King Fahd University of Petroleum and Minerals

Department of Electrical Engineering

EE-315-Probabilistic Methods in Electrical Engineering
SECOND SEMESTER 2010-2011 (102)

TEXT BOOK:

Peebles, P. Z. "*Probability, Random Variables, and Random Signal Principles*", McGraw-Hill, 4th Edition, 2001.

HW # 3	2.5-3, 2.6-3, 3.1-11, 3.1-15, 3.2.-23, 3.2-24, 3.4-2, 3.4-10, **Due Date: 04/04/2011**

2.5-3. We use the Rayleigh distribution, given by (2.5-7) with $a = 0$ and $b = 400$, for probability.

(a) $P\{X \le 1\} = F_X(1) = 1 - e^{-1/400} \approx 0.0025$ or 0.25%.

(b) $P\{X > 52\} = 1 - F_X(52) = 1 - [1 - e^{-(52)^2/400}]$
$= e^{-(52)^2/400} \approx 0.00116$ or 0.12%.

★ 2.6-3. The expressions given in Problem 2.6-2 apply. Specifically, let $a = 20$ and $b = \infty$ so

$$f_X(x \mid 20 < X) = 0, \qquad x < 20$$

$$= \frac{f_X(x)}{\int_{20}^{\infty} f_X(x)\,dx}, \qquad 20 \le x < \infty. \qquad (1)$$

However, here

$$f_X(x) = \frac{x}{200} e^{-x^2/400}, \qquad 0 \le x$$

$$= 0, \qquad x < 0$$

so

$$\int_{20}^{\infty} f_X(x)\,dx = 1 - \int_{-\infty}^{20} f_X(x)\,dx = 1 - F_X(20) = e^{-(20)^2/400} = e^{-1}. \quad (2)$$

Since the probability of system lifetime being larger than 26 weeks, given that it has survived beyond 20 weeks, is $P\{X > 26 \mid X > 20\}$, we use (2) with (1) to obtain

$$P\{X > 26 \mid X > 20\} = \int_{26}^{\infty} f_X(x \mid X > 20)\,dx$$

$$= e \int_{26}^{\infty} \frac{x}{200} e^{-x^2/400}\,dx.$$

*2.6-3. (Continued)

Let $\xi = x^2/400$, $d\xi = x\,dx/200$ and use (C-45) to get

$$P\{X > 26 \mid X > 20\} = e \int_{(26)^2/400}^{\infty} e^{-\xi}\,d\xi = e\left[-e^{-\xi}\Big|_{(26)^2/400}^{\infty}\right]$$

$$= e^{-0.69} \approx 0.5016.$$

3.1-11. $E[g(X)] = E[4X^2] = \int_{-\infty}^{\infty} 4x^2 f_X(x)\,dx = 4\int_{-\pi/2}^{\pi/2} \frac{x^2}{2}\cos(x)\,dx$

$$= \pi^2 - 8 \approx 1.8696.$$

(3.1-15.) Here $Y = g(X) = 5X^2$. Thus, $E[Y] = E[g(X)]$

$$= \int_{-\infty}^{\infty} 5x^2 \frac{e^{-x^2/2\sigma_X^2}}{\sqrt{2\pi}\,\sigma_X} dx.$$ Let $\xi = x/\sigma_X$, $d\xi = dx/\sigma_X$.

$$E[Y] = 5\sigma_X^2 \underbrace{\int_{-\infty}^{\infty} \xi^2 \frac{e^{-\xi^2/2}}{\sqrt{2\pi}} d\xi}_{\text{Variance of zero-mean Gaussian r.v. with variance} = 1}, \text{ where } \sigma_X^2 = 9.$$

Variance of zero-mean Gaussian r.v. with variance $= 1$ so $E[Y] = 45$.

