King Fahd University of Petroleum and Minerals

Department of Electrical Engineering

EE-315-Probabilistic Methods in Electrical Engineering
SECOND SEMESTER 2010-2011 (102)

TEXT BOOK:
Peebles, P. Z. "*Probability, Random Variables, and Random Signal Principles*", McGraw-Hill, 4^{th} Edition, 2001.

HW # 2	2.1-1, 2.1-7, 2.1-11, 2.2-5, 2.3-2, 2.3-15, 2.4-1, 2.4-6, 2.4-14, **Due Date: 14/03/2011**

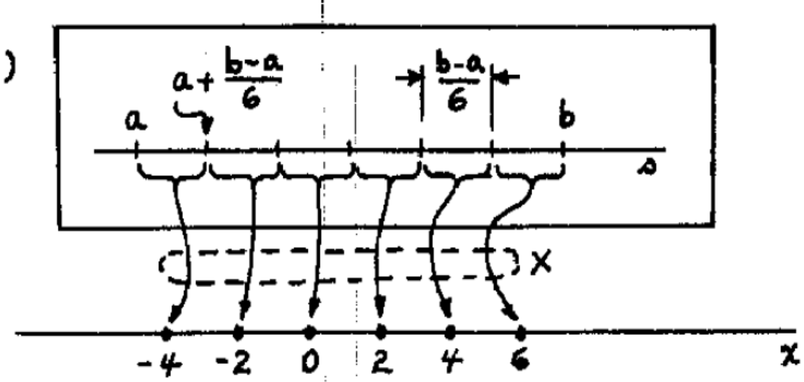
2.1-1. Let S_X denote the set of values that X can have. (a) For $X = 2s$: $S_X = \{0, 2, 5, 12\}$.

(b) For $X = 5s^2 - 1$: $S_X = \{-1, 4, 30.25, 179\}$.

(c) For $X = \cos(\pi s)$: $S_X = \{1, -1, 0\}$

(d) For $X = 1/(1-3s)$: $S_X = \{1, -0.5, -1/6.5, -1/17\}$.

2.1-7. (a)



(b) From (a) it is obvious that

$(b-a)/6 = 2$ and $a + [(b-a)/12] = -4$. Solving for

a and b gives $a = -5$ and $b = 7$.

(2.1-11.) For $180\,\Omega$: $E_2 = 12\left(\frac{180}{820+180}\right) = 2.16$ V.

For $470\,\Omega$: $E_2 = 12\left(\frac{470}{820+470}\right) \approx 4.372$V.

For $1000\,\Omega$: $E_2 = 12\left(\frac{1000}{820+1000}\right) = 6.593$ V.

For $2200\,\Omega$: $E_2 = 12\left(\frac{2200}{820+2200}\right) = 8.742$ V.

Thus,

$$E_2 = \{2.16, 4.372, 6.593, 8.742\}.$$

Since all resistor values are equally probable so are the voltage values. The set P of the probabilities is $P = \{1/4, 1/4, 1/4, 1/4\}$.

(2.2-5.) $G_X(x)$ must satisfy (2.2-2 a, b, d and f) to be valid. (a) $G_X(-\infty) = 0$, $G_X(\infty) = 1$, $G_X(x_2) > G_X(x_1)$ if $x_2 > x_1$ as seen from a sketch which also shows $G_X(x^+) = G_X(x)$. Therefore $G_X(x)$ is a valid distribution. (b) From calculations and a sketch $G_X(x)$ is a valid distribution. (c) $G_X(\infty) \neq 1$ so it is not a valid distribution.

