King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals and Systems (043)

Major Exam II

August 9, 2005
04:45 PM-06:15PM
Building 19-416

Serial #
0

-2 points for not writing yonr serial #

Name: KEY
ID:

Sec. (1) 9:20-10:20

Question	Mark
1	/18
2	/12
3	/10
Total	/40

Instructions:

1. This is a closed-books/notes exam.
2. The duration of this exam is one and half hours.
3. Read the questions carefully. Plan which question to start with.
4. Write explicitly the formulas that you use in your solution (e.g. by KVL … by KCL). No credit will be given if you do not show your formulas.
5. Work in your own.
6. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
7. Strictly no mobile phones are allowed.

Good luck

Dr. Ali Muqaibel

Problem 1: (18 points)

a. True or False, +1 for correct answer and -0.5 for wrong answer.
Fill the table below: **(5 points)**

1. The energy spectral density of a signal $x(t)$ is given by $G_x(f) = |X(f)|^2$.
2. It is true in general that a pulse's bandwidth and rise time are directly proportional.
3. Ideal filters can be constructed using resistors ~~and~~ capacitor ~~but not inductors~~. , and inductor.
4. Comparing Laplace Transform with Fourier transform, Fourier transform is more general.
5. **Both** 1 and u(t) have the same single sided Laplace transform.

Q	1	2	3	4	5
T or F	T	F	F	F	T

b. Find the Fourier Transform of the following signals (Tables are attached)
Indicate the property or pair number that you use **(6 points)**

1. $x_1(t) = \sin \pi t + 1$

$\boxed{p.9}$ $1 \leftrightarrow \delta(f)$

$\boxed{p.13}$ $\sin 2\pi f_c t \leftrightarrow \frac{1}{2j}\delta(f-f_c) - \frac{1}{2j}\delta(f+f_c)$ $f_c = \frac{1}{2}$

$X_1(f) = \frac{1}{2j}[\delta(f-\frac{1}{2}) - \delta(f+\frac{1}{2})] + \delta(f)$

2. $x_2(t) = \text{sgn}\left(\frac{t}{2}\right)$

$\boxed{p.15}$ $\text{Sgn}\, t \leftrightarrow (j\pi f)^{-1}$ $\boxed{Th\ 3a}$. $x(at) \leftrightarrow |a|^{-1} X(\frac{f}{a})$ $a = \frac{1}{2}$

$X_2(f) = 2\,(j\pi 2f)^{-1} = \frac{2}{j\pi 2f} = (j\pi f)^{-1}$ not time scale did not change the sgn.

3. $x_3(t) = \delta(t-2) + \text{sinc}\,(2\pi t)$

$\boxed{p.10}$ ~~$\delta(t) \leftrightarrow 1$~~ $\delta(t-t_0) \leftrightarrow \exp(-j2\pi f t_0)$ $t_0 = 2$

$\boxed{p.2}$ $2W\text{sinc}\, 2Wt \leftrightarrow \Pi(\frac{f}{2W})$ $W = \pi$ & scale by $\frac{2\pi}{2\pi}$

$X_3(f) = \exp(-j4\pi f) + \frac{1}{2\pi}\Pi(\frac{f}{2\pi})$

c. Solve the following differential equation by means of the Laplace transform: (7 points)

$$\frac{d^2x(t)}{dt^2} + 6\frac{dx(t)}{dt} + 5x(t) = e^{-7t} \qquad \text{with } x(0)\text{=0 and } \left[\frac{dx(t)}{dt}\right]_{t=0} = 1$$

$\frac{d^2x(t)}{dt^2} = s^2X(s) - s\cancel{x(0^-)} - x^{(1)}(0^-) = s^2X(s) - 1$ 1

