KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

ELECTRICAL ENGINEERING DEPARTMENT

EE 315

MAJOR 1

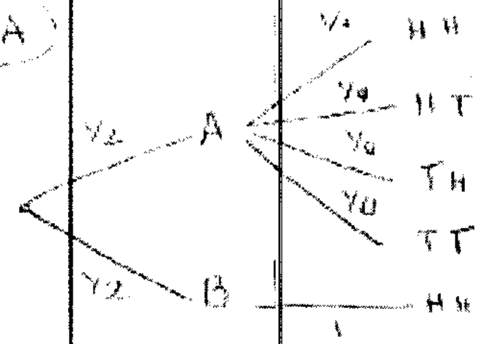
DATE: March 26, 2011

TIME: 7:00 PM-8:45 PM

ID#	
Name	
Section#	

	Maximum	Score
Problem 1 A Problem 2 A	8 4	
Problem 2	10	
Problem 3 A Problem 3 B	8 4	
Problem 4	6	
TOTAL	40	

1A



1. $P(HH/A) = \frac{1}{4}$ (2)

2. $P(B/HH) = \frac{P(B \cap HH)}{P(HH)}$ (1)

$P(HH) = P(HH/B)\,P(B) + P(HH/A) \cdot P(A)$ (2)

$= 1 \cdot \frac{1}{2} + \frac{1}{4} \cdot \frac{1}{2} = \frac{5}{8}$

$P(B \cap HH) = P(HH/B) \cdot P(B) = \frac{1}{2}$

$\longrightarrow P(B/H) = \frac{1/2}{5/8} = \frac{4}{5}$ (4) (1)

3. $P(B/TT) = \frac{P(B \cap TT)}{P(TT)}$

$P(B \cap TT) = P(TT/B) \cdot P(B) = 0$

$\longrightarrow P(B/TT) = 0$ (2)

Pb 1B

$p = \frac{2}{3} = P(\text{"Hit Target"}) = P(HT)$

5 trial till hit target (1)

$\longrightarrow$ prob of sequence

(1) $\bar{H}\,\bar{H}\,\bar{H}\,\bar{H}\,H$ $\quad P(\bar{H}\bar{H}\bar{H}\bar{H}H) = (1-p)^4 p$ (2)

$= \left(\frac{1}{3}\right)^4 \frac{2}{3} = \frac{2}{3^5} = \frac{2}{243} = \boxed{8.2\,1E^{3}}$ (1)

Problem 2: / 10

A random variable X has a probability density function (pdf) defined by:

$$f_X(x) = \begin{cases} cx(1-x), & 0 \le x \le 1, \\ 0, & \text{elsewhere} \end{cases}$$

1. Find c such that $f_X(x)$ is a valid pdf.
2. Find $F_X(x)$ and sketch it.
3. Find b such that $P[|X| < b] = \frac{5b}{8}$.

1- The value of c such that $f_X(x)$ is a valid pdf:

$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1$$

$\boxed{0.5}$

$$\int_0^1 cx(1-x)\,dx = 1$$

$$c\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$$

$\boxed{1.5}$

$$\frac{c}{6} = 1 \Rightarrow \boxed{c = 6}$$ 2

2- $F_X(x)$ and its graph:

* For $x < 0$:

$f_X(x) = 0$

$$F_X(x) = \int_{-\infty}^{x} f_X(\varepsilon)\,d\varepsilon = 0$$ 1

* For $0 \le x \le 1$:

$$F_X(x) = \int_{-\infty}^{x} f_X(\varepsilon)\,d\varepsilon = \int_{-\infty}^{0} \cancel{f_X(\varepsilon)}^{\,0}\,d\varepsilon + \int_0^x f_X(\varepsilon)\,dx$$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]$$ 1.5

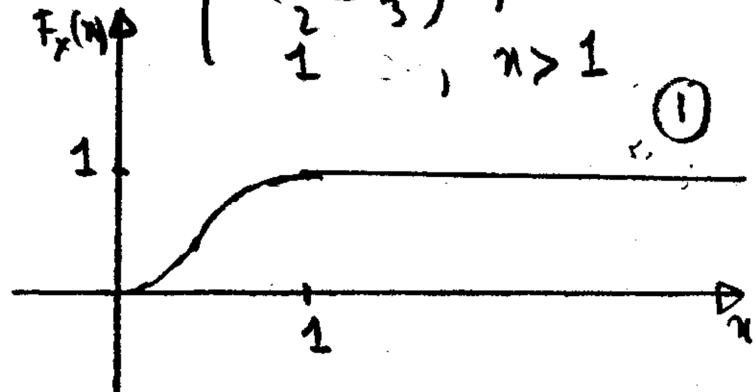
* For $x > 1$:

$$F_X(x) = \int_{-\infty}^{x} f_X(\varepsilon)\,d\varepsilon = \int_{-\infty}^{0} \cancel{f_X(\varepsilon)}^{\,0}\,d\varepsilon + \int_0^1 f(\varepsilon)\,d\varepsilon + \int_1^x \cancel{f_X(\varepsilon)}^{\,0}\,d\varepsilon$$

$$= 1$$ 1

Therefore:

$$F_X(x) = \begin{cases} 0, & x < 0 \\ 6\left(\frac{x^2}{2} - \frac{x^3}{3}\right), & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$ 0.5



