بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

Department of Electrical Engineering
EE 207 Signals and Systems
First Semester (111)

Exam II
Saturday, 3 December 2011
7:00 pm – 8:30 pm

Name: ________________________________

ID: ________________________________

Instructors:

Dr. Khurram Qureshi	**(Sections 1 & 5)**
Dr. Husain Al-Jamed	**(Section 2)**
Dr. Ali Muqaibel	**(Section 3)**
Dr. Wajih Abu-Al-Saud	**(Section 4)**

Problem	Score	Out of
1		35
2		20
3		15
4		30
Total		100

Good luck!

Problem 1:

a) In each case of the following, circle ALL words between parentheses that describe the statement:

i) Ideal lowpass filters cannot be implemented because they are non-causal. (True) (False)

ii) The Fourier transform does not exist for some signals. (True) (False)

iii) The signals 1 and $u(t)$ have different single-sided Laplace transforms. (True) (False)

iv) If $x(t)$ is real and even function of time, then $X(f)$ is:
(Real) (Imaginary) (Complex) (Odd) (Even)

v) The spectrum of a non-period signal is (discrete) (continuous)

b) Using the tables of Fourier transform, find the Fourier transform of the signals

i) $x(t) = \text{sinc}(4t) + \Pi\left(\frac{t-3}{6}\right)$

From Table 4-2.1 and 4-2.2

$$\text{sinc}(4t) \xleftrightarrow{FT} \frac{1}{4}\Pi\left(\frac{f}{4}\right)$$

$$\Pi\left(\frac{t}{6}\right) \xleftrightarrow{FT} 6\cdot\text{sinc}(6f)$$

Using the Time Delay (Table 4-1.2) on the second relation gives

$$\Pi\left(\frac{t-3}{6}\right) \xleftrightarrow{FT} 6\cdot\text{sinc}(6f)e^{-j2\pi f(3)}$$

Using Superposition (Table 4-1.1) to combine the two terms

$$\text{sinc}(4t) + \Pi\left(\frac{t-3}{6}\right) \xleftrightarrow{FT} \frac{1}{4}\Pi\left(\frac{f}{4}\right) + 6\cdot\text{sinc}(6f)e^{-j2\pi f(3)}$$

ii) $x(t) = 2 \cdot \text{sinc}\left(\frac{t}{3}\right)\delta(t)$

Method 1:

From Table 4-2.2 and 4-2.8

$$\text{sinc}\left(\frac{t}{3}\right) \xleftrightarrow{FT} 3 \cdot \Pi(3f)$$
$$2 \cdot \delta(t) \xleftrightarrow{FT} 2$$

Using the Multiplication (Table 4-1.9) gives

$$2 \cdot \text{sinc}\left(\frac{t}{3}\right)\delta(t) \xleftrightarrow{FT} 3 \cdot \Pi(3f) * 2$$

Let us evaluate the convolution:

$$3 \cdot \Pi(3f) * 2 = 6 \underbrace{\int_{-\infty}^{\infty} \Pi(3\lambda) \cdot 1 \, d\lambda}_{\frac{1}{3}} = 6 \cdot \frac{1}{3} = 2$$

So,

$$2 \cdot \text{sinc}\left(\frac{t}{3}\right)\delta(t) \xleftrightarrow{FT} 2$$

Method 2:

We note that

$$x(t) = 2 \cdot \text{sinc}\left(\frac{t}{3}\right)\delta(t) = 2 \cdot \text{sinc}\left(\frac{0}{3}\right)\delta(t)$$
$$= 2\delta(t)$$

So,

$$2 \cdot \text{sinc}\left(\frac{t}{3}\right)\delta(t) \xleftrightarrow{FT} 2$$

iii) $x(t) = \dfrac{\cos(2\pi 5t)}{3 + j2\pi t}$

From Table 4-2.4

$$e^{-3t}u(t) \xleftrightarrow{FT} \frac{1}{3 + j2\pi f}$$

Applying Duality (Table 4-1.4) on the above gives

$$\frac{1}{3 + j2\pi t} \xleftrightarrow{FT} e^{3f}u(-f)$$

Using Modulation (Table 4-1.5b) on the above gives

$$\frac{\cos(2\pi 5t)}{3 + j2\pi t} \xleftrightarrow{FT} \frac{1}{2}e^{3(f-5)}u\left(-(f-5)\right) + \frac{1}{2}e^{3(f+5)}u\left(-(f+5)\right)$$

c) Using the tables of Fourier transform, find the inverse Fourier transform of the signal

$$X(f) = \frac{4}{5 + j2\pi(f+50)} + \frac{4}{5 + j2\pi(f-50)}$$

We note that the two terms are similar to each other with the difference that one of them is shifted by +50 while the other is shifted by -50.

