KING ABDULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY
ELECTRICAL ENGINEERING DEPARTMENT
Fall 2012
EE 242 Digital Communications and Coding
Home Work #2
(*due Oct. 8, 2012*)

Q1. Prove that vectors $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$ form a basis in $\mathbb{R}^3$.

Q2. Let $s_1, \ldots, s_N$ be a basis in a vector space $\mathcal{V}$ and let v be a vector in $\mathcal{V}$. Prove that the representation of v using the basis is unique, i.e. if

$$v = c_1 s_1 + c_2 s_2 + \cdots + c_N s_N \text{ and}$$
$$v = c'_1 s_1 + c'_2 s_2 + \cdots + c'_N s_N$$

then $c_1 = c'_1, c_2 = c'_2, \cdots, c_N = c'_N$.

Q3. Consider the three signals $s_1(t), s_2(t)$ and $s_3(t)$ shown in Fig. 1.

a. Find the norm of $s_1(t)$ and $s_2(t)$, the inner product of the two signals, and the angle between the two signals.

b. Prove that $s_1(t)$, $s_2(t)$ and $s_3(t)$ are independent.

c. Find a signal $s_4(t)$ that is orthogonal to both $s_1(t)$ and $s_2(t)$.

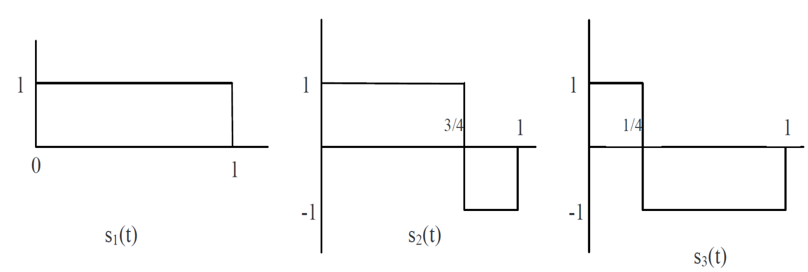

Figure 1: Signals $s_1(t), s_2(t)$ and $s_3(t)$

Q4. Consider the following modulated waveforms.

$$x_0(t) = \begin{cases} \sqrt{2}(\cos(2\pi t) + \sin(2\pi t)) & \text{if } t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$
$$x_1(t) = \begin{cases} \sqrt{2}(\cos(2\pi t) + 3\sin(2\pi t)) & \text{if } t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$
$$x_2(t) = \begin{cases} \sqrt{2}(3\cos(2\pi t) + \sin(2\pi t)) & \text{if } t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$
$$x_3(t) = \begin{cases} \sqrt{2}(3\cos(2\pi t) + 3\sin(2\pi t)) & \text{if } t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$
$$x_4(t) = \begin{cases} \sqrt{2}(\cos(2\pi t) - \sin(2\pi t)) & \text{if } t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$
$$x_5(t) = \begin{cases} \sqrt{2}(\cos(2\pi t) - 3\sin(2\pi t)) & \text{if } t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$
$$x_6(t) = \begin{cases} \sqrt{2}(3\cos(2\pi t) - \sin(2\pi t)) & \text{if } t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$
$$x_7(t) = \begin{cases} \sqrt{2}(3\cos(2\pi t) - 3\sin(2\pi t)) & \text{if } t \in [0,1] \\ 0 & \text{otherwise} \end{cases}$$
$$x_{i+8}(t) = -x_i(t) \quad i = 0, \cdots, 7$$

The signals have the following probabilities

$$p(x_0) = p(x_4) = p(x_8) = p(x_{12}) = \frac{1}{8} \text{ and}$$
$$p(x_i) = \frac{1}{24} \quad i = 1,2,3,5,6,7,9,10,11,13,14,15$$

a. Find a basis for this set of signals and find the vector representation of the signals using this basis.

b. Plot the signal constellation.

c. Compute the average energy of the constellation and the minimum distance.

Q5. Consider the channel shown in Fig. 2, where $s_1 = 1$ and $s_2 = -0.5$. $p(r|s_1)$ and $p(r|s_2)$ are

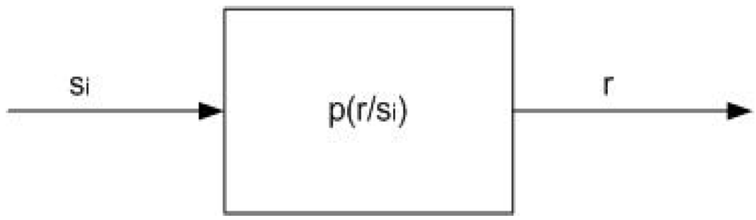

Figure 2: Channel

as shown in Fig. 3.

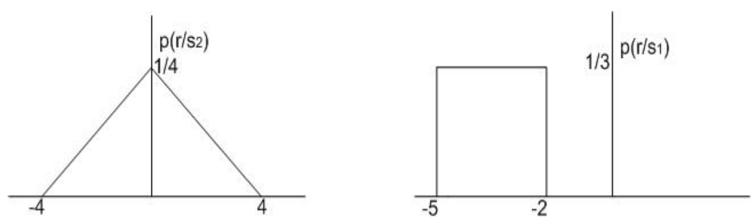

Figure 3: $p(r|s_1)$ and $p(r|s_2)$

a. Assume the two messages are equiprobable. Determine the received signal when

(i) $r = -2$, **(ii)** $r = -1$, **(iii)** $r = 3$, and **(iv)** $r = -7$

b. Repeat **a.** if $p(s_1) = 0.75$ and $p(s_2) = 0.25$.

c. Determine the decision regions and probabilities of error for part **b.**