King Fahd University of Petroleum and Minerals

Department of Electrical Engineering

EE-315-Probabilistic Methods in Electrical Engineering

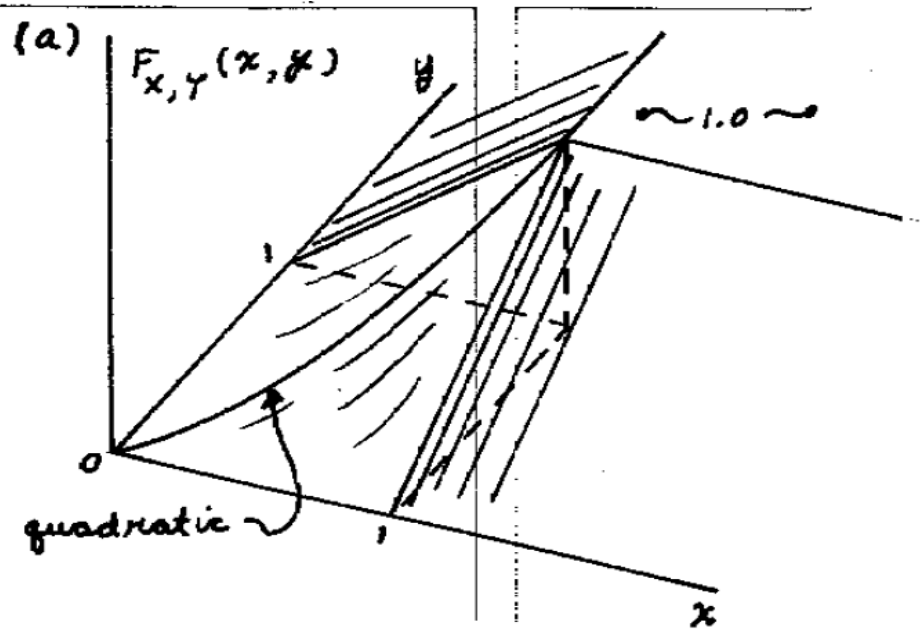
SECOND SEMESTER 2010-2011 (102)

<u>TEXT BOOK:</u>

Peebles, P. Z. "*Probability, Random Variables, and Random Signal Principles*", McGraw-Hill, 4th Edition, 2001.

HW # 4	4.2-8, 4.2-11, 4.3-8, 4.3-11, **Due Date: 18/04/2011**

4.2-8. (a)



(b) $F_X(x) = F_{X,Y}(x,\infty) = 0, \quad x < 0$

$= x, \quad 0 \le x < 1$

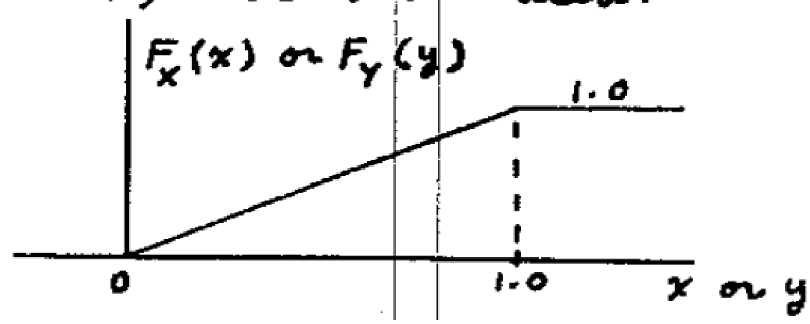
$= 1, \quad 1 \le x.$

$F_Y(y) = F_{X,Y}(\infty, y) = 0, \quad y < 0$

$= y, \quad 0 \le y < 1$

$= 1, \quad 1 \le y.$

Here (4.2-6f) has been used.



(4.2-11.) (a)

$$F_X(x) = F_{X,Y}(x,\infty) = \begin{cases} 0, & x<0 \\ \frac{5x}{4(x+1)}, & 0 \le x < 4 \\ 1, & x \ge 4 \end{cases}$$

$$F_Y(y) = F_{X,Y}(\infty,y) = \begin{cases} 0, & y<0 \\ 1+\frac{1}{4}e^{-5y^2} - \frac{5}{4}e^{-y^2}, & y \ge 0 \end{cases}$$

(b) $P\{3 < X \le 5, 1 < Y \le 2\} = F_{X,Y}(5,2) + F_{X,Y}(3,1) - F_{X,Y}(3,2)$
$- F_{X,Y}(5,1) = [1+\frac{1}{4}e^{-5(4)} - \frac{5}{4}e^{-4}] + \frac{5}{4}[\frac{3+e^{-4}}{4} - e^{-1}]$
$-\frac{5}{4}[\frac{3+e^{-4(4)}}{4} - e^{-4}] - [1+\frac{1}{4}e^{-5} - \frac{5}{4}e^{-1}] = 0.004039.$

(4.3-8.) (a) The requirement $f_{X,Y}(x,y) \ge 0$ means tha[t] $b>0$ is necessary. Since area must be unity

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx\,dy = \int_{y=0}^{\infty}\int_{x=0}^{a} b\,e^{-x}e^{-y}\,dx\,dy = b[1-e$$

$= 1$, $\quad b = [1-e^{-a}]^{-1}$.

(b) $F_{X,Y}(x,y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f_{X,Y}(\alpha,\beta)\,d\alpha\,d\beta$

$$= \int_{-\infty}^{y}\int_{-\infty}^{x} u(\alpha)\,u(\beta)\,u(a-\alpha)\,\frac{e^{-\alpha}e^{-\beta}}{(1-e^{-a})}\,d\alpha\,d\beta$$

This function is zero when $x<0$ or $y<0$. Straightforward solution for the remaining cases produces

$$F_{X,Y}(x,y) = 0 \qquad x<0 \text{ or } y<0$$

$$= \frac{(1-e^{-x})(1-e^{-y})}{(1-e^{-a})}, \quad 0 \le x < a \text{ and } 0 \le y$$

$$= 1-e^{-y}, \quad a \le x \text{ and } 0 \le y.$$

★ 4.3-11. Here $f_{X,Y}(x,y)$ exists (is nonzero) over the area of the xy plane within a circle of radius $\sqrt{b}$ that is centered on the origin.

(a) $\int\int [(x^2+y^2)/8\pi]\,dx\,dy = \int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{b}} \frac{r^2}{8\pi}\, r\,dr\,d\theta = 1$

over circular area in xy plane

Here $r = \sqrt{x^2+y^2}$, $r\,dr\,d\theta = dx\,dy$ and $\theta = \tan^{-1}(y/x)$ define a change from rectangular to polar coordinates. The integrals reduce to

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{\sqrt{b}} \frac{r^3}{8\pi}\,dr\,d\theta = b^2/16 \overset{\text{must}}{=} 1 \quad \text{so} \quad b = 4.$$

(b) The set $\{0.5b < X^2+Y^2 \le 0.8b\}$ corresponds to the annulus defined by $\sqrt{b/2} < r \le \sqrt{0.8b}$ and $0 < \theta \le 2\pi$. Thus,

$$P\{0.5b < X^2+Y^2 \le 0.8b\} = \int_{\theta=0}^{2\pi}\int_{r=\sqrt{0.5b}}^{\sqrt{0.8b}} \frac{r^3}{8\pi}\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{r^4}{32\pi}\Big|_{\sqrt{0.5b}}^{\sqrt{0.8b}}\,d\theta = 0.39\,b^2/16 = 0.39.$$