King Fahd University of Petroleum and Minerals

Department of Electrical Engineering

EE-315-Probabilistic Methods in Electrical Engineering
SECOND SEMESTER 2010-2011 (102)

TEXT BOOK:
Peebles, P. Z. "*Probability, Random Variables, and Random Signal Principles*", McGraw-Hill, 4^{th} Edition, 2001.

HW # 1	1.1-5, 1.2-3, 1.3-7, 1.3-11, 1.4-5, 1.4-11, 15-6, 1.7-1 **Due Date: 28/02/2011**

1.1-5. $\{\cdot\}$ (null set); $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$; $\{a,b\}$, $\{a,c\}$, $\{a,d\}$, $\{b,c\}$, $\{b,d\}$, $\{c,d\}$; $\{a,b,c\}$, $\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$; $\{a,b,c,d\}$.

1.2-3. (a) $\bar{A} = S - A = \{6, 8, 12\}$. (b) $A - B = \{2\}$; $B - A = \{6, 8\}$. (c) $A \cup B = \{2, 4, 6, 8, 10\}$. (d) $A \cap B = \{4, 10\}$. (e) $\bar{A} \cap B = \{6, 8\}$.

1.3-7. $P(\text{black}) = 75/500$, $P(\text{green}) = 150/500$, $P(\text{red}) = 175/500$, $P(\text{white}) = 70/500$ and $P(\text{blue}) = 30/500$.

1.3-11.

$\Omega \backslash W$	0.25	0.50	1.0
10	0.08	0.10	0.01
22	0.20	0.26	0.05
48	0.12	0.15	0.03
	0.40	0.51	0.09

$S = \{\text{all resistors}\}$

All resistors span S.

From table: (a) $P(48\Omega \text{ and } 0.25W) = 0.12$, (b) $P(48\Omega \text{ and } 0.5W) = 0.15$, (c) $P(48\Omega \text{ and } 1.0W) = 0.03$.

1.4-5. (a) $P(0.01\mu F \mid \text{box } 2) = 95/210$.

(b) $P(0.01\mu F \mid \text{box } 3)\, P(\text{box } 3) = P(\text{box } 3 \mid 0.01\mu F)\, P(0.01\mu F)$

Thus,

$$P(\text{box } 3 \mid 0.01\mu F) = \frac{P(0.01\mu F \mid \text{box } 3)\, P(\text{box } 3)}{P(0.01\mu F)}$$

From the total probability theorem:

$$P(0.01\mu F) = P(0.01\mu F \mid \text{box } 1)\, P(\text{box } 1) + P(0.01\mu F \mid \text{box } 2)\, P(\text{box } 2) + P(0.01\mu F \mid \text{box } 3)\, P(\text{box } 3)$$

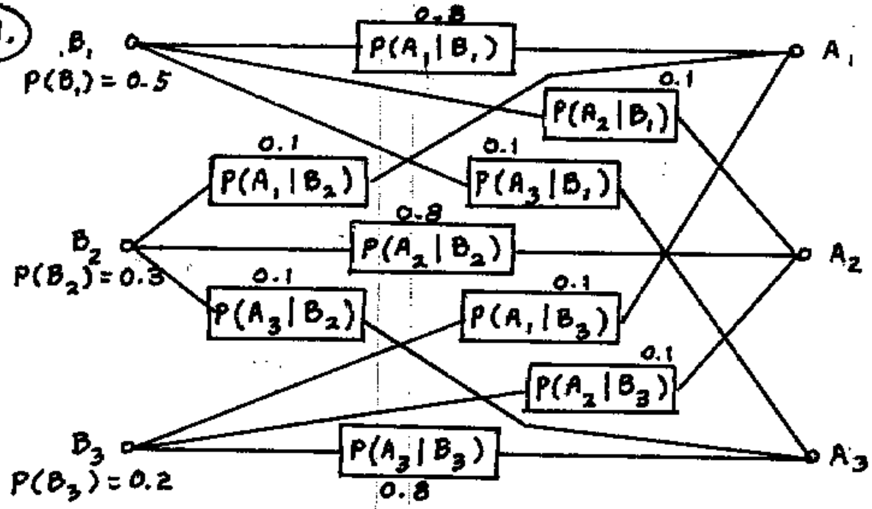
$$= \frac{20}{145}\left(\frac{1}{3}\right) + \frac{95}{210}\left(\frac{1}{3}\right) + \frac{25}{245}\left(\frac{1}{3}\right) = \frac{5903}{3(29)42(7)}.$$

