Joint Probing and Scheduling in Wireless Communications Systems

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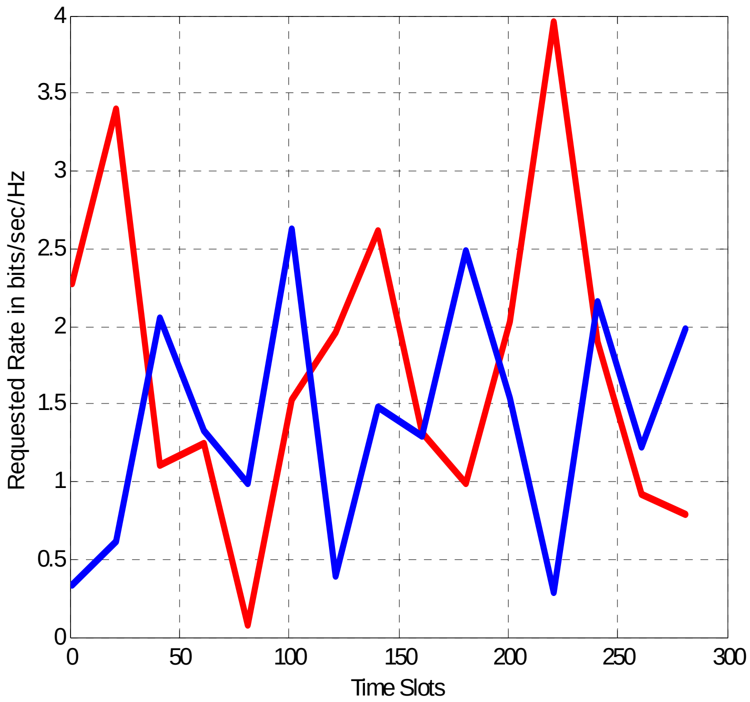
Outline

- The concept of Multiuser Diversity.
- Literature Review
- System Model
- Algorithm 1
- Algorithm 2
- Conclusions

Basic Concept of Multiuser Diversity

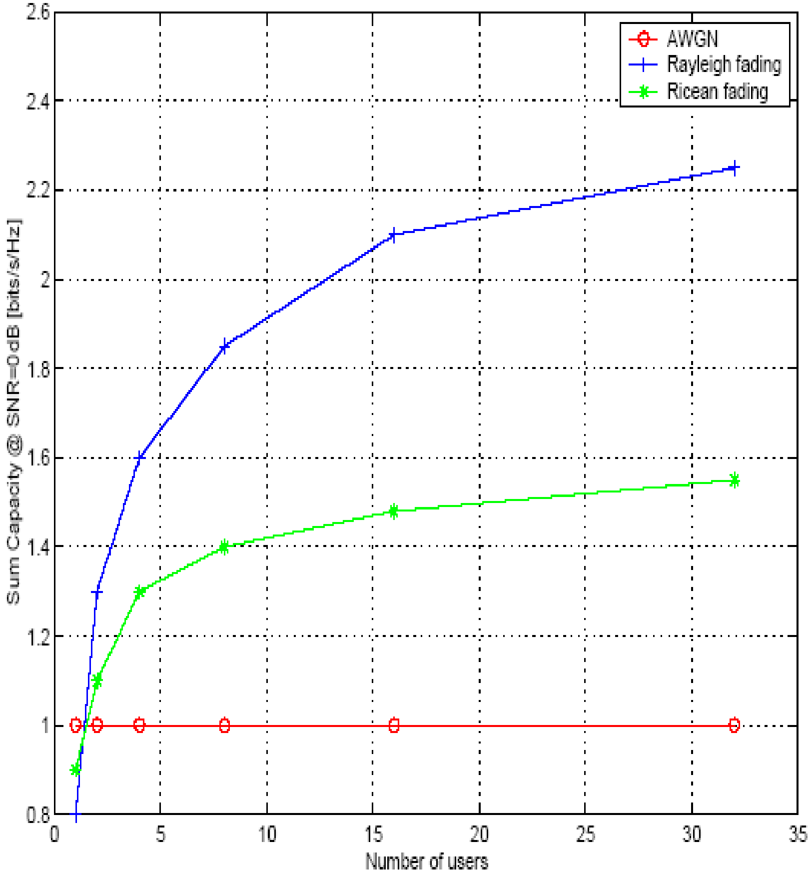
- In a large system, there is likely to be a user with a very good channel at any time.
- Long term total system throughput can be maximized Uplink [Knopp and Humblet "1995"], and Downlink [Tse "1997"].
- Scheduling schemes that exploit the time-varying channel conditions are "***opportunist***".