From Table 4-2.4

$$4e^{-5t}u(t) \xleftrightarrow{FT} \frac{4}{5 + j2\pi f}$$

Using Modulation (Table 4-1.5b) on the above gives

$$8e^{-5t} \cdot \cos(2\pi 50t) \cdot u(t) \xleftrightarrow{FT} \frac{4}{5 + j2\pi(f+50)} + \frac{4}{5 + j2\pi(f-50)}$$

So,

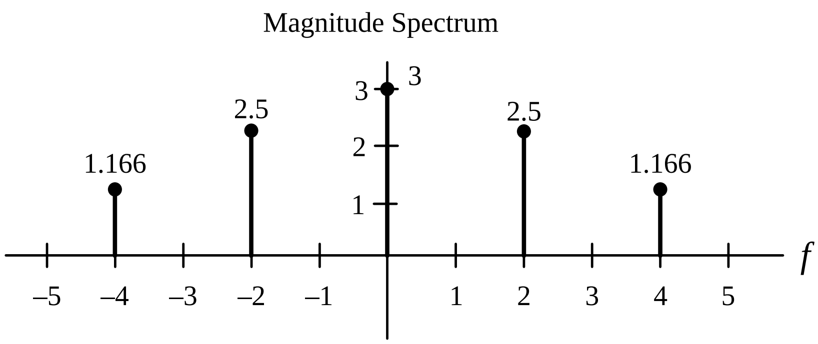
$$x(t) = 8e^{-5t} \cdot \cos(2\pi 50t) \cdot u(t)$$

Problem 2:

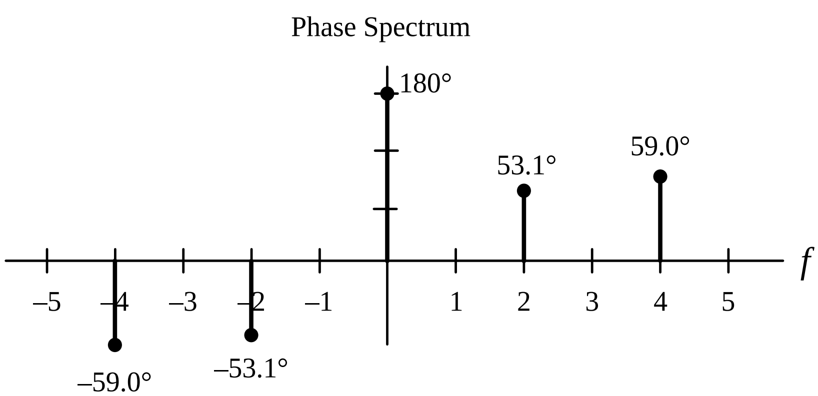
A periodic signal $x(t)$ with period $T_0 = 0.5$ s has complex exponential Fourier series coefficients

$$X_n = \begin{cases} -3, & n = 0 \\ \dfrac{3}{n^2+1} + j\dfrac{2}{n}, & n \neq 0 \end{cases}$$

a) Sketch the double-sided magnitude spectrum of $x(t)$ over the range –5 Hz to 5 Hz showing all important value on the x- and y-axis.



b) Sketch the double-sided phase spectrum of $x(t)$ over the range –5 Hz to 5 Hz.



c) Find the average power of $x(t)$ contained in the frequency range –5 Hz to 5 Hz.

Over the range –5 Hz to 5 Hz, the average power is

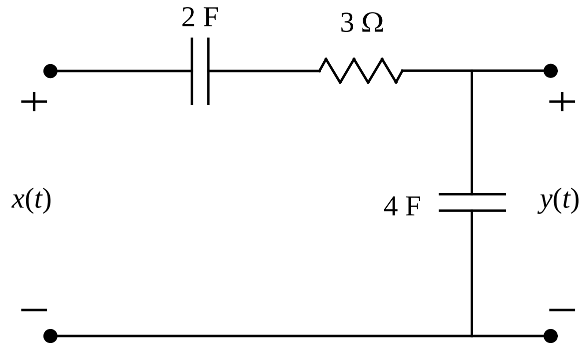
$$P_{Avg} = (3)^2 + 2(2.5)^2 + 2(1.166)^2$$

$$= 9 + 2(6.25) + 2(1.3596)$$

$$= 24.2191 \text{ W}$$

Problem 3:

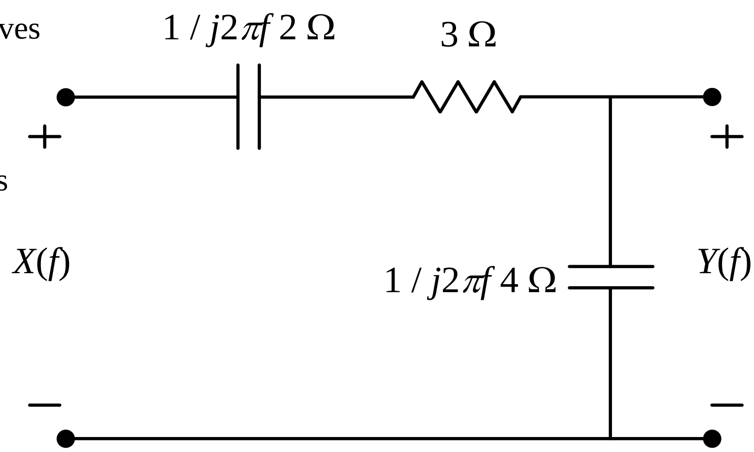
For the system represented by the circuit shown to the right, where $x(t)$ is the input and $y(t)$ is the output. Find:



a) The frequency response $H(f)$ of the system.

