Problem 4.16 :

$$u(t) = X\cos 2\pi ft - Y\sin 2\pi ft$$
$$E\left[u(t)\right] = E(X)\cos 2\pi ft - E(Y)\sin 2\pi ft$$

and :

$$\begin{aligned}\phi_{uu}(t,t+\tau) &= E\left\{\left[X\cos 2\pi ft - Y\sin 2\pi ft\right]\left[X\cos 2\pi f(t+\tau) - Y\sin 2\pi f(t+\tau)\right]\right\} \\ &= E\left(X^2\right)\left[\cos 2\pi f(2t+\tau) + \cos 2\pi f\tau\right] + E\left(Y^2\right)\left[-\cos 2\pi f(2t+\tau) + \cos 2\pi f\tau\right] \\ &-E\left(XY\right)\sin 2\pi f(2t+\tau)\end{aligned}$$

For $u(t)$ to be wide-sense stationary, we must have : $E\left[u(t)\right]$ =constant and $\phi_{uu}(t, t{+}\tau) = \phi_{uu}(\tau)$. We note that if $E(X) = E(Y) = 0$, and $E(XY) = 0$ and $E(X^2) = E(Y^2)$, then the above requirements for WSS hold; hence these conditions are necessary. Conversely, if any of the above conditions does not hold, then either $E\left[u(t)\right] \neq$constant, or $\phi_{uu}(t, t+\tau) \neq \phi_{uu}(\tau)$. Hence, the conditions are also necessary.

Problem 4.22 :

(a) $I_n = a_n - a_{n-2}$, with the sequence $\{a_n\}$ being uncorrelated random variables (i.e $E(a_{n+m}a_n) = \delta(m)$). Hence :

$$\begin{aligned}
\phi_{ii}(m) &= E[I_{n+m}I_n] = E[(a_{n+m} - a_{n+m-2})(a_n - a_{n-2})] \\
&= 2\delta(m) - \delta(m-2) - \delta(m+2) \\
&= \left\{ \begin{array}{rr} 2, & \text{m} = 0 \\ -1, & \text{m} = \pm 2 \\ 0, & \text{o.w.} \end{array} \right\}
\end{aligned}$$

(b) $\Phi_{uu}(f) = \frac{1}{T}|G(f)|^2 \Phi_{ii}(f)$ where :

$$\begin{aligned}
\Phi_{ii}(f) &= \sum_{m=-\infty}^{\infty} \phi_{ii}(m) \exp(-j2\pi f m T) = 2 - \exp(j4\pi f T) - \exp(-j4\pi f T) \\
&= 2[1 - \cos 4\pi f T] = 4 \sin^2 2\pi f T
\end{aligned}$$

and

$$|G(f)|^2 = (AT)^2 \left(\frac{\sin \pi f T}{\pi f T}\right)^2$$

Therefore :

$$\Phi_{uu}(f) = 4A^2T\left(\frac{\sin \pi fT}{\pi fT}\right)^2 \sin^2 2\pi fT$$

(c) If $\{a_n\}$ takes the values (0,1) with equal probability then $E(a_n) = 1/2$ and $E(a_{n+m}a_n) = \left\{\begin{array}{ll} 1/4, & \mathrm{m} \neq 0 \\ 1/2, & \mathrm{m} = 0 \end{array}\right\} = [1+\delta(m)]/4$. Then :

$$\begin{array}{rcl} \phi_{ii}(m) & = & E\left[I_{n+m}I_n\right] = 2\phi_{aa}(0) - \phi_{aa}(2) - \phi_{aa}(-2) \\ & = & \frac{1}{4}\left[2\delta(m) - \delta(m-2) - \delta(m+2)\right] \end{array}$$

and

$$\Phi_{ii}(f) = \sum_{m=-\infty}^{\infty} \phi_{ii}(m)\exp(-j2\pi fmT) = \sin^2 2\pi fT$$

$$\Phi_{uu}(f) = A^2T\left(\frac{\sin \pi fT}{\pi fT}\right)^2 \sin^2 2\pi fT$$

Thus, we obtain the same result as in (b) , but the magnitude of the various quantities is reduced by a factor of 4 .

Problem 5.20 :

The constellation of Fig. P5-20(a) has four points at a distance $2A$ from the origin and four points at a distance $2\sqrt{2}A$. Thus, the average transmitted power of the constellation is:

$$P_a = \frac{1}{8}\left[4 \times (2A)^2 + 4 \times (2\sqrt{2}A)^2\right] = 6A^2$$

The second constellation has four points at a distance $\sqrt{7}A$ from the origin, two points at a distance $\sqrt{3}A$ and two points at a distance A. Thus, the average transmitted power of the second constellation is:

$$P_b = \frac{1}{8}\left[4 \times (\sqrt{7}A)^2 + 2 \times (\sqrt{3}A)^2 + 2A^2\right] = \frac{9}{2}A^2$$

Since $P_b < P_a$ the second constellation is more power efficient.

Problem 5.28 :

For 4-phase PSK ($M = 4$) we have the following realtionship between the symbol rate $1/T$, the required bandwith W and the bit rate $R = k \cdot 1/T = \frac{log_2 M}{T}$ (see 5-2-84):

$$W = \frac{R}{log_2 M} \rightarrow R = W log_2 M = 2W = 200 \text{ kbits/sec}$$

For binary FSK ($M = 2$) the required frequency separation is $1/2T$ (assuming coherent receiver) and (see 5-2-86):

$$W = \frac{M}{log_2 M} R \rightarrow R = \frac{2W log_2 M}{M} = W = 100 \text{ kbits/sec}$$

Finally, for 4-frequency non-coherent FSK, the required frequency separation is $1/T$, so the symbol rate is half that of binary coherent FSK, but since we have two bits/symbol, the bit ate is tha same as in binary FSK :

$$R = W = 100 \text{ kbits/sec}$$