Chapter I: Signal and System Modeling Concepts

Note Title

Learning Objectives:

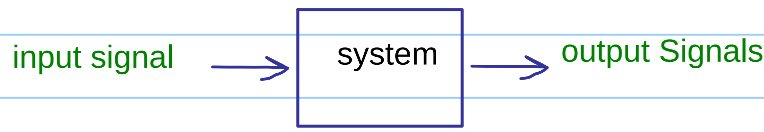
Define signals and systems
Examples of systems
Systems and subsystems
Signal Models (Signal Classification)
- Determinestic/ random
- Continuous/ discrete Time
- Continuous/ discrete Amplitude (Analog/Digital)
- Periodic/ non eriodic
- Power/ energy

Important Signals (delta, step, unit sawtooth, sinusoidal, singularity...)
Power and Energy of signals
Represeantaiton of signals
- Time domain
- Phasore and Frequency domain

1.1 Introduction

System:
Combination and interconnecting of several components to perform a desired task. (linear Systems).

Signal:
A function of time that represents a physical variable of interest associated with a system.

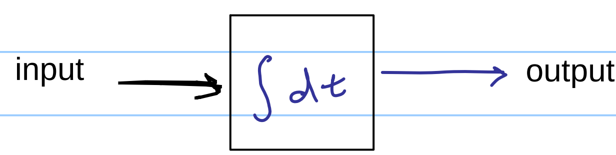


The concept is general for Mechanical Engineering, Electrical...etc.
However, most signals are converted to voltage & current before processing.

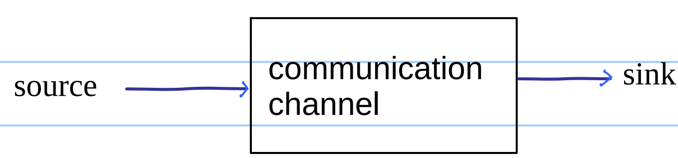
1.2 Examples of Systems

Examples in the text are not very relavent concentrate on Examples (1-2) & (1-3)
You do not have to understand all the details in this section.

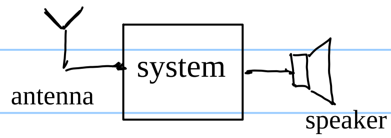
integrator



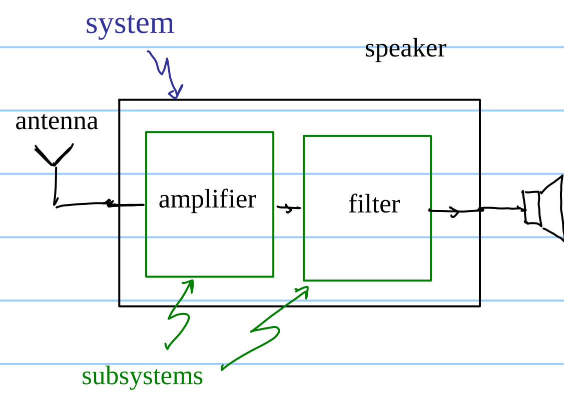
Another Example
communication link



Systems and subsystems



Radio



Understanding the systems help in **design and modeling**

1.3 Signal Models (Signal Classifications)

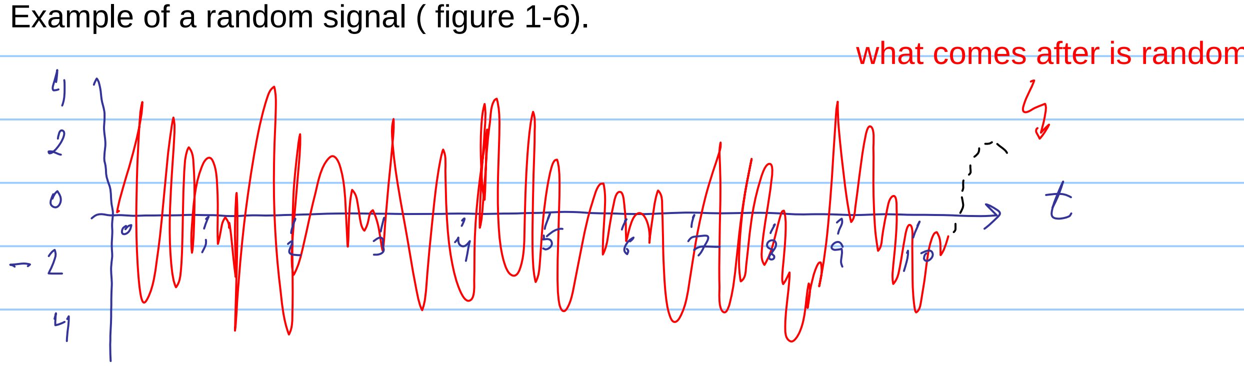
Deterministic signals are modeled as completly specified functions of time.

