King Fahd University of Petroleum & Minerals

Electrical Engineering Department

EE207: Signals and Systems (043)

Final Exam

Tuesday, August 23, 2005

12:30 PM-03:00PM

Building 14-108

Serial #

Name:_____________________________

ID:________________________________

Sec. (1) 9:20-10:20, Dr. Muqaibel (2) 10:30-11:30, Dr. Andalusi

Question	Mark
1	/10
2	/10
3	/10
4	/10
5	/10
Total	/50

Instructions:

1. This is a closed-books/notes exam.
2. Read the questions carefully. Plan which question to start with.
3. Write explicitly the formulas that you use in your solution (e.g. by KVL … by KCL). No credit will be given if you do not show your formulas.
4. Work in your own.
5. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
6. Strictly no mobile phones are allowed.

Good luck

Dr. Ali Muqaibel & Dr. Adnan Andalusi

Problem 1: **(10 points)**

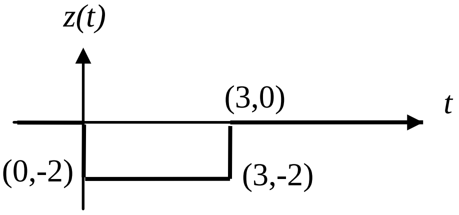
Choose (Circle) the correct answer (or) answers: **(5 points)**

a. If $y(t)$ is a real function of time then the magnitude of its Fourier transform is **(even, odd)** function of frequency.

b. The spectrum of a discrete time sampled signal is **(Periodic in frequency, non-periodic in frequency)**.

c. The shown signal can be represented by

($-u(t)+u(t-3)$, $-2u(t)\times u(-t+3)$, $\Pi(\frac{t}{3}-0.5)$, $-2\,sinc\,(3t)$)



e. The signal $z(t)$, shown in the previous figure, is
(energy signal, power signal, neither energy nor power)

d. A system which is represented by $y(t) = \cos(t) + x^2(t)$, where $x(t)$ is the input signal and $y(t)$ is the output signal is
(time varying, time invariant, linear, non-linear)

(5 points)

2. State whether the following statements are True or False: (fill the table below with T or F)
+1 for any correct answer, and -0.5 for any wrong answer. Maximum=5, Minimum=0

a. $\int_{-\infty}^{\infty} u(t)dt = r(t)$.

b. The ramp response of a causal system is zero for $t<0$.

c. The z-transform of the sequence $x[(n-1)T_s]$ is $z^{-1}X(z)$, where $X(z)$ is the z-transform of $x[n\,T_s]$.

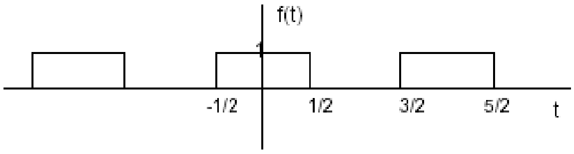
d. The convolution of two signals $x(t)$ and $y(t)$ in the time domain is equivalent to convolving $X(f)$ and $Y(f)$ in the frequency domain.

e. The bandwidth of a pulse is inversely proportional to its time duration.

Q	a	b	c	d	e
T or F					

Problem 2:

Consider the following periodic signal $f(t)$:



a) What is the period and fundamental frequency of this signal?
b) Show that the Fourier series of this signal is given by:
   $f(t) = 1/2 + (2/\pi)*[\cos(\pi t) - 1/3 \cos(3\pi t) + 1/5 \cos(5\pi t) - 1/7 \cos(7\pi t) + \ldots]$
c) Give the exponential form of this Fourier series.
d) Give a simple plot of the double-sided line spectra of this signal (both magnitude and phase), and specify which one is even or odd.

Problem 3: **(10 points)**

Consider the following signal which is sampled using ideal impulse train at a rate of 10 samples/second.

$$x(t) = 1 + 4\cos(4\pi t)$$

(1 point) a) Calculate the power of $x(t)$.

(1 point) b) For the first 4 samples fill in the following table:

	Sampling Time	***Sampled Value***
n	$\boldsymbol{nT_s}$	$\boldsymbol{x(nT_s)}$
0		
1		
2		
3		

(3 points) c) Find **and** sketch the magnitude of $X(f)$, which represents the magnitude spectrum of $x(t)$.

