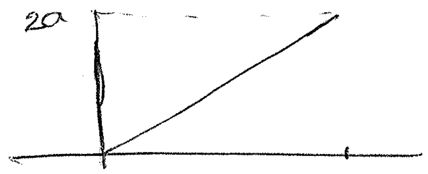
KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
ELECTRICAL ENGINEERING DEPARTMENT
SPRING 2010 (102)
EE 315 Probabilistic Methods in Electrical Engineering
QUIZ #2

Name: **ID:** **Roaster no:**

Q1. Let be X be a random variable and let the following function be its probability density function (pdf).

$$f_X(x) = 2x\,[u(x) - u(x-a)]$$



where $a > 0$.

a) Find a to make $f(x)$ a valid pdf.

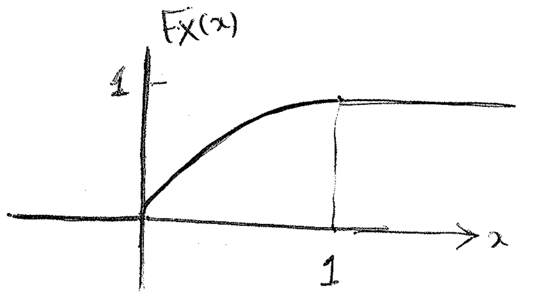
$f(x)$ is positive & increasing. So the only property left to investigate is that $\int_{-\infty}^{\infty} f(x)dx = 1$. Now $\int_{-\infty}^{\infty} f(x)\,dx = \frac{\cancel{2}a^2}{\cancel{2}} \Rightarrow$ we need to choose $\boxed{a=1}$ for $f_X(x)$ to be a valid pdf.

b) Find the corresponding CDF and sketch it.

For $x \leq 0$ $\quad F_X(x) = \int_{-\infty}^{x} f(y)\,dy = \int_{-\infty}^{x} 0 \cdot dy = 0$

For $x \geq 1$ $\quad F_X(x) = \int_{-\infty}^{x} f(y)dy = \int_{0}^{1} f(y)dy$

For $0 \leq x \leq 1$ $\quad F_X(x) = \int_{-\infty}^{x} f(y)dy = \int_{0}^{x} 2y\,dy = x^2$



c) Let A be the event $A = \{-1 < X \leq \frac{1}{2}\}$. Find $P(A)$.

$$P(A) = \int_{-1}^{1/2} f_X(x)\,dx = \int_{0}^{1/2} f_X(x)\,dx = \int_{0}^{1/2} 2x = x^2\Big|_0^{1/2} = 1/4$$

Another solution

$$P(A) = F(1/2) - F(-1) = x^2\Big|_{1/2} - 0 = 1/4.$$

$P(A) = 1/4$.

d) Find the conditional CDF $F_X(x|A)$.

$F_X(x|A) = P\{X \leq x | A\} = P\{X \leq x \cap -1 \leq X \leq 1/2\} / P(A)$

We have 4 cases:

$x < -1 \Rightarrow$ ~~$F_X(x|A)$~~ $P\{X \leq x \cap -1 \leq X \leq 1/2\} = P(\emptyset) = 0$.

$-1 \leq x \leq 0 \Rightarrow P\{X \leq x \cap -1 \leq X \leq 1/2\} = P\{-1 \leq x \leq 0\} = 0$

$0 \leq x \leq 1/2 \Rightarrow P\{X \leq x \cap -1 \leq X \leq 1/2\} = P\{0 \leq x\} =$

Q2. Let X be a Gaussian random variable with $a_x = 4$ and $\sigma_x = 2$. Find $P(|X| > 2)$.

$P(\cancel{|X| > 2}) \cancel{= P(X > 2 \cup X < -2)}$

$\cancel{= P(X > 2) + P(X < -2)}$ ~~as the two sets $P(X > 2)$ & $P(X < -2)$ are disjoint~~

$\cancel{= 1}$

$$P(|X| > 2) = 1 - P(|X| \leq 2)$$

$$= 1 - P(-2 \leq X \leq 2)$$

$$= 1 - (F_X(2) - F_X(-2))$$

$$= 1 - \left(F\left(\frac{2-4}{2}\right) - F\left(\frac{-2-4}{2}\right)\right)$$

$$= 1 - (F(-1) - F(-3))$$

$$= 1 - ((1 - F(1)) - (1 - F(3)))$$

$$= 1 - (F(3) - F(1))$$

get from tables