(2.3-2.) This is a Bernoulli trials problem where $N=6$ and $p=0.4$. Here

$$P(0W) = \binom{6}{0}(0.4)^0(0.6)^6 \approx 0.0467$$

$$P(0.5W) = \binom{6}{1}(0.4)^1(0.6)^5 \approx 0.1866$$

$$P(1.0W) = \binom{6}{2}(0.4)^2(0.6)^4 \approx 0.3110$$

$$P(1.5W) = \binom{6}{3}(0.4)^3(0.6)^3 \approx 0.2765$$

$$P(2.0W) = \binom{6}{4}(0.4)^4(0.6)^2 \approx 0.1382$$

$$P(2.5W) = \binom{6}{5}(0.4)^5(0.6)^1 \approx 0.0369$$

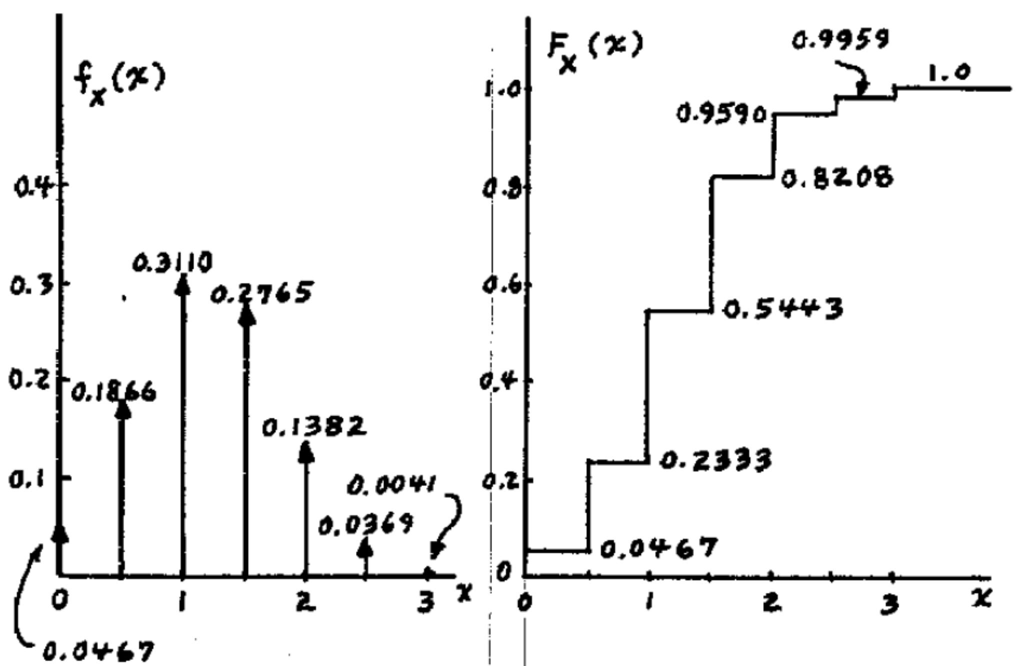
$$P(3.0W) = \binom{6}{6}(0.4)^6(0.6)^0 \approx 0.0041.$$

(a) Let X be a random variable representing power delivered. From (2.3-5) and (2.2-6):

$$f_X(x) = 0.0467\,\delta(x) + 0.1866\,\delta(x-0.5) + 0.3110\,\delta(x-1.0) + 0.2765\,\delta(x-1.5) + 0.1382\,\delta(x-2.0) + 0.0369\,\delta(x-2.5) + 0.0041\,\delta(x-3.0)$$

$$F_X(x) = 0.0467\,u(x) + 0.1866\,u(x-0.5) + 0.3110\,u(x-1.0) + 0.2765\,u(x-1.5) + 0.1382\,u(x-2.0) + 0.0369\,u(x-2.5) + 0.0041\,u(x-3.0)$$

(2.3-2.) (Continued)



(b) P(Overload) = P(2.5 W) + P(3.0 W) = 0.041 or 4.1%.

2.3-15. From use of (C-39) and (C-41):

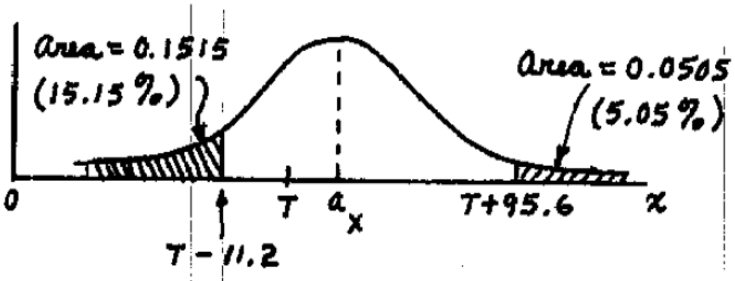
$$\int_{-\infty}^{\infty} f_X(x)\,dx = K\int_{-1}^{1}(1-x^2)\cos(\pi x/2)\,dx = K\left[\frac{4}{\pi}-\frac{8}{\pi^3}\left(\frac{\pi^2}{2}-4\right)\right] = K\frac{32}{\pi^3}$$

$\underset{\text{must}}{=} 1$ so $K = \pi^3/32 \approx 0.9689$.

2.4-1. (a) $P\{|X|>2\} = P\{2<X\} + P\{X<-2\} = 1 - P\{X \le 2\} + P\{X<-2\}$. From (2.4-3) this equals $P\{|X|>2\} = 1-F(2) + F(-2) = 1-F(2)+1-F(2) = 2-2F(2)$. From Appendix B: $P\{|X|>2\} = 2-2(0.9772) = 0.0456$.

(b) $P\{X>2\} = 1-P\{X\le 2\} = 1-F(2) = 1-0.9772 = 0.0228$.

(2.4-6.) We use the sketch as a guide where T = target.



$$P\{X > T+95.6\} = 0.0505 = 1 - F\left(\frac{T+95.6-a_X}{\sigma_X}\right)$$

occurs when

$$\frac{T+95.6-a_X}{\sigma_X} = 1.64 \qquad (1)$$

from Appendix B.

$$P\{X \le T-11.2\} = 0.1515 = F\left(\frac{T-11.2-a_X}{\sigma_X}\right)$$

$= 1 - F\left(-\frac{T-11.2-a_X}{\sigma_X}\right)$ occurs when

$$-\frac{T-11.2-a_X}{\sigma_X} = 1.03. \qquad (2)$$

On solving (1) and (2) for a_X and σ_X we get $a_X = T + 30$ m and $\sigma_X = 40$ m.

(2.4-14.) (a) $P\{1.4 < X \le 2.0\} = F_X(2.0) - F_X(1.4) = F\left(\frac{2.0-1.6}{0.4}\right) - F\left(\frac{1.4-1.6}{0.4}\right)$

$= F(1.0) - F(-0.5) = F(1.0) - [1 - F(0.5)] = 0.8413 + 0.6915 - 1.0 = 0.5328.$

(b) $P\{-0.6 < (X-1.6) \le 0.6\} = P\{1.0 < X \le 2.2\} = F_X(2.2) - F_X(1.0)$

$= F\left(\frac{2.2-1.6}{0.4}\right) - F\left(\frac{1.0-1.6}{0.4}\right) = F(1.5) - F(-1.5) = 2F(1.5) - 1 = 2(0.9332)$

$-1.0 = 0.8664.$