Random signals take random values at any given time instant and most be modeled probabilistically.

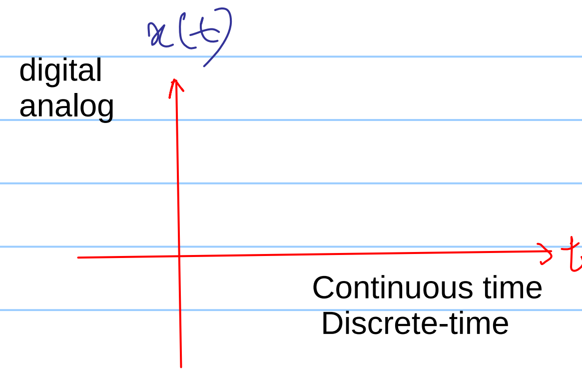
Examples for deterministic signals

$$x(t) = \frac{At^2}{B+t^2} \quad -\infty < t < \infty, \quad x(t) = t, \ldots$$

Example of a random signal (figure 1-6).



Continuous-time
Discrete-time



Digital (quantized signal)
Analog

a quantized signal is one whose values may assume only a countable number of values (levels).

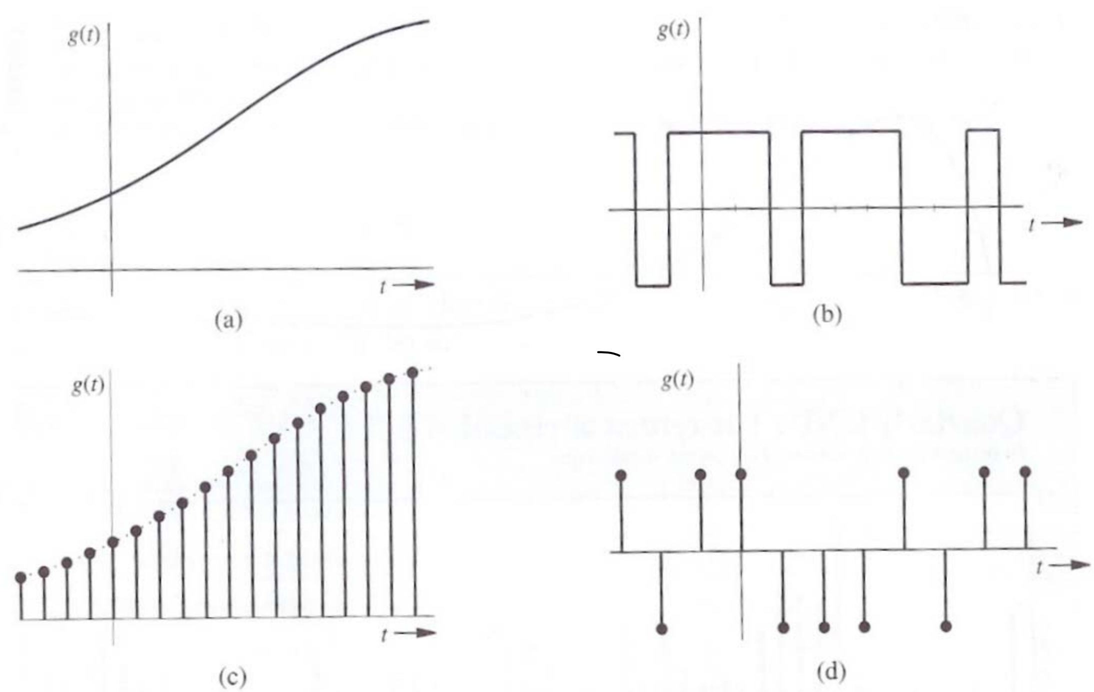


Figure 2.5 Examples of signals. (a) Analog, continuous time. (b) Digital, continuous time. (c) Analog, discrete time. (d) Digital, discrete time.

