Digital Transmission over AWGN Channel

Contents

Pulse Modulator

The stream of 0's and 1's is mapped into a sequence of pulses. For the case of binary transmission, the widely used signaling are:

1) **bipolar (or PAM):** where a "1" is represented by a pulse of amplitude *A* and duration T_b, and a "0" is represented by amplitude –*A*.
2) **Unipolar (OOK):** "1" is a pulse of amplitude *A*, no pulse transmission for "0"

For ***M*-ary** pulse transmission (like multi-amplitude M-ary PAM or orthogonal M-FSK). A block of *k* bits, called a symbol, is represented by one of the $M = 2^k$ symbols. For a bit rate of R_b, the symbol duration $T = kT_b$.

Baseband vs Passband

Geometric Representations of Signal Waveforms

Such a representation provides a **compact** characterization of signal sets for transmitting information over a channel and **simplifies the analysis** of their performance.

Suppose we have a set of M signal waveforms $s_m(t)$ where $1 \le m \le M$ which are to be used for transmitting information over a communication channel. These M signals can be represented in a N dimensional space $(N \le M)$ defined by the set of orthonormal basis functions $\psi_i(t)$, $1 \le i \le N$.

Each signal waveform may be represented by the vector $\boldsymbol{s_m} = (s_{m1}, s_{m2}, \dots \dots, s_{mN})$ or equivalently as a point in the *N*-dimensional signal space with coordinates $\{s_{mn}, 1 \le i \le N\}$, the elements of the s_{mn} are the projection of $s_m(t)$ on to $\psi_n(t)$.

$$s_{mn} = \int_0^T s_m(t)\psi_n(t)dt$$

Note that the energy of the m^{th} signal , E_m is given by

$$E_m = \int_{-\infty}^{\infty} s_m^2(t)dt = \sum_{n=1}^{N} s_{mn}^2$$

Is simply the square of the Euclidean distance from the origin to the point in the *N*-dimensional space.

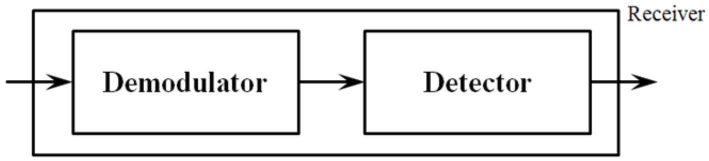
Gram Schmidt Orthogonalization Procedure

Optimum Receiver for Pulse-Modulated Signals in AWGN

$$r(t) = S_m(t) + n(t)$$

Sample function if AWGN process with power spectral density $S_n(f) = \frac{N_0}{2}$

Objective : To design a receiver that is optimum in the sense that it minimizes the probability of making error.



- The demodulator converts $r(t)$ to N-dimension observation vector $r(r_1, r_2, \ldots, r_N)$.
- The detector finds out which of the M possible signal waveforms was transmitted.

What is then the optimum demodulator & optimum detector?

For AWGN (Matched filter demodulator, *see your text book*)

Optimum Demodulator: is the matched filter, a filter whose impulse response $h(t)$ is matched to the signal $s(t)$.

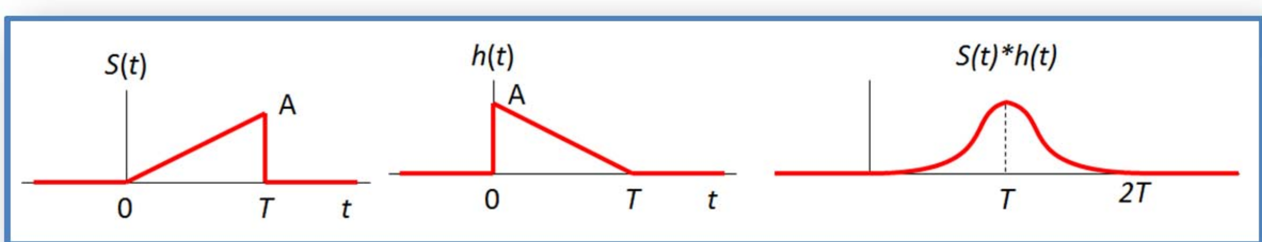
$h(t) = S(T - t)$, where T is the pulse period