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3- $P[|X| < b] = \frac{5b}{8}$, $0 \le b \le 1$

$P[-b < X < b] = \frac{5b}{8}$

$$\int_{-b}^{b} f_X(x)\,dx = \frac{5b}{8}, \quad 0 < x \le 1$$ 1.5

$$\int_0^b 6(x - x^2)\,dx = \frac{5b}{8}$$ 1

$6\left(b^2 - b^3\right)$ $5b$ $b \neq 0$ (trivial)

Problem 3: 18

A. A coin is tossed four times. It is known that this coin is biased and P{Head}=0.4. Let X be a random variable that represents the number of heads obtained in 4 tosses.

1. Find the probabilities of the different values the random variable X takes {P(X=k)}.
2. Find P{X > 3}.

1. $P[X=k]$, $0 \le k \le 4$

$P[X=0] = \binom{4}{0} 0.4^0 \, 0.6^4$

$= 0.1296$ ①

$P[X=1] = \binom{4}{1} 0.4^1 \, 0.6^3$

$= 0.3456$ ①

$P[X=2] = \binom{4}{2} 0.4^2 \, 0.6^2$

$= 0.3456$ ①

$P[X=3] = \binom{4}{3} 0.4^3 \, 0.6^1$

$= 0.1536$ ①

$P[X=4] = \binom{4}{4} 0.4^4 \, 0.6^0$

$= 0.0256$ ①

$f_X(x) = \sum_{k=0}^{4} P[X=k] \, \delta(x-k)$

or

$F_X(x) = \sum_{k=0}^{4} P[X=k] \, u(x-k)$

2. $P[X>3] = 1 - P[X \le 3]$

$= 1 - F_X(3)$

$= 1 - 0.9744$

$= 0.0256$ ✓

or

$P[X>3] = P[X=4]$

$= 0.0256$ ✓

③

$X \sim G(100, \sigma^2)$

$P(80 < X < 120) = 0.9$ (1)

$\longrightarrow P(X < 120) - P(X < 80) = 0.9$

$\longrightarrow \Phi\left(\frac{120-100}{\sigma}\right) - \Phi\left(\frac{80-100}{\sigma}\right) = 0.9$ (1)

$\Phi\left(\frac{20}{\sigma}\right) - \Phi\left(-\frac{20}{\sigma}\right) = 0.9$

$\longrightarrow \Phi\left(\frac{20}{\sigma}\right) - \left[1 - \Phi\left(\frac{20}{\sigma}\right)\right] = 0.9$

$\longrightarrow 2\Phi\left(\frac{20}{\sigma}\right) = 1.9 \longrightarrow \Phi\left(\frac{20}{\sigma}\right) = 0.95$ (1)

$\longrightarrow$ From table: $\frac{20}{\sigma} = 1.645$

$\longrightarrow \underline{\underline{\sigma = 12.158}}$ (1)

(students can also use 1.64 or 1.65).

Problem 4:

A random voltage V has a density function $f_V(v) = \frac{1}{4}u(v)e^{-v/4}$ (u(t) is the unit step function)

1. Calculate the mean value of V. Show your work.
2. If the voltage is passed through a device that generates the voltage $Y=V^3$, then calculate the expected value of Y.

The mean of V is

$$E[V] = \int_{-\infty}^{\infty} v f_V(v)\, dv$$

$$= \int_0^{\infty} v \frac{1}{4} e^{-v/4} \quad 1$$

From table with $a = 1/4$

$$= \frac{1}{4} e^{-1/4 v}\left[4v - 16\right]\Big|_0^{\infty}$$

$$= \lim_{v\to\infty} \frac{1}{4} e^{-1/4 v}\left[4v-16\right] \xrightarrow{} 0$$

$$- \frac{1}{4}\, 1\, [0 - 16]$$

$$= +4 \quad 2$$

$$E[Y^3] =$$

$$\int_{-\infty}^{\infty} v^3 f_V(v)\, dv$$

$$= \int_0^{\infty} \frac{v^3}{4} e^{-v/4} \quad 1$$

From table with $a = 1/4$

$$E[Y^3] =$$

$$e^{-v/4}\left[4v^3 - 3\times16v^2 + 6\times64v - 6\times4^4\right]\Big|_0^{\infty} \quad 0.5$$

$$= \lim_{v\to\infty} e^{-v/4}\left[4v^3 - 3\times16v^2 + 6\times64v - 6\times4^4\right] \to 0 \quad 0.5$$

$$- 1 \times [-6\times 4^4]$$

$$= 6\times 256$$

$$= 1536 \quad 1$$

$$\frac{1536}{4} = 384$$