Periodic / Aperiodic

A signal is periodic if and only if

$$x(t+T_0) = x(t) \qquad -\infty < t < \infty$$

T_o: *fundamental period* is the smallest value that satisfies the equation.

example of, a periodic signal is

$$x(t) = A\sin(2\pi f_0 t + \theta) \qquad -\infty < t < \infty$$

$$\boxed{T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}}$$

f_0 : frequency in hertz (cycle per second)

ω_0 : angular frequency in rad/s

Verify

$$x\left(t + \frac{2\pi}{\omega_0}\right) = x(t), \quad \text{all } t$$

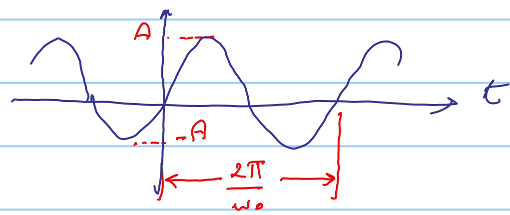
$$x\left(t + \frac{2\pi}{\omega_0}\right) = A\sin\left[\omega_0\left(t + \frac{2\pi}{\omega_0}\right) + \theta\right]$$

$$= A\sin(\omega_0 t + 2\pi + \theta)$$

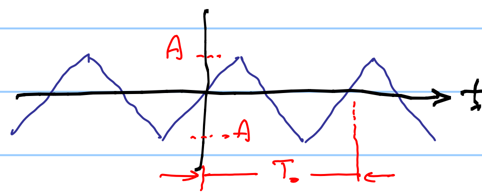
We know that adding 2pi does not change the Sin or Cos functions

Popular examples of periodic signals (figure 1.8)

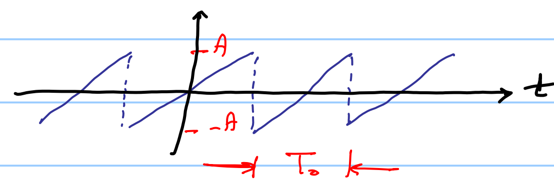
a) Sinusoidal



b) Triangular signal



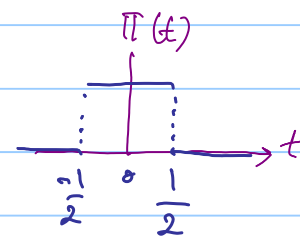
c) Sawtooth



Important functions

1. Unit Pulse function "rect"

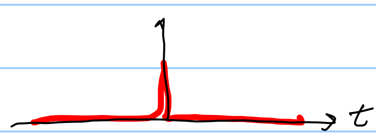
$$\Pi(t) = \begin{cases} 1 & |t| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$



2. Unit impulse function

$\delta(t)$ $\qquad \delta(t) = 0 \ , \ t \neq 0$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

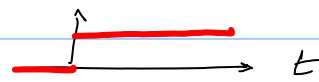


The discrete version

$$\delta[n] = \begin{cases} 1 & , \ n = 0 \\ 0 & , \ \text{otherwise} \end{cases}$$

3. Unit Step function

$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$



continuous

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$

discrete

Is the sum of two or more sinusoids periodic?