$$H(f) = S^*(f)e^{-j2\pi fT}$$

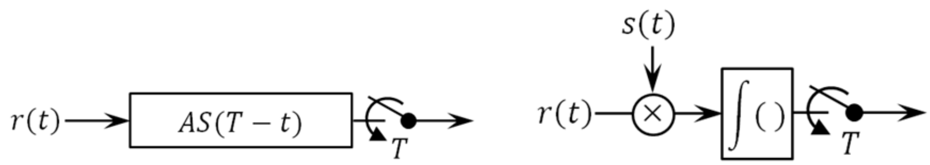
Matched filter has two interesting properties:

1) Maximizes the output SNR $\left(\frac{S}{N_0}\right)$
2) The output (maximum) SNR obtained is $\frac{2E_s}{N_0}$

Depends on the energy of the waveform, $s(t)$ not on its detailed characteristics



The matched filter can be realized by a product integrator or correlator.



Optimum Detector

if all $\{S_m\}$ are equi-probable, & for AWGN, the optimum detector is the *minimum distance detector*. Select the signal $\boldsymbol{S_m}$ that is closest in Euclidian distance to the received vector $\boldsymbol{r}$.

Make sure that you know the following detectors

MAP : Maximum Aposteriori Probabilities

ML : Maximum Likelihood

MLSE : Maximum Likelihood Sequence Estimator

Probability of Error for Signals in AWGN

$$Q(\sqrt{x}) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{x}{2}}\right)$$

1) Unipolar Signaling \FSK

$$P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

where E_b is the received average energy per bit.

2) Bipolar \PSK

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

- To simplify the notation, call $\frac{E_b}{N_0} = \gamma_b$; SNR per bit, Remember that E_b is the received energy/bit.
- If the channel has attenuation α then $E_b = \alpha^2 E_{b_{transmitted}}$
  In what follows, we are dropping α i.e.$\alpha = 1$

3) For the digital transmission via carrier modulation

(Coherent) PSK, (Coherent)FSK, M-ary PAM, M-ary PSK ($M = 2$& $M = 4$)

(Coherent) PSK: $P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

(Coherent) FSK: $P_b = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$

M-ary PAM: $P_M = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6(\log_2 M)E_b}{(M^2-1)N_0}}\right)$

Note that:

- $E_{av} = (\log_2 M)E_b$ is the average energy per symbol
- when $M = 2$, M-ary PAM reduces to BPSK.
- For M-ary PSK, no analytical solution is available except for $M = 2$ (BPSK given above), and $M = 4$
- When $M = 4$ we have in effect two BPSK signals in phase quadrature, with coherent demodulation. There is no interference between the signals in the quadrature carriers and hence P_b is identical to $M = 2$. Symbol error can be calculated as :

$$P_{e,4} = 1 - (1 - P_2)^2 = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - \frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]$$

(Coherent) PSK, (Coherent)FSK, M-ary PAM, M-ary PSK ($M = 2$& $M = 4$)

QAM

$$P_M \le 4Q\left(\sqrt{\frac{3kE_b}{(M-1)N_0}}\right) \cong 2Q\left(\sqrt{2k\frac{E_b}{N_0}}\sin\frac{\pi}{M}\right)$$

Binary DPSK

$$P_o = \frac{1}{2}e^{-\frac{E_b}{N_0}}$$

Non Coherent MFSK

$$\boldsymbol{P_M = \sum_{n=1}^{M-1}(-1)^{n+1}\binom{M-1}{n}\frac{1}{n+1}e^{-(\frac{nkE_b}{N_0})/(n+1)}}$$

Binary FSK

$$P_o = \frac{1}{2}e^{-\frac{E_b}{2N_0}}$$

which is 3dB worse than DPSK.

