Evaluation of Robust Estimators for Bad-Data Detection in Power Systems

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State Estimators in Power Systems

- Estimators are used when unknown states (or parameters) in a given mathematical model must be determined from available measurements

- In power systems, there are more measurements than are strictly needed to define the unknowns and the problem is called over-determined

State Estimators in Power Systems

- This type of problem is variously referred to as state estimation, parameter estimation, multivariate regression, and curve fitting

- This presentation illustrates and explains five robust state estimators for bad-data detections in power systems

Least Squares (LS) Estimator

- The well known least squares estimate can be found by
- $\text{Min}\, \underline{e}^T \underline{e}$
- Subject to
- $A\underline{x} = \underline{b} + \underline{e}$
- In other words, choose values for the unknown elements of $\underline{x}$ and unknown measurement errors $\underline{e}$ that give a minimum sum of squared errors.

Least Squares (LS) Estimator

- If the measurements have normally-distributed errors, the method of Least Squares (LS), or more generally Weighted Least Squares (WLS), provides an optimal solution

- However, if some of the measurements are statistical outliers (i.e. have unexpected very large errors) then the LS estimate becomes unreliable

Least Absolute Values (LAV) Estimator

- An efficient algorithm for LAV estimation is via the solution of the following linear program
- $\text{Min} \quad S\,(e_i + f_i)$
- Subject to: $\quad A\underline{x} - \underline{e} + \underline{f} = \underline{b}$
- $\underline{e} \geq \underline{o}\,,\ \underline{f} \geq \underline{o}$
- where $\underline{e}$ and $\underline{f}$ are non-negative vectors of unknown measurement errors

Least Absolute Values (LAV) Estimator

- By taking the absolute value (or modulus) of the residual, the effect of outliers on the estimate is reduced.

- A property of LAV estimates is that at least ‘n’ of the measurements will be fitted exactly (with zero residuals)

Least Median of Squares (LMS) Estimator

- A characterisation of an LMS estimate is that it seeks a regression that minimises the value of a tolerance ‘t’ whereby the majority of the measurements fall within tolerance

- $\text{Min} \quad t$
- Subject to: $\underline{b} - t - M\,\underline{k} \le A\underline{x} \le \underline{b} + t + M\,\underline{k}$
- $k_1 + k_2 + \ldots + k_m \le K$

Least Median of Squares (LMS) Estimator

- This estimator is a generalisation of the idea that the median of a set of real values is a more robust estimate than the mean

- For example, if we measure temperature using five different thermometers and obtain the readings 12.7, 12.5, 19.8, 12.6, 12.8, the median (12.7) is a more robust estimate than the mean (14.08)

Least Trimmed Squares (LTS) Estimator

- This estimator considers the sum of squared errors for the (m-K) smallest residuals only

- Equivalently, the K largest residuals are rejected and the remaining residuals are considered in a least squares objective

- $$\text{Min} \qquad \underline{e}^T\underline{e}$$
- Subject to: $\underline{b} - M\,\underline{k} \le A\underline{x} - \underline{e} \le \underline{b} + M\,\underline{k}$
- $$k_1 + k_2 + \ldots + k_m \le K$$

Least Measurements Rejected (LMR) Estimator

- This estimator requires the user to pre-specify a tolerance for each measurement and then seeks a regression that minimises the number of measurements unable to satisfy their tolerance

- $$\text{Min} \qquad \text{S } k_i$$
- Subject to: $\underline{b} - M\,\underline{k} - \underline{t} \le A\underline{x} \le \underline{b} + M\,\underline{k} + \underline{t}$

IEEE 14-Bus Example

- the IEEE 14-bus DC example is used to illustrate and evaluate the solutions of the above five estimators

- Six linear regression cases (case 1, case 2, case 3a, case 3b, case 3c, and case 3d) have been considered

- Cases 1 and 2 are intended to be a straightforward problem with low and high redundancy measurements, respectively

IEEE 14-Bus Example

- Case 3a introduces three independent bad-data to case 2: P1, P10, and P7-8
- Case 3b introduces two confirming bad-data: P1, P1-2
- Case 3c introduces two leverage bad-data: P2, P5
- Case 3d introduces a leverage bad-data P2 and a confirming bad-data P1-2

IEEE 14-Bus Example

- Table below shows the summation of the absolute of the difference between the estimated and the actual values of all cases.

	LS	LAV	LMS	LTS	LMR
Case 1	0.0920	0.1216	0.1076	0.1344	0.1077
Case 2	0.0905	0.0819	0.0889	0.0902	0.0813
Case 3a	1.6892	0.6613	0.0813	0.0593	0.0815
Case 3b	0.4176	0.3665	0.0743	0.0573	0.0564
Case 3c	0.5022	0.1181	0.6215	1.1125	0.3962
Case 3d	0.3590	0.0829	0.0878	0.4549	0.1568

IEEE 14-Bus Example

- It can be observed that all the measurements can be fitted reasonably well by all five estimators for the first two cases

- However, better fitting has been achieved when the redundancy of the system is high as in Case 2

IEEE 14-Bus Example

- In Case 3a, the LTS, and LMR estimators have successfully rejected the three independent bad-data measurements
- The LTS estimator performed better than the LMR estimator in terms of the estimated values
- The LAV and LMS estimators failed to reject the third bad-data measurement P7-8

IEEE 14-Bus Example

- In Case 3b, the LMS, LTS, and LMR estimators have all successfully rejected the two confirming bad-data measurements
- The LMR estimator performed better than the LMS and LTS estimators

IEEE 14-Bus Example

- In Case 3c, the LAV estimator is the only estimator that rejected successfully the two leverage bad-data measurements

- The LS, LMS, LTS, and LMR estimators have failed to reject the two leverage bad-data measurements

IEEE 14-Bus Example

- In Case 3d, the LAV, LMS and LMR estimators have rejected successfully the leverage and the confirming bad-data measurements

- However, the LAV and LMS estimators have the best estimated values

Conclusions

- It is difficult to draw a general conclusion on the performance of a particular estimator
- They all depend on the type and number of bad-data measurements

Thank You