Two users

Multiuser Diversity (cont'd)

- The a mount of multiuser diversity gain depends crucially on the tail of the fading distribution.
- The heavier the tail, the more likely there is a user with very strong channel, and the larger the multiuser diversity gain.



Literature Review

- In [D. Zheng *et al* “2007”], an optimal stopping algorithm was used to obtain the optimal number of probing for opportunistic scheduling in ad-hoc networks.
- In [N. Chang *et al* “2007”], a study of the optimal strategy for joint probing and transmission in multiple channel was conducted. The goal is to decide which channel to probe and in which order.
- Similar study was conducted by [S. Guha *et al* “2006”]
- In [P. Chaporkar *et al* “2008”], optimal strategies were provided to optimize the joint probing and scheduling.
- In [A. Sabharwal *et al* “2007”], opportunistic band selection is considered and analysis of the optimal skipping rule was derived.

System Description

- Let $\Gamma = \{\gamma_i : i = 1, 2, \ldots, K\}$ be i.i.d. rv's with $F_\Gamma(\gamma)$ distribution function
- Scheduling based on the following yields optimal channel capacity (MUDiv):

$$\begin{cases} \gamma^\star = \max_{i=1,2,\ldots,K} \gamma_i, \\ R^\star = \max_{i=1,2,\ldots,K} \log_2(1+\gamma_i). \end{cases}$$

- Unfortunately, the is a cost, which we call "***probing load***"

$$T(x) = (1 - \beta x) \log_2(1 + \gamma_x).$$

number of probes

System Description (cont'd)

- Let $C_i = \log_2(1+\gamma_i)$, then

$$M_k = \max_{i=1,2,\ldots,k} C_i,$$

- A paper by G. Song *et al* found an asymptotic result for the throughput using Extreme Value Theorem. They showed that the distribution of the throughput belongs to the domain of attraction of the Gumbel distribution.

$\exists\ a_k, b_k : \frac{M_k - a_k}{b_k} \to Y,\ Y \sim \exp(-e^{-y})$ (Gumbel distribution), where

$$a_k = \log_2\left(1 + F_\Gamma^{-1}\left(1 - \frac{1}{k}\right)\right),$$

$$b_k = \log_2\left(\frac{1 + F_\Gamma^{-1}\left(1 - \frac{1}{(k)(e)}\right)}{1 + F_\Gamma^{-1}\left(1 - \frac{1}{k}\right)}\right).$$

System Description (cont'd)

- Therefore,

$$M_k = b_k Y + a_k$$
$$= \log_2\left(1 + \frac{\bar{\gamma}}{1+\bar{\gamma}\ln k}\right)Y + \log_2\left(1 + \bar{\gamma}\,\underline{\ln k}\right).$$

MUDiv gain

- and

If $k \to \infty,\ a_k \to \infty,\ b_k \to 0$

$$M_k \approx \log_2(1 + \bar{\gamma}\ln k).$$

System Description (cont'd)

- Therefore, the maximization become (average domain)

$$\max_{i=1,2,\dots,k}(1-\beta i)M_i,$$

or

$$\max_{i=1,2,\dots,k} e^{-\beta i}M_i.$$

- Given the number of users (***n***), the maximum average throughput is:

$$\bar{T}^{\star}(n) = (1-\beta n)\log_2(1+\bar{\gamma}\ln n).$$

- d*T*/d*n* = 0 to find ***n*** that maximizes the above equation.