$\frac{dx(t)}{dt} = sX(s) - \cancel{x(0^-)} = sX(s)$ 1

Equation in Laplace $s^2X(s) - 1 + 6sX(s) + 5X(s) = \frac{1}{s+7}$ 1

$X(s)[s^2 + 6s + 5] = \frac{1}{s+7} + 1 = \frac{s+8}{s+7}$

1 $\Rightarrow X(s) = \frac{s+8}{(s+7)(s^2+6s+5)} = \frac{s+8}{(s+7)(s+5)(s+1)} = \frac{K_1}{s+7} + \frac{K_2}{s+5} + \frac{K_3}{s+1}$

2 $K_1 = \frac{s+8}{(s+5)(s+1)}\Big|_{s=-7} = \frac{1}{(-2)(-6)} = 1/12$, $K_2 = \frac{s+8}{(s+7)(s+1)}\Big|_{s=-5} = \frac{3}{(2)(-4)} = -3/8$

$K_3 = \frac{s+8}{(s+7)(s+5)}\Big|_{s=-1} = \frac{7}{(6)(4)} = 7/24$, By inverse Laplace Transform.

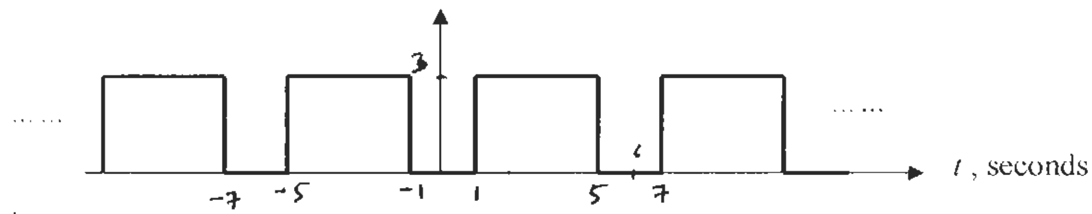
1 $x(t) = [\frac{1}{12}e^{-7t} - 3/8\, e^{-5t} + 7/24\, e^{-t}]u(t)$

Check $x(0)$ & $t=0$
$\frac{1}{12} - \frac{3}{8} + \frac{7}{24} = \frac{2-9+7}{24} = 0$

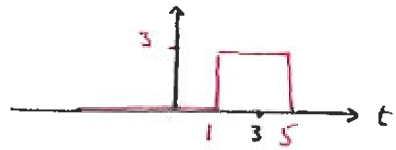
Check d/dt
$-7/12 + 15/8 - 7/24 = \frac{-14+45-7}{24} = 1$

Problem 2:

a. Obtain the Fourier transforms of the periodic waveform shown. *Hint: first find the Fourier transform for the non-periodic signal then use the proper formula to get the transform of the periodic signal* **(6 points)**



$$\sum_{m=-\infty}^{\infty} p(t-mT_s) \leftrightarrow \sum_{n=-\infty}^{\infty} f_s\, P(nf_s)\, \delta(f-nf_s)$$



$3\Pi\left(\frac{t-3}{4}\right)$

2 $$X(f)_{\text{Periodic}} = \sum_{n=-\infty}^{\infty} \frac{1}{6}\left(\cancel{4}\,12\,\text{sinc}\left(\frac{2}{3}n\right)\right) e^{-jn\pi}\, \delta\left(f-\frac{n}{6}\right)$$

$X(f) = \cancel{4}\,12\,\text{sinc}\, 4f\; e^{-j6\pi f}$ 2

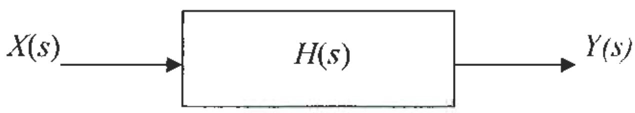
1 $$= \frac{2}{\cancel{3}} \sum_{n=-\infty}^{\infty} \text{Sinc}\left(\frac{2n}{3}\right) e^{-jn\pi}\, \delta\left(f-\frac{n}{6}\right)$$

It could be simplified more!

