_2}^2$$

$$= \min_w \|y - Hw\|_{\alpha_1 W_1 + \alpha_2 W_2}^2$$

$A^* A$ ?

Can we write  $\alpha_1 W_1 + \alpha_2 W_2$  as

Question:

Note that  $W = \alpha_1 W_1 + \alpha_2 W_2$  is Hermitian.  
eigenvalue decomposition of  $W$ .

Also, since  $W_1$  &  $W_2$  are the definite &  $\alpha_1, \alpha_2 > 0$ ,  
then  $W > 0$ .

So, we can have

Prove it!