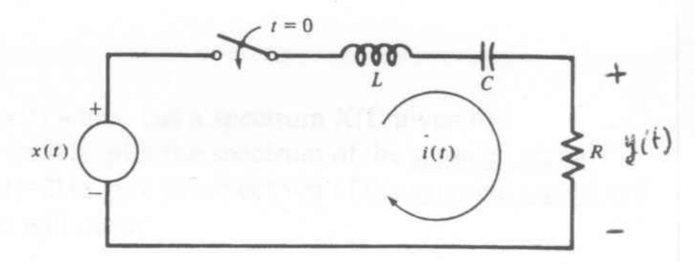
(3 points) d) Find and sketch $|X_s(f)|$, which represents the magnitude spectrum of the sampled signal $x_s(t)$, Your sketch should show the range of frequencies $-25<f<25$ Hz.

(1 point) e) Suggest a method to reconstruct $x(t)$ from $x_s(t)$?

(1 point) f) What is the minimum required sampling frequency to avoid aliasing?

<u>Problem 4:</u> (10 points)

Consider the following circuit:

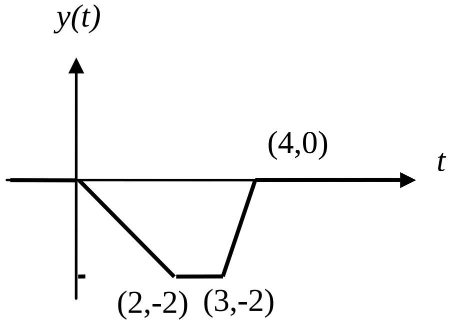


1) Write the equivalent Laplace transform model for this circuit. You can assume that the initial conditions are all zero ($v_C(0)=0$ and $i_L(0)=0$).
2) Assume the output Y(s) is the voltage $V_R(s)$. Find the transfer function Y(s)/X(s).
3) Assume that C=1F, L=1H, R=2Ω, and the input is a step: x(t) = u(t). Find the output function y(t).
4) Suppose now that C=1/8 F, L=1H, R=6Ω, and the input is x(t)=exp(-t).u(t). Find the output function y(t).

<u>Problem 5:</u> (10 points)

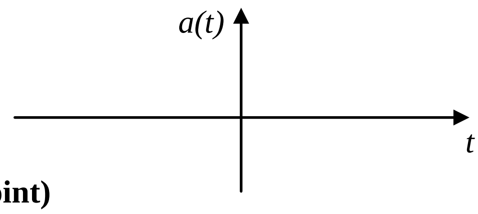
A. Find the Fourier transform of the following signals (Tables are attached, If needed)

Hint: you may use successive differentiation **(4 points)**



B. A linear fixed (time-invariant) system has the following **<u>step</u>** response **(6 points)**

$$a(t) = 2e^{-4t}u(t+1)$$



a. Sketch the step response as function of time. **(1 point)**

b. Is this system causal or non-causal? <u>Justify your answer</u>. **(1 point)**

c. Find the impulse response, $h(t)$? **(1 point)**

d. What is the output if the input is $x(t) = \delta(t-2) + 3u(t+4)$ **(1 point)**

e. If the input is now given by $x(t) = \Pi\left(\frac{t-1}{4}\right)$, what would be the output of the system. **(2 points)**

Name:
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TABLE 4-1
Fourier Transform Theorems[a]

Name of Theorem		
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f)e^{-j2\pi f t_0}$
3a. Scale change	$x(at)$	$\lvert a\rvert^{-1}X\left(\frac{f}{a}\right)$
b. Time reversal	$x(-t)$	$X(-f) = X^*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t)e^{j\omega_0 t}$	$X(f - f_0)$
b. Modulation	$x(t)\cos\omega_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\,dt'$	$(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt' = \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df' = \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df'$

[a] $\omega_0 = 2\pi f_0$; $x(t)$ is assumed to be real in 3b.