Algorithm 1

- Given ***K*** users in the system. Find $x = n^*$
- Probe the users and stop if one of the following occurs:
  - ***x*** users have been probed, or
  - a user is found such that

$$T_p(i) - \bar{T}^\star(n) \geq 0,$$

where,

$$T_p(i) = (1 - \beta i)\log_2(1 + \max_{j=1,2,\ldots,i} \gamma_j).$$

Numerical Example (Algorithm 1)

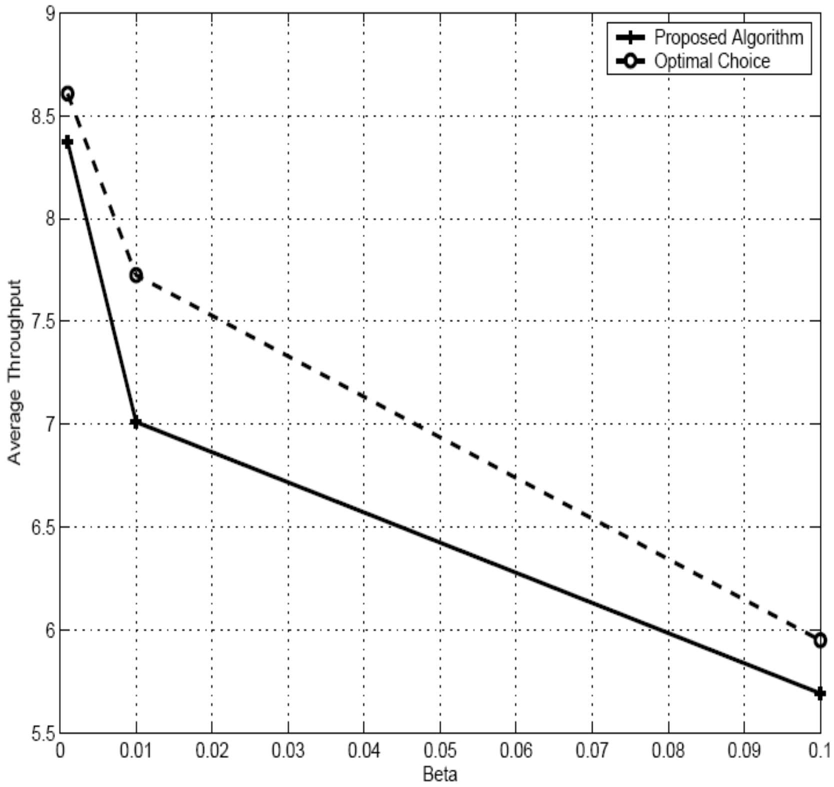


Figure 1: Average throughput for different value is β ($\bar{\gamma} = 20$dB).