The sum is periodic if their periods can be expressed as a rational number **or** their frequencies are commensurable

(There is f_o such that $f_1 = n_1 f_0$ & $f_2 = n_2 f_0$ n_1, n_2 integers

Example 1.6

(a) $x_1(t) = \sin 10\pi t$

(b) $x_2(t) = \sin 20\pi t$

(c) $x_3(t) = \sin 31 t$

(d) $x_4(t) = x_1(t) + x_2(t)$

(e) $x_5(t) = x_1(t) + x_3(t)$

only (e) is not periodic

$$T_0 = \frac{2\pi}{\omega_0}$$

Phasor Signals and Spectra

Physical systems interact with real signals. Complex quantities are used for representation. We can use phasors to represent Sinusoidal quantities

$$\vec{X} = A e^{j\theta} = A \angle \theta$$

all means

$$x(t) = Re\left(\vec{X} e^{j\omega_c t}\right) = A \cos(\omega_0 t + \theta) \quad -\infty < t < \infty$$

the complex quantity is called ***rotating phasor*** $\tilde{x}(t) = \vec{X} e^{j\omega_c t}$

It is characterized by three quantities

amplitud A, phase θ, and frequency $\omega_0 > 0$

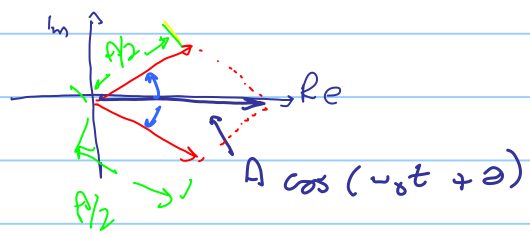
Euler Theorem $$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

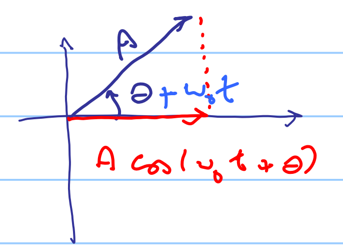
$$\cos\theta = \frac{1}{2}\left[e^{j\theta} + e^{-j\theta}\right]$$ add

$$\sin\theta = \frac{1}{2j}\left[e^{j\theta} - e^{-j\theta}\right]$$ subtract

We can also write the real part as $x(t) = \frac{1}{2}\tilde{x}(t) + \frac{1}{2}\tilde{x}^*(t)$

$\tilde{x}^*(t)$ is the conjugate of $\tilde{x}(t)$





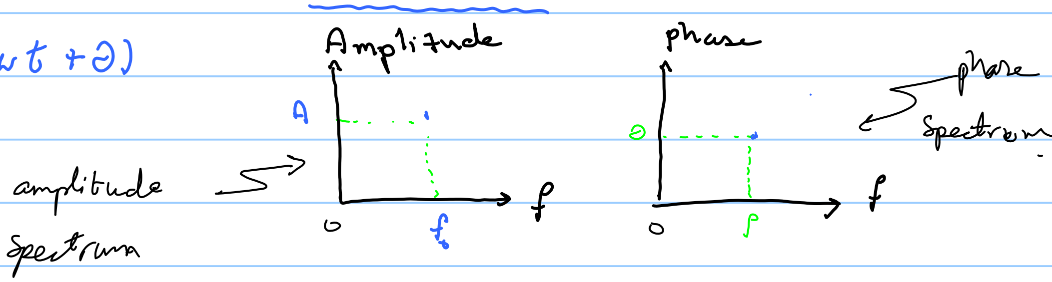
We can use Cartesian representation

$$X = a + jb = R\,e^{j\theta} = R\cos\theta + jR\sin\theta$$

Cartesian representation is good for addition and subtraction
while, polar is good for multiplication and division.

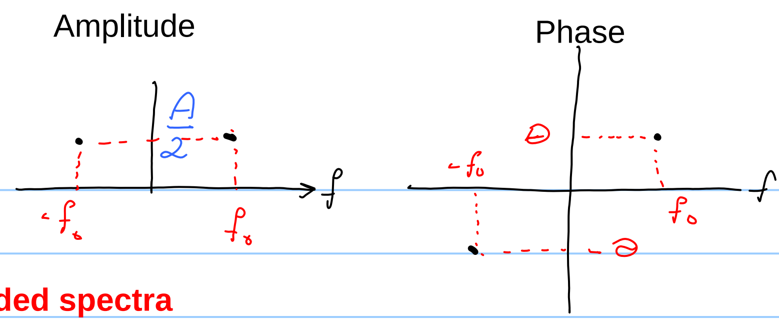
An alternative representation is the **frequency domain** representation

$A\cos(\omega t + \theta)$



Single Sided Spectra

Double Sided Spectra



Important points about double-sided spectra

1) we use (negative frequency) or double sided spectra to illustrate the fact that we need to add two conjugates to get the real function

2) for any real signal, amplitude spectrum has even symmetry.
phase spectrum has odd symmetry.