TABLE 4-2
Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}\,\tau f$	Direct evaluation
2.	$2W\,\mathrm{sinc}\,2Wt$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1, Example 4-7
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}^2\,\tau f$	Convolution using pair 1
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp(-\alpha\lvert t\rvert),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$	Direct evaluation
8.	$\delta(t)$	1	Example 4-9
9.	1	$\delta(f)$	Duality with pair 7
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	Shift and pair 7
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$	
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
15.	$\mathrm{sgn}\,t$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
16.	$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$	Duality with pair 14
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t - \lambda}\,d\lambda$	$-j\,\mathrm{sgn}(f)X(f)$	Convolution and pair 15
18.	$\sum_{m=-\infty}^{\infty}\delta(t - mT_s)$	$f_s\sum_{m=-\infty}^{\infty}\delta(f - mf_s),\ f_s = T_s^{-1}$	Example 4-10

TABLE 5-2
Laplace Transform Theorems

Name	Operation in Time Domain	Operation in Frequency Domain
1. Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(s) + a_2X_2(s)$
2. Differentiation	$\dfrac{d^n x(t)}{dt^n}$	$s^n X(s) - s^{n-1}x(0^-) - \cdots - x^{(n-1)}(0^-)$
3. Integration	$\int_{-\infty}^{t} x(\lambda)\, d\lambda$	$\dfrac{X(s)}{s} + \dfrac{x^{(-1)}(0^-)}{s}$
4. s-shift	$x(t)\exp(-\alpha t)$	$X(s+\alpha)$
5. Delay	$x(t-t_0)u(t-t_0)$	$X(s)\exp(-st_0)$
6. Convolution	$x_1(t) * x_2(t) = \int_0^{\infty} x_1(\lambda)x_2(t-\lambda)\, d\lambda$	$X_1(s)X_2(s)$
7. Product	$x_1(t)x_2(t)$	$\dfrac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X_1(s-\lambda)X_2(\lambda)\, d\lambda$
8. Initial value (provided limits exist)	$\lim_{t\to 0^+} x(t)$	$\lim_{s\to\infty} sX(s)$
9. Final value (provided limits exist)	$\lim_{t\to\infty} x(t)$	$\lim_{s\to 0} sX(s)$
10. Time scaling	$x(at), \quad a > 0$	$a^{-1}X\left(\dfrac{s}{a}\right)$

TABLE 5-3
Extended Table of Single-Sided Laplace Transforms

Signal	Laplace Transform	Comments on Derivation
1. $\delta^{(n)}(t)$	s^n	Direct evaluation with aid of (1-66)
2. 1 or $u(t)$	$\dfrac{1}{s}$	Direct evaluation
3. $\dfrac{t^n \exp(-\alpha t)u(t)}{n!}$	$\dfrac{1}{(s+\alpha)^{n+1}}$	Differentiation applied to pair 3, Table 5-1
4. $\cos\omega_0 t\, u(t)$	$\dfrac{s}{s^2+\omega_0^2}$	Example 5-1
5. $\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{s^2+\omega_0^2}$	Example 5-1
6. $\exp(-\alpha t)\cos\omega_0 t\, u(t)$	$\dfrac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$	s-shift and pair 4
7. $\exp(-\alpha t)\sin\omega_0 t\, u(t)$	$\dfrac{\omega_0}{(s+\alpha)^2+\omega_0^2}$	s-shift and pair 5
8. Square wave: $u(t) - 2u\left(t - \dfrac{T_0}{2}\right) + 2u(t-T_0) - \cdots$	$\dfrac{1}{s}\dfrac{1-e^{-sT_0/2}}{1+e^{-sT_0/2}}$	Example 5-5
9. $(\sin\omega_0 t - \omega_0 t\cos\omega_0 t)u(t)$	$\dfrac{2\omega_0^3}{(s^2+\omega_0^2)^2}$	Example 5-12, pair 5, and convolution
10. $(\omega_0 t\sin\omega_0 t)u(t)$	$\dfrac{2\omega_0^2 s}{(s^2+\omega_0^2)^2}$	Pair 4 and convolution
11. $\omega_0 t\exp(-\alpha t)\sin\omega_0 t\, u(t)$	$\dfrac{2\omega_0^2(s+\alpha)}{[(s+\alpha)^2+\omega_0^2]^2}$	s-shift and pair 10
12. $\exp(-\alpha t)(\sin\omega_0 t - \omega_0 t\cos\omega_0 t)u(t)$	$\dfrac{2\omega_0^3}{[(s+\alpha)^2+\omega_0^2]^2}$	s-shift and pair 9