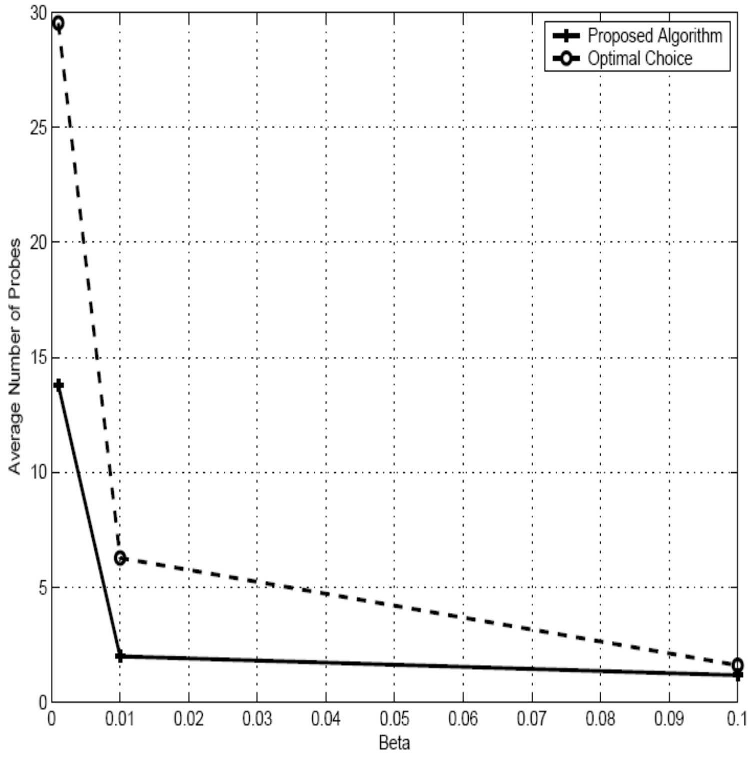


Figure 2: Average number of probes (probes users) for different value is β ($\bar{\gamma} = 20$dB).

Numerical Example (Algorithm 1)

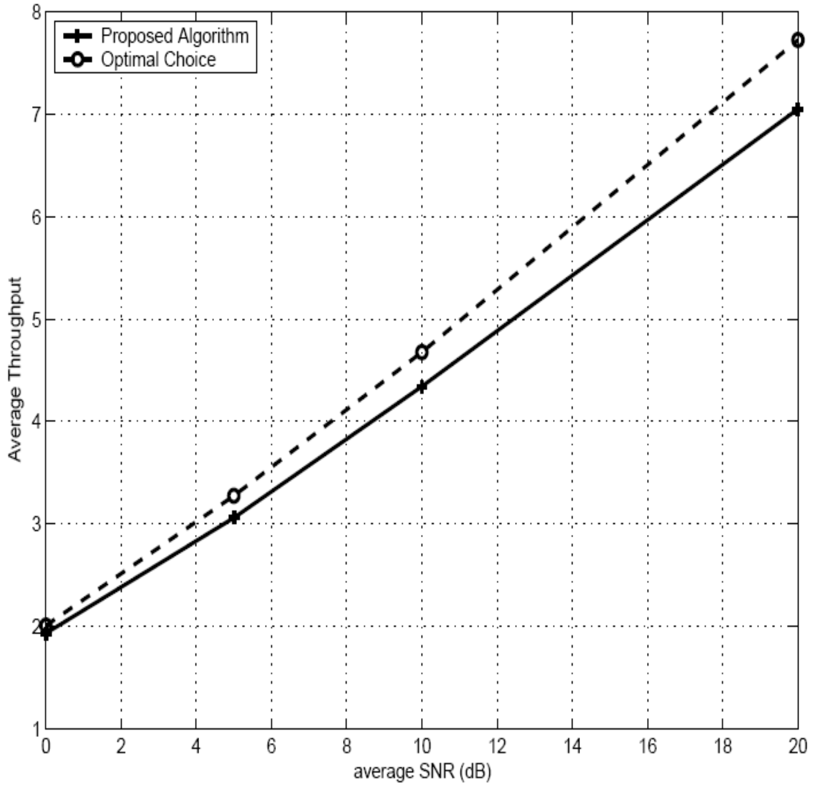


Figure 3: Average throughput for different value is $\bar{\gamma}$ ($\beta = 0.01$).