3) double side spectrum is directly related to exponential representation.
single side is related to trigonometric representation

To represent any other signal in frequency domain, we have to convert the signal into cosine or sum of cosines.

for Example if we want to represent $\sin(\omega_0 t+\theta)$, use

$$\boxed{\sin(\omega_0 t+\theta) = \cos\left(\omega_0 t+\theta-\frac{\pi}{2}\right)}$$

EXAMPLE 1-7

We wish to sketch the single-sided and double-sided amplitude and phase spectra of the signal

$$x(t) = 4\sin\left(20\pi t - \frac{\pi}{6}\right), \qquad -\infty < t < \infty \tag{1-30}$$

To sketch the single-sided spectra, we write $x(t)$ as the real part of a rotating phasor and plot the amplitude and phase of this phasor as a function of frequency for $t = 0$. Noting that $\cos(u - \pi/2) = \sin u$, we find that

$$\begin{aligned} x(t) &= 4\cos\left(20\pi t - \frac{\pi}{6} - \frac{\pi}{2}\right) \\ &= 4\cos\left(20\pi t - \frac{2\pi}{3}\right) \\ &= \mathrm{Re}\left\{4\exp\left[j\left(20\pi t - \frac{2\pi}{3}\right)\right]\right\} \end{aligned} \tag{1-31}$$

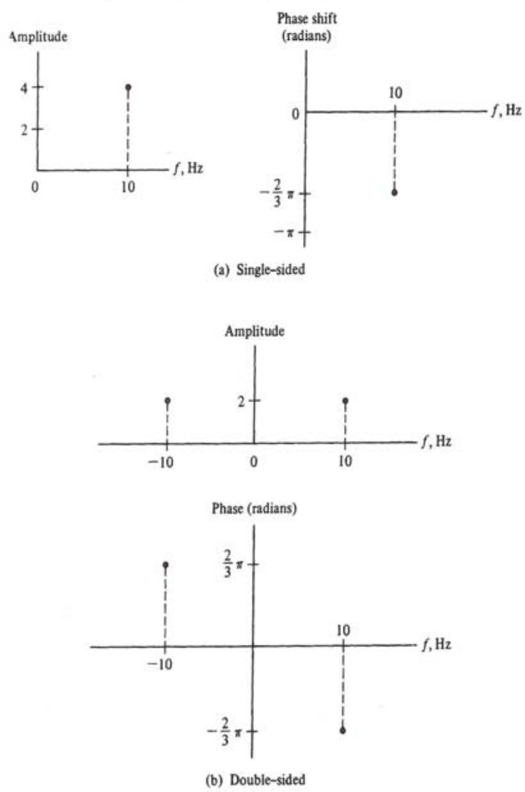


FIGURE 1-11. Amplitude and phase spectra for Example 1-6.