(3.2-23.) (a) $\bar{X} = \int_{-4}^{4} \frac{\pi}{16} x \cos\left(\frac{\pi x}{8}\right) dx$ ← Let $\alpha = \pi x/8$, $d\alpha = \pi\, dx/8$

$= \frac{4}{\pi}\int_{-\pi/2}^{\pi/2} \alpha \cos(\alpha)\, d\alpha = 0$ from (C-40). (b) $\overline{X^2} =$

$\int_{-4}^{4} \frac{\pi}{16} x^2 \cos\left(\frac{\pi x}{8}\right) dx = \frac{32}{\pi^2}\int_{-\pi/2}^{\pi/2} \alpha^2 \cos(\alpha)\, d\alpha = 16\left(1 - \frac{8}{\pi^2}\right).$

(c) $\sigma_X^2 = \overline{X^2} - \bar{X}^2 = \overline{X^2} = 16\left[1 - (8/\pi^2)\right]$.

(3.2-24.) With the dc signal present it acts as a shift in the density of the random noise. The shift becomes the mean of a gaussian random variable. Thus,

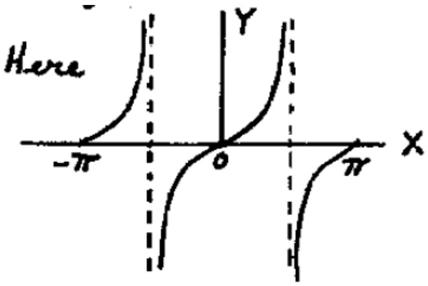
$P\{0 < X\} = 0.2514 = 1 - F_X(0) = 1 - F\left(\frac{0 - \bar{X}}{\sigma_X}\right)$ and

$F(-\bar{X}/\sigma_X) = 1 - 0.2514 = 0.7486$. From Table B-1

this occurs where $-\bar{X}/\sigma_X = 0.67$ so $\bar{X} = -0.67\sigma_X$

$= -0.67(10^{-3}) = -670\,\mu V$.

(3.4-2.) A sketch is helpful. Here $Y = T(X)$ is not monotonic.



We use (3.4-11).

When $y < 0$:

$$-\pi/2 < x < \tan^{-1}(y/a)$$

and $\pi/2 < x < \pi + \tan^{-1}(y/a)$ so

$$F_Y(y) = \int_{-\pi/2}^{\tan^{-1}(y/a)} \frac{dx}{2\pi} + \int_{\pi/2}^{\pi+\tan^{-1}(y/a)} \frac{dx}{2\pi}$$

$$= \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(y/a), \quad y < 0. \qquad (1)$$

When $y \geq 0$:

$$-\pi < x < -\pi + \tan^{-1}(y/a)$$

and $-\pi/2 < x < \tan^{-1}(y/a)$

and $\pi/2 < x < \pi$ so

$$F_Y(y) = \int_{-\pi}^{-\pi+\tan^{-1}(y/a)} \frac{dx}{2\pi} + \int_{-\pi/2}^{\tan^{-1}(y/a)} \frac{dx}{2\pi} + \int_{\pi/2}^{\pi} \frac{dx}{2\pi}$$

$$= \frac{1}{2} + \frac{1}{\pi}\tan^{-1}(y/a), \quad y \geq 0. \qquad (2)$$

(3.4-2.) (Continued)

Combining (1) and (2)

$$F_Y(y) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{y}{a}\right), \quad -\infty < y < \infty.$$

By differentiation

$$f_Y(y) = \frac{dF_Y(y)}{dy} = \frac{a/\pi}{a^2 + y^2}, \quad -\infty < y < \infty.$$

3.4-10. (a) Y is discrete. $P\{Y=-4\} = P\{X<-1\} = F_X(-1)$

$= F\left(\frac{-1-0.6}{0.8}\right) = F(-2) = 1 - F(2) = 1 - 0.9772 = 0.0228.$

$P\{Y=-2\} = P\{-1 \le X < 0\} = F\left(\frac{0-0.6}{0.8}\right) - F\left(\frac{-1-0.6}{0.8}\right) =$

$-F(0.75) + F(2.0) = 0.9772 - 0.7734 = 0.2038.$

$P\{Y=2\} = P\{0 \le X < 1\} = F\left(\frac{1-0.6}{0.8}\right) - F\left(\frac{0-0.6}{0.8}\right) = 0.7734$

$-1.0 + 0.6915 = 0.4649.$ $P\{Y=4\} = P\{1 \le X < \infty\}$

$= 1 - F\left(\frac{1-0.6}{0.8}\right) = 1 - 0.6915 = 0.3085.$