P_M and P_b

1) Orthogonal Signaling (e.g. MFSK)

All signals have the same distance between each other.

P_M: Symbol Error Probability.

Selecting any of (M-1) error is equi-probable with probability=$\frac{P_M}{M-1}$

- consider a symbol of k bits,

  there are $\binom{k}{m}$ ways in which m bits out of k bits may be in error. Since all k-tuple are used.

  ⇒ The average number of bits error per k-bit Symbol.

$$\sum_{m=1}^{k} m\binom{k}{m}\frac{P_M}{M-1} = k\frac{2^{k-1}}{2^k-1}P_M$$

⇒ For the average P_b divided by k ⇒ $P_b = \frac{2^{k-1}}{2^k-1}P_M$

2) For MPSK

- Nearest to two other points assumed to be most dominant error.
- if adjacent symbols differs by one bit only

$$P_b = \frac{P_M}{k}$$

Comparison of Digital Signaling Methods (Spectral Efficiency)

See your text book

- SNR to achieve P_e is not sufficient for accurate comparison.
- The most compact and meaningful comparison.
- Normalized data rate $\frac{R}{W}$ (bit /sec /Hz of BW) (BW efficiency) versus $\frac{E_b}{N_0}$ to achieve a given error rate.
- Suppose we fix R $\left(\frac{\text{bits}}{\text{sec}}\right)$, what is BW? $T_b = \frac{1}{R}$

  for k-bits symbol $T_s = \frac{k}{R}$
- The minimum BW required to transmit a pulse of width T is $\frac{1}{2T}$ (from Nyquist Criteria for zero ISI true for baseband, SSB pass band, or quadrature modulation)

$$w = \frac{R}{2k}\text{ Hz} \Rightarrow \frac{R}{w} = 2k = 2\log_2 M \text{ bits/sec/Hz}$$

- Doing so for other modulation schemes

M-PAM	$2\log_2 M$
QAM	$\log_2 M$
M-PSK	$\log_2 M$

Orthogonal $\frac{2\log_2 M}{m}$ (MFSK) where, $m = \frac{R}{w}$

- See attached figure, comparison of different modulation schemes at $P_s = 10^{-5}$.
- Explain: given M find γ_b to achieve $\frac{R}{w}$ at P_e(symbol)=10^{-5}.
- Comments on the figure below (Spectral Efficiency)
  1) PAM, QAM, PSK and DPSK are bandwidth-efficient, ⇒For fixed BW, the rate ↑ logarithmically with M (number of waveforms)
     The cost of R ↑ is $\frac{E_b}{N_0}$ to fix $P_e = 10^{-5}$
  2) Orthogonal signals make inefficient use of BW $\left(\frac{R}{W} < 1\right)$.
     - Trades BW for a reduction in $\frac{E_b}{N_0}$ to achieve P_s
  3) Normalized Channel Capacity limit

$$\frac{C}{W} = \log_2\left(1 + \frac{C}{W}\frac{E_b}{N_0}\right)$$

     - There is a large gap between the channel capacity and the given performance ⇒ Coding
  4) Different modulation schemes evaluated have different spectral characteristics.