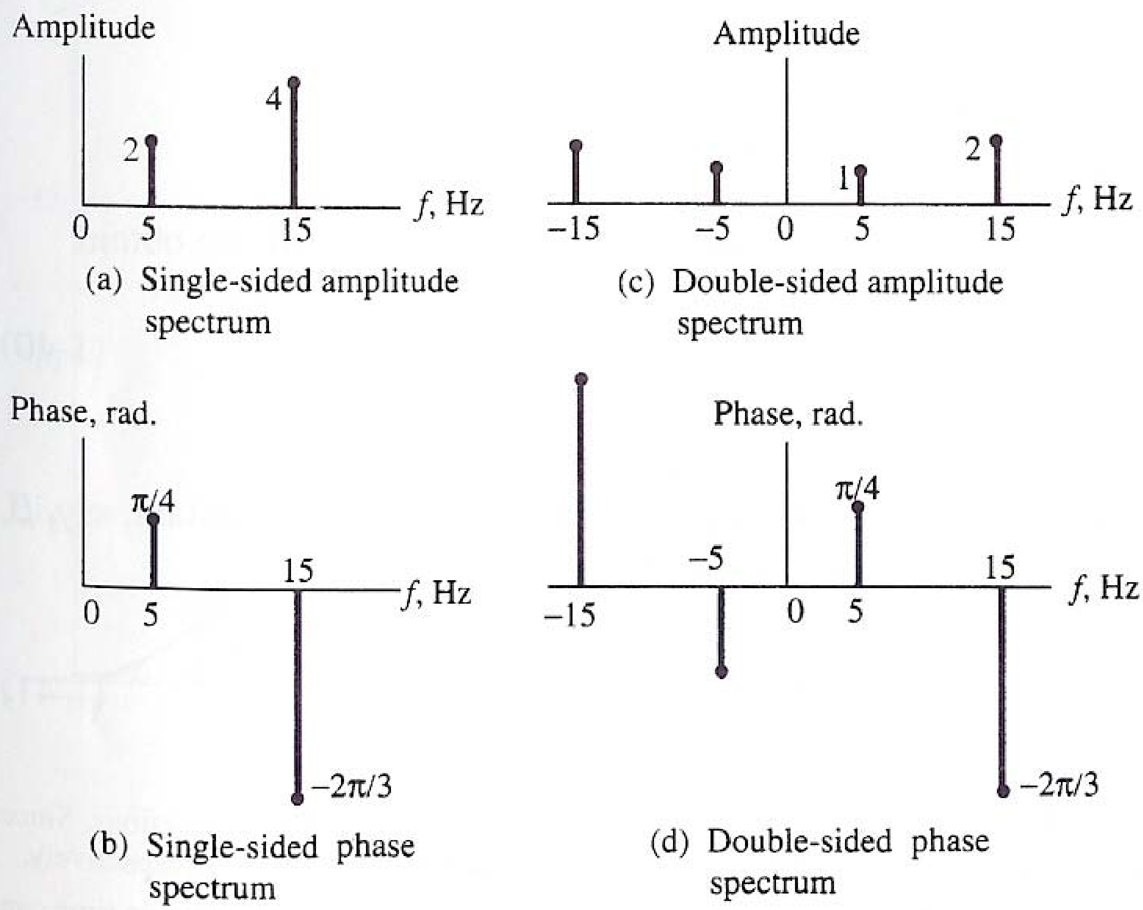
which results in the amplitude and phase spectral plots shown in Figure 1-11a. To plot the double-sided amplitude and phase spectra, we write $x(t)$ as the sum of complex conjugate rotating phasors. Recalling that $2\cos u = \exp(ju) + \exp(-ju)$, we obtain

$$x(t) = 2\exp\left[j\left(20\pi t - \frac{2\pi}{3}\right)\right] + 2\exp\left[-j\left(20\pi t - \frac{2\pi}{3}\right)\right] \tag{1-32}$$

from which the double-sided amplitude and phase spectral plots of Figure 1-11b result.

Example 1.8 (see ~~book~~ text book p. 16)

$x(t) = 2\cos(10\pi t + \pi/4) + 4\sin(30\pi t - \pi/6)$



1.8

FIGURE 1-12. Spectra for Example 1-7.

Singularity Functions

(aperiodic subclass of signals)

1) Unit step function

$$u(t) = u_{-1}(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases} \quad \text{at } t=0, \text{ finite !}$$

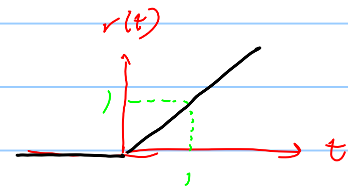
$$u_{i-1}(t) = \int_{-\infty}^{t} u_i(\lambda)\, d\lambda \quad , \quad i = \ldots, -2, -1, 0, 1, 2, \ldots$$

or

$$u_{i+1}(t) = \frac{du_i(t)}{dt}$$

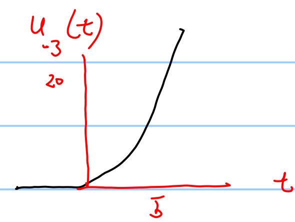
2) Unit ramp Function

$$u_{-2}(t) = r(t) = \begin{cases} t & t \geq 0 \\ 0 & t<0 \end{cases}$$



3) Unit parabolic function

$$u_{-3}(t) = \begin{cases} t^2/2 & t \geq 0 \\ 0 & t<0 \end{cases}$$



Time Operations

Time shifting

$t \leftarrow t - \frac{1}{2}$ shift to the right.