1 $T_o = 6$

$f_s = 1/6$

b. A system has a transfer function given by $H(s) = \frac{1}{s^2+2s+1}$, if the input signal is $x(t) = 18e^{-4t}u(t)$, find the output signal $y(t)$. **(6 points)**



$X(s) = \frac{18}{s+4}$ 1

$$Y(s) = X(s)\,H(s) = \frac{18}{(s+4)(s^2+2s+1)} = \frac{18}{(s+4)(s+1)^2}$$ 1

By partial fraction Expansion.

$$Y(s) = \frac{K_0}{s+4} + \frac{K_{11}}{s+1} + \frac{K_{12}}{(s+1)^2}$$

$$K_0 = \left.\frac{18}{(s+1)^2}\right|_{s=-4} = \frac{18}{9} = 2, \quad K_{12} = \left.\frac{18}{s+4}\right|_{s=-1} = 6$$ 1

$$K_{11} = \frac{d}{ds}\left[18(s+4)^{-1}\right]\Big|_{s=-1} = -18(s+4)^{-2}\Big|_{s=-1} = \frac{-18}{9} = -2$$ 2

$$Y(s) = \frac{2}{s+4} + \frac{-2}{s+1} + \frac{6}{(s+1)^2}$$

By Inverse Laplace Transform.

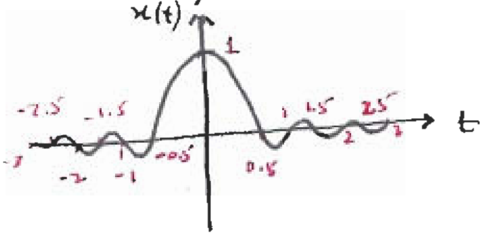
$$= \left[2e^{-4t} - 2e^{-t} + 6te^{-t}\right] u(t)$$ 1

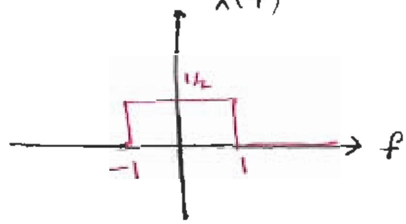
Problem 3: **(10 points)**

a. Sketch the following signal $x(t)$ = sinc $(2t)$ and its magnitude spectrum.**(4 points)**

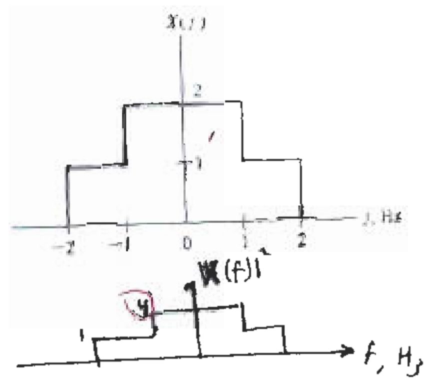
Compressed sinc by factor of 2

$\text{sinc}(2t) \leftrightarrow \frac{1}{2}\Pi\left(\frac{f}{2}\right)$





b. The **magnitude spectrum** of a signal $x(t)$ is shown in the figure **(6 points)**



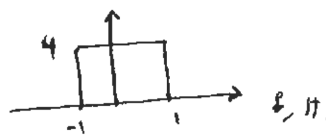
② 1. Find the total energy of the signal.

$$E = \int_{-\infty}^{\infty} |X(f)|^2 df$$

area under square of the curve.

$= 1 + 8 + 1 = 10$

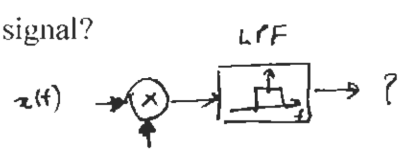
① 2. If the signal, $x(t)$, is passed through an ideal low-pass filter of cut off frequency =1 Hz, what is the energy of the output signal?



$E = 8$

③ 3. If the signal, $x(t)$,is first multiplied by $\cos(4\pi t)$ ~~cos (4t)~~ and then passed through the same ideal filter of cut off frequency =1, what is the energy of the output signal?

1 Hz



$f_0 = 2$ Hz

Multiplying by cos 4π will result in two images with ½ amplitude shifted by ±2 Hz

after the filter

E = area under square of the curve = 0.25 (2) = 0.5