b) The impulse response $h(t)$ of the system.

a) Converting the circuit to Frequency Domain gives



The frequency response $H(f)$ of the system is

$$H(f)=\frac{\frac{1}{j2\pi f\,4}}{\frac{1}{j2\pi f\,4}+3+\frac{1}{j2\pi f\,2}}$$

$$=\frac{1}{\frac{j2\pi f\,4}{j2\pi f\,4}+3j2\pi f\,4+\frac{j2\pi f\,4}{j2\pi f\,2}}$$

$$=\frac{1}{1+3j2\pi f\,4+2}$$

$$\boxed{H(f)=\frac{1}{3+j2\pi f\,12}}$$

b) The frequency response $H(f)$ can be reformatted by dividing numerator and denominator by 12 to give:

$$H(f)=\frac{\frac{1}{12}}{\frac{1}{4}+j2\pi f}$$

So,

$$h(t)=\frac{1}{12}e^{-\frac{1}{4}t}u(t)$$

Problem 4:

a) Using the tables of Laplace transform, find the Laplace transform of the signal

$$x(t) = \delta(t-10) + e^{-2t}u(t)$$

From Table 5-3.1 and 5-3.3

$$\delta(t) \xleftrightarrow{L.T.} 1$$

$$e^{-2t}u(t) \xleftrightarrow{L.T.} \frac{1}{s+2}$$

Note that $\delta(t-10) = \delta(t-10)\cdot u(t-10)$

Using the Time Delay (Table 5-2.5) on the first relation gives

$$\delta(t-10) \xleftrightarrow{L.T.} e^{-10s}$$

Using Linearity (Table 5-2.1) to combine the two terms

$$\delta(t-10) + e^{-2t}u(t) \xleftrightarrow{L.T.} e^{-10s} + \frac{1}{s+2}$$

$$X(s) = e^{-10s} + \frac{1}{s+2}$$

b) Using the tables of the Laplace transform, find the signal $x(t)$ whose single-sided Laplace transform is

$$X(s) = \frac{1}{s} + \frac{e^{-2s}}{s+5}$$

From Table 5-3.1 and 5-3.3

$$u(t) \overset{LT}{\longleftrightarrow} \frac{1}{s}$$

$$e^{-5t}u(t) \overset{LT}{\longleftrightarrow} \frac{1}{s+5}$$

Using the Time Delay (Table 5-2.5) on the second relation gives

$$e^{-5(t-2)}u(t-2) \overset{LT}{\longleftrightarrow} \frac{e^{-2s}}{s+5}$$

Using Linearity (Table 5-2.1) to combine the two terms

$$u(t) + e^{-5(t-2)}u(t-2) \overset{LT}{\longleftrightarrow} \frac{1}{s} + \frac{e^{-2s}}{s+5}$$

So,

$$x(t) = u(t) + e^{-5(t-2)}u(t-2)$$

c) Find the initial and final values (assuming they exist) of the signal $x(t)$ with the Laplace transform given below.

$$X(s) = \frac{2s+5}{s^2+3s}$$

Initial Value

$$x(0^+) = \lim_{s\to\infty}\left((s)\frac{2s+5}{s^2+3s}\right)$$

$$= \lim_{s\to\infty}\left(\frac{2s^2+5\not{s}}{s^2+3\not{s}}\right) = 2$$

Final Value

$$x(\infty) = \lim_{s\to 0}\left((s)\frac{2s+5}{s^2+3s}\right)$$

$$= \lim_{s\to 0}\left(\frac{2\not{s}^2+5s}{\not{s}^2+3s}\right) = \frac{5}{3}$$

d) A signal $x(t)$ has the Laplace transform $X(s) = \frac{s}{s^2+2}$. Find the Laplace transform $Y(s)$ of the signal

$$y(t) = 2x\left(\frac{t}{4}\right) + 3x(5t) + x(t) * x(t)$$

without finding $x(t)$.

Apply the Laplace transform of both sides of the above equation

$$Y(s) = 8\cdot X(4s) + \frac{3}{5}\cdot X\left(\frac{s}{5}\right) + X(s)\cdot X(s)$$

$$= 8\cdot\frac{4s}{16s^2+2} + \frac{3}{5}\cdot\frac{\frac{s}{5}}{\frac{s^2}{25}+2} + \left(\frac{s}{s^2+2}\right)^2$$