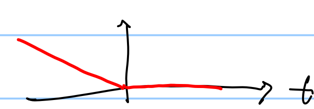
$t \leftarrow t + \frac{1}{2}$ shift to the left.

$$u\left(t-\frac{1}{2}\right) = \begin{cases} 0 & t-\frac{1}{2}<0 \\ 1 & t-\frac{1}{2}>0 \end{cases}$$

$\Rightarrow$ we can write $\Pi(t) = u\left(t+\frac{1}{2}\right) - u\left(t-\frac{1}{2}\right)$

Time revisal (folding)

Example $r(-t) = \begin{cases} -t & t \leq 0 \\ 0 & t>0 \end{cases}$



Summary of time operations

$$x(\beta t + \alpha) = x\left[\beta\left(t + \alpha/\beta\right)\right]$$

if $\frac{\alpha}{\beta} = t_0$ is positive shift lift.
negative shift right.

$\beta < 0$ time reversal or reflection

if $|\beta| > 1$, $x(t)$ is compressed
$|\beta| < 1$ $x(t)$ is expanded.

Example 1.9 p. 20

sketch

(a) $x_1(t) = \Pi(2t+6)$

(b) $x_2(t) = \cos(20\pi t - 5\pi)$

(c) $x_3(t) = r(-0.5t+2)$

See figure 1.14 p. 21

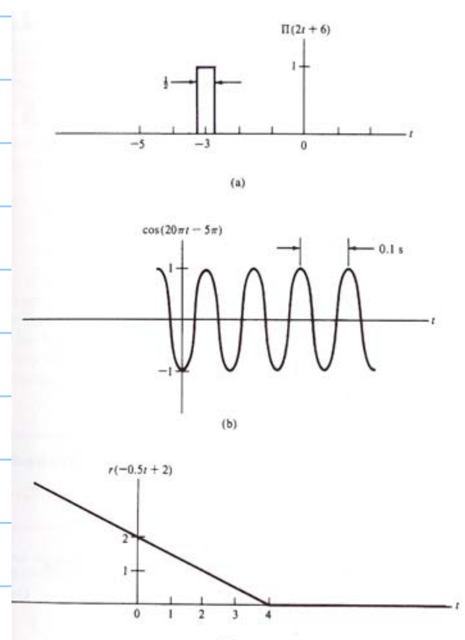


FIGURE 1-14. Signals relating to Example 1-9.

Example 1.10

Express the signals shown in terms of singularity functions

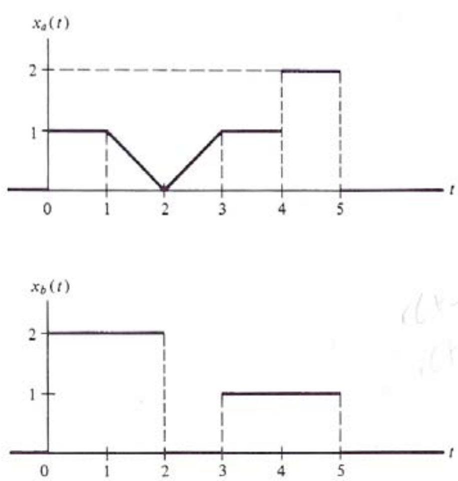


FIGURE 1-15. Signal to be expressed in terms of singularity functions.

$$x_a(t) = u(t) - r(t-1) + 2r(t-2) - r(t-3) + u(t-4) - 2u(t-5)$$

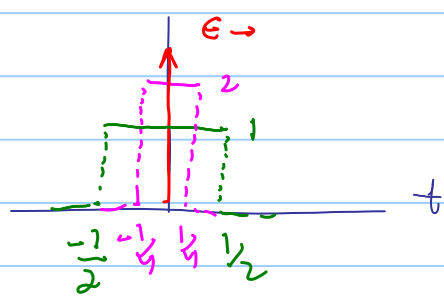
Some properties of unit impulse function

$$\delta(t) = 0 \, , \quad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$ to represent very short events