Thus,

$$P(\text{box } 3 \mid 0.01\mu F) = \frac{\frac{25}{245}\left(\frac{1}{3}\right)}{\frac{5903}{3(29)42(7)}} = \frac{870}{5903} \approx 0.1474.$$

★ 1.4-11.

(a)



(b) $P(A_1) = P(A_1|B_1)P(B_1) + P(A_1|B_2)P(B_2) + P(A_1|B_3)P(B_3)$

$= 0.8(0.5) + 0.1(0.3) + 0.1(0.2) = 0.45$, $P(A_2) =$

$0.1(0.5) + 0.8(0.3) + 0.1(0.2) = 0.31$, $P(A_3) = 0.1(0.5)$

$+ 0.1(0.3) + 0.8(0.2) = 0.24$. (c)

(c) $P(B_1|A_1) = 0.8(0.5)/0.45 = 0.8889$, Bayes' rule.

$P(B_1|A_2) = 0.1(0.5)/0.31 = 0.1613$

$P(B_1|A_3) = 0.1(0.5)/0.24 = 0.2083$

1.4-11. (Continued)

$P(B_2|A_1) = 0.1(0.3)/0.45 = 0.0667$

$P(B_2|A_2) = 0.8(0.3)/0.31 = 0.7742$

$P(B_2|A_3) = 0.1(0.3)/0.24 = 0.1250$

$\underline{P(B_3|A_1) = 0.1(0.2)/0.45 = 0.0444}$

$P(B_2|A_2) = 0.1(0.2)/0.31 = 0.0645$

$P(B_3|A_3) = 0.8(0.2)/0.24 = 0.6667$

(d) When $P(B_i) = 1/3$, $i = 1, 2, 3$, then $P(A_1) = 1/3[$ $P(A_1|B_1) + P(A_1|B_2) + P(A_1|B_3)] = \frac{1}{3}[0.8 + 0.1 + 0.1] = 1/3$. Similarly $P(A_2) = P(A_3) = P(A_1) = 1/3$ and also $P(B_i|A_k) = 0.1$, $k \neq i$, and $P(B_i|A_i) = 0.8$, $i = 1, 2, 3$.

1.5-6.

P{signal does not arrive} = P{(upper path fails) and (lower path fails)} = P{(upper path fails) $\cap$ (lower path fails)} = P(upper path fails) P(lower path fails). But P(upper path fails) $= P(R_1) + P(R_2 \cap R_3) - P(R_1 \cap R_2 \cap R_3) = p_1 + p_2^2 - p_1 p_2^2 = 5.0995(10^{-3})$, and P(lower path fails) $= p_2 + p_3^2 - p_2 p_3^2 = 12.475(10^{-3})$. Thus,

P{signal does not arrive} $= 5.0995(10^{-3})\,12.475(10^{-3}) = 63.6163(10^{-6})$.

1.7-1. This is a Bernoulli trials experiment with $N=4$,

$p = P(\text{a can is out of tolerance}) = 0.03$.

(a) $P(4 \text{ out of tolerance}) = \binom{4}{4}(0.03)^4(1-0.03)^0 = 8.1(10^{-7})$.

(b) $P(2 \text{ out of tolerance}) = \binom{4}{2}(0.03)^2(1-0.03)^2$

$$= \frac{4!}{2!\,2!}(9)10^{-4}(0.97)^2 \approx 5.081\,(10^{-3}).$$

(c) $P(\text{all in tolerance}) = P(\text{none is out of tolerance})$

$$= \binom{4}{0}(0.03)^0(1-0.03)^4 = (0.97)^4 \approx 0.8853.$$