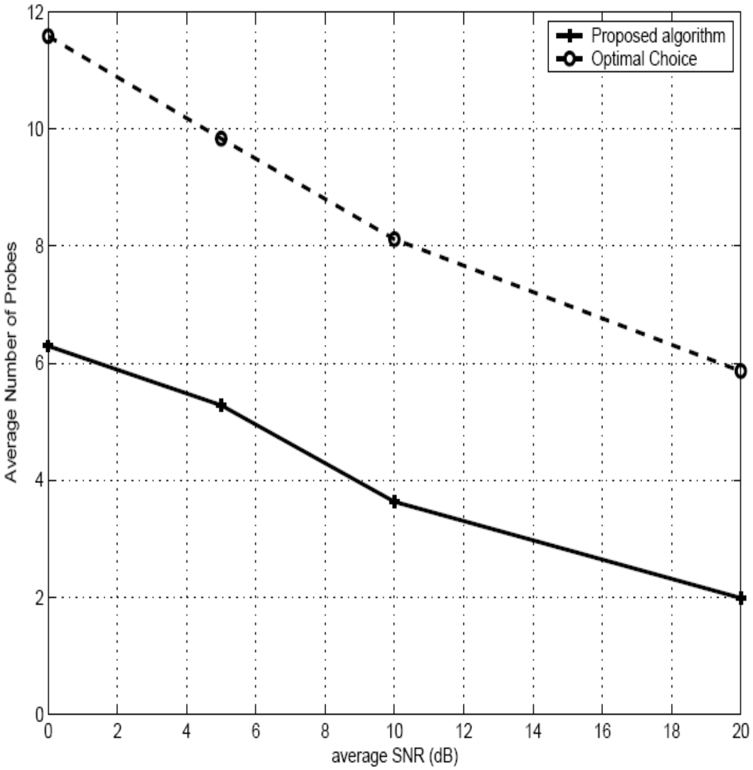


Figure 4: Average number of probes (probes users) for different value is $\bar{\gamma}$ ($\beta = 0.01$).

Stopping Rule Problem

- The Theory of optimal stopping rule is concerned with the problem of choosing a time to take a given action based on sequentially observed rv's in order to maximize an expected payoff or to minimize an expected cost.
- **Definition:** Stopping rule problems are defined by two objects,
  - a sequence of rv's, $X_1, X_2, \ldots,$ whose joint distribution is assumed know, and
  - a sequence of real-valued reward functions,

$$y_o, y_1(x_1), y_2(x_1, x_2), y_3(x_1, x_2, x_3), \ldots, y_\infty(x_1, x_2, \ldots)$$

- **The 1-stage look-ahead rule.** For stopping rule problems, the 1-sla is described by the stopping time,

$$N^* = \min\{n \geq 0 : Y_n \geq x^*\}.$$

Algorithm 2

- Problem statement:

$$\max_{i=1,2,\ldots,k}(1-\beta i)M_i, \qquad \text{or} \qquad \max_{i=1,2,\ldots,k} e^{-\beta i}M_i.$$

- We consider the second maximization and find the 1-sla threshold,

$$e^{-\beta k}M_k \geq E[e^{-\beta(k+1)}(M_k \vee C_{k+1})]$$
$$M_k \geq e^{-\beta}E[M_k \vee C_{k+1}].$$

- The optimal threshold is,

$$x^* = e^{-\beta}E[x^* \vee C_1].$$

- Stop if,

$$\log_2(1+\max_{j=1,2,\ldots,i}\gamma_j) \geq x^*.$$

Iterative Computation of the threshold

- Assume $C_1, C_2, \ldots, C_n$ $i.i.d$ rv's with $\overline{C} = \mu$ and distribution function $F_C(x)$
- Initially, set $v_1 = \mu$
- Iteratively calculate:

$$v_{l+1} = e^{-\beta} E[v_l \vee C_1]$$

where

$$E[v_l \vee C_1] = \int_0^{v_l} v_l f_C(y) dy + \int_{v_l}^{+\infty} y f_C(y) dy$$
$$= v_l + \int_{v_l}^{+\infty} (1 - F_C(y)) dy.$$

Numerical Example

Parameters beta=0.01, mean channel gain = 7dB, K=100 users	**Algorithm 1**	**Algorithm 2**	**Optimal**
Throughput (bps/Hz)	3.5177	3.7354	3.8561
Probes (users)	14	11.1170	

Parameters beta=0.1, mean channel gain = 7dB, K=100 users	**Algorithm 1**	**Algorithm 2**	**Optimal**
Throughput (bps/Hz)	2.2124	2.5805	2.6885
Probes (users)	4	2.8690	

Open Questions

- What is the optimal stopping rule of the first maximization?
- What about 2-sla or 3-sla?
- Does this scheme stabilizes the queues? The stability region?

Thanks