King Fahd University of Petroleum and Minerals

Department of Electrical Engineering

EE-315-Probabilistic Methods in Electrical Engineering
SECOND SEMESTER 2010-2011 (102)

TEXT BOOK:

Peebles, P. Z. "*Probability, Random Variables, and Random Signal Principles*", McGraw-Hill, 4th Edition, 2001.

HW # 5	4.4-3, 4.5-7, 4.6-10, 5.1-4, 5.1-12, 5.1-19, 5.1-34, 5.3-4, 5.4-1, **Due Date: 02/05/2011**

(4.4-3.) From Example 4.4-2 $f_Y(y|X=x) = u(x)u(y)\,x\,e^{-xy}$.

Thus, $P\{Y \le 2 | X=1\} = \int_{-\infty}^{2} f_Y(y|X=1)\,dy = \int_0^2 e^{-y}\,dy$

$= 1 - e^{-2} \approx 0.8647.$

(4.5-7.) From the solution to Problem 4.3-17 we have

$f_X(x)\,f_Y(y) = f_{X,Y}(x,y)$ so X and Y are independent.

(4.6-10.) $f_W(w) = \int_{-\infty}^{\infty} f_X(w-y)\,f_Y(y)\,dy = \int_{-\infty}^{\infty} 5e^{-5(w-y)}\,u(w-y)\,u(y)$

$\cdot\, 2e^{-2y}\,dy = 10\int_0^{w} e^{-5w-(2-5)y}\,dy, \quad w>0$

$= \frac{10}{3}\,u(w)\left[e^{-2w} - e^{-5w}\right].$

(5.1-4.) From (5.1-1): $E\left[e^{-2(X^2+Y^2)}\right] = \int_0^\infty \int_0^\infty e^{-2(x^2+y^2)}$
$\cdot 16 e^{-4(x+y)} dx\, dy = 16 \int_0^\infty e^{-2x^2-4x} dx \int_0^\infty e^{-2y^2-4y} dy$
$= 16 e^4 \int_0^\infty e^{-2(x+1)^2} dx \int_0^\infty e^{-2(y+1)^2} dy$.

(5.1-12.) (a) $m_{20} = \int_{-3}^{3} \int_{-1}^{1} \frac{x^2(x+y)^2}{40} dx\, dy = \frac{9}{25} = 0.36$.

$m_{02} = \int_{-3}^{3} \int_{-1}^{1} \frac{y^2(x+y)^2}{40} dx\, dy = \frac{129}{25} = 5.16.$

$m_{11} = \int_{-3}^{3} \int_{-1}^{1} \frac{xy(x+y)^2}{40} dx\, dy = \frac{6}{10} = 0.6.$

(b) $\sigma_X^2 = m_{20} - (m_{10})^2$, $\sigma_Y^2 = m_{02} - (m_{01})^2$

$m_{10} = \int_{-3}^{3} \int_{-1}^{1} \frac{x(x+y)^2}{40} dx\, dy = 0$, $m_{01} = \int_{-3}^{3} \int_{-1}^{1} \frac{y(x+y)^2}{40} dx\, dy = 0.$

$\sigma_X^2 = m_{20} = 0.36$, $\sigma_Y^2 = m_{02} = 5.16$.

(c) Use (5.1-17): $\rho_{XY} = E[(X-\bar{X})(Y-\bar{Y})]/\sigma_X\sigma_Y = C_{XY}/\sigma_X\sigma_Y =$
$[R_{XY} - \bar{X}\bar{Y}]/\sigma_X\sigma_Y = [m_{11} - m_{10}m_{01}]/\sqrt{m_{20}m_{01}} \approx 0.440.$

(5.1-19.) (a) Use (5.1-5). $m_{10} = E[X] = \int_0^3 \int_0^2 x \frac{2}{43} (x+0.5y)^2 dx\,dy$

$= 57/43 \approx 1.326$. $m_{01} = E[Y] = \int_0^3 \int_0^2 y \frac{2}{43} (x+0.5y)^2 dx\,dy$

$= 321/172 \approx 1.866$. $m_{20} = E[X^2] = \int_0^3 \int_0^2 x^2 \frac{2}{43} (x+0.5y)^2 dx\,dy$

$= 432/215 \approx 2.009$. $m_{02} = E[Y^2] = \int_0^3 \int_0^2 y^2 \frac{2}{43} (x+0.5y)^2 dx\,dy$

$= 888/215 \approx 4.130$. $m_{11} = E[XY] = \int_0^3 \int_0^2 xy \frac{2}{43} (x+0.5y)^2 dx\,dy$

$= 417/172 \approx 2.424$.

(b) From (5.1-14) $C_{XY} = R_{XY} - E[X]E[Y] = m_{11} - m_{10} m_{01}$

$= \frac{417}{172} - \frac{57}{43}\left(\frac{321}{172}\right) = \frac{-366}{(43)^2 4} \approx -0.0495$.

(c) X and Y are not uncorrelated because $C_{XY} \neq 0$.

(5.1-34.) $\overline{W} = \overline{2X+Y} = 2\bar{X} + \bar{Y} = 0 - 1 = -1$, $\overline{U} = \overline{-X-3Y} = -\bar{X} - 3\bar{Y} = 3$.

$\overline{W^2} = \overline{(2X+Y)^2} = 4\overline{X^2} + 4\overline{XY} + \overline{Y^2} = 4(2) + 4(-2) + 4 = 4$, $\overline{U^2} =$

$\overline{(-X-3Y)^2} = \overline{X^2} + 6\overline{XY} + 9\overline{Y^2} = 2 + 6(-2) + 9(4) = 26$, $R_{WU} = \overline{WU} =$

$\overline{(2X+Y)(-X-3Y)} = -2\overline{X^2} - 7\overline{XY} - 3\overline{Y^2} = -2$, $\sigma_X^2 = \overline{X^2} - \bar{X}^2 = 2 - 0 = 2$,

$\sigma_Y^2 = \overline{Y^2} - \bar{Y}^2 = 4 - 1 = 3$,

(5.3-4.) Use (5.3-11): $\rho = \frac{\sigma_X^2 - \sigma_Y^2}{2\sigma_X \sigma_Y} \tan(2\theta) = \frac{9-4}{2\sqrt{9}\sqrt{4}} \tan\left(\frac{-\pi}{4}\right) = \frac{-5}{12}$.

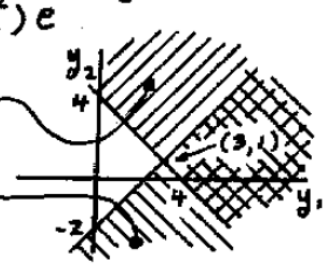
★ 5.4-1. (a) From Example 5.4-3 with $a = 1$, $b = 1$, $c = 1$ and $d = -1$:

$$f_{Y_1,Y_2}(y_1,y_2) = \frac{1}{|-2|} f_{X_1,X_2}\left(\frac{y_1+y_2}{2}, \frac{y_1-y_2}{2}\right)$$

$$= \frac{1}{6}\, u\left(\frac{y_1+y_2-4}{2}\right) u\left(\frac{y_1-y_2-2}{2}\right)(y_1-y_2)(y_1^2-y_2^2)\, e^{4-\frac{1}{2}(y_1^2-y_2^2)}$$

(b) $u\left(\frac{y_1+y_2-4}{2}\right) > 0$ only for $y_2 > 4 - y_1$

$u\left(\frac{y_1-y_2-2}{2}\right) > 0$ only for $y_2 < y_1 - 2$



Applicable points shown crosshatched.