Approximated with square pulse

$$\delta_\epsilon(t) = \frac{1}{2\epsilon} \Pi\left(\frac{t}{2\epsilon}\right) = \begin{cases} \frac{1}{2\epsilon} \, , & |t| \leq \epsilon \\ 0 \, , & |t| > \epsilon \end{cases}$$



Note, area is 1

$$\int_{-\infty}^{t} \delta_\epsilon(\lambda)\, d\lambda = \begin{cases} 0 & t < -\epsilon \\ 1 & t > \epsilon \end{cases}$$

sifting property:

$$\boxed{\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)}$$

Other properties

$$\int_{t_1}^{t_2} x(t)\,\delta(t-t_0)\,dt = \begin{cases} x(t_0) & t_1 < t_0 < t_2 \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{x(t)\,\delta(t-t_0) = x(t_0)\,\delta(t-t_0)}$$

compare

$$\delta(t) = \delta(-t) \qquad \delta(t) = \frac{d\,u(t)}{dt}$$

$$\delta(at) = \frac{1}{|a|}\,\delta(t)$$

In general

$$\int_{t_1}^{t_2} x(t)\,\delta^{(n)}(t-t_0)\,dt = (-1)^n\,x^{(n)}(t_0) \qquad t_1 < t_0 < t_2$$

Do exercise p 27 (also do (b) & (c))

Evaluate the following Integrals

(a) $\displaystyle\int_{-\infty}^{\infty} e^{-\alpha t^2}\,\delta(t-10)\,dt$

(d) $\displaystyle\int_{-\infty}^{\infty} \left[5\,\delta(t) + e^{-(t-1)}\,\dot{\delta}(t) + \cos 5\pi t\,\delta(t) + e^{-t^2}\,\dot{\delta}(t)\right] dt$

(a) $e^{-100\alpha}$ sifting property

(d) $5 + (-1)^1\left[-(t-1)\,e^{-(t-1)}\right]\Big|_{t=0} + 1 + (-1)^1\left[-2t\,e^{-t^2}\right]\Big|_{t=0} = 5+e+1+0 = 6+e$

1.4 Energy and Power Signals

power is the rate of energy per time

lets assume that $e(t)$ is the voltage signal applied across a resistor R and producing a current $i(t)$

$$p(t) = \frac{e(t)\, i(t)}{R} = i^2(t)$$

per ohm = normalized R = R = 1

for a signal $x(t)$, the total Energy normalized to a unit resistance

$$E \triangleq \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt \quad \text{Joules}$$

$$P \triangleq \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\, dt \quad \text{watts}$$

1* $x(t)$ is an energy signal if and only if $0 < E < \infty \Rightarrow P = 0$

2* $x(t)$ is a power signal if and only if $0 < P < \infty \Rightarrow E = \infty$

3* $x(t)$ could beniether energy nor power signal; but it can not be both at the same time.

Example 1-11 $x_1(t) = A e^{-\alpha t} u(t)$ $\alpha > 0$ $E = \frac{A^2}{2\alpha}$

1-12 $x(t) = A \cos(\omega_0 t + \theta)$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} A^2 \cos^2(\omega_0 t + \theta)\, dt$$

use $\cos^2\theta = \frac{1}{2}[1 + \cos 2\theta]$

& $\lim \frac{A^2}{4T} \int_{-T}^{T} \cos 2(\omega_0 t + \theta)\, dt = 0$

integrate cos over 4 periods ?

Average Power of Periodic Signals

$$P = \frac{1}{T_0} \int_{t_0}^{t_0+T} |x_p(t)|^2 \, dt$$

one period

see Example 1-13 p 30

power of rotating pharos signal $A e^{j(\omega_0 t + \theta)} = A^2$

Using Euler's theorem $\boxed{e^{j\theta} = \cos(\theta) + j\sin(\theta)}$

$\Rightarrow$ 1.6 is about Matlab p 32

In this section (1.6) you will find Matlab functions for unit step, unit impulse & unit ramp.

There is a good summary at the end of the Chapter (See p. 35)