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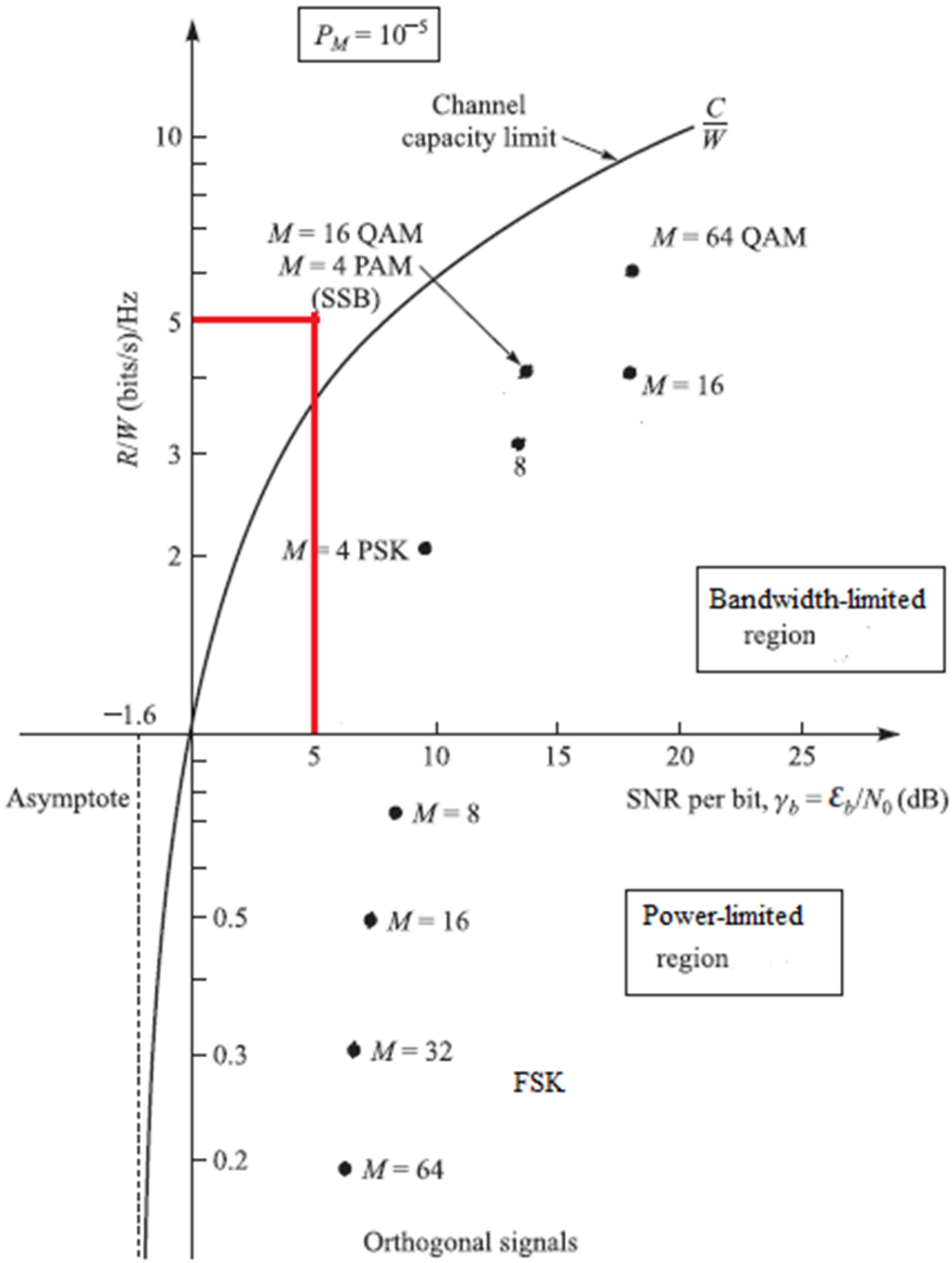
http://faculty.kfupm.edu.sa/ee/muqaibel/Courses/courses.htm


$P_M = 10^{-5}$
Channel capacity limit
$\frac{C}{W}$
10
$M = 16$ QAM
$M = 4$ PAM
(SSB)
$M = 64$ QAM
5
$M = 16$
3
8
2
$M = 4$ PSK
Bandwidth-limited region
R/W (bits/s)/Hz
−1.6
5
10
15
20
25
Asymptote
SNR per bit, $\gamma_b = \mathcal{E}_b/N_0$ (dB)
$M = 8$
0.5
$M = 16$
Power-limited region
0.3
$M = 32$
FSK
0.2
$M = 64$
Orthogonal signals