Xterzn of B-L xnnels

Freq response $C(f) = |C(f)| e^{j\theta(f)}$

$|C(f)|$ = Amp $\theta(f)$ = phase-response

- Delay $\tau(f) = -\frac{1}{2\pi}\frac{d\theta(f)}{df}$

A xnnel is said to be non-distorting or ideal if d amp response $|C(f)|$ is const $\forall\ |f| \le W$ & $\theta(f)$ is a linear fn of freq.

Other impairements:

1. Non-linear distorn: From non-linear amp
2. Freq offset: less dan 5Hz, results from d use of carrier equip in d telephone xnnl.
3. Phase jitter: a low-index freq modln of d xmitted signal $\bar{c}$ d low freq harmonics of d power line freq (50-60Hz).

- d degree to wc one must be concerned $\bar{c}$ dese xnnel impairements depends on d xmission rate over d xnnel & d modln techs.

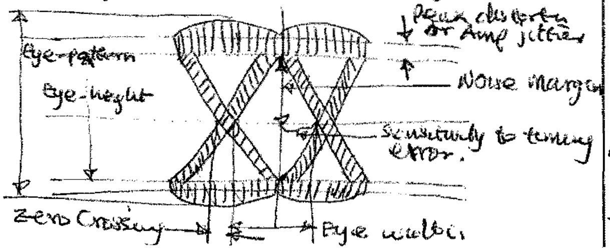
Signal design for B-L xnnels

LP xmitted signal is
$$v(t) = \sum_{n=0}^{\infty} I_n g(t-nT)$$
∴ Rxed signal is
$$r_l(t) = \sum_{n=0}^{\infty} I_n h(t-nT) + z(t)$$
d o/p of d Rxg filter
$$y(t) = \sum_{n=0}^{\infty} I_n x(t-nT) + v(t)$$
Now if $y(t)$ is samples @ $t = kT + \tau_0$
$$\Rightarrow y(kT+\tau_0) = y_k = \sum_{n=0}^{\infty} I_n x(kT-nT+\tau_0) + v(kT+\tau_0)$$
or $y_k = \sum_{n=0}^{\infty} I_n x_{k-n} + v_k \qquad k = 0, 1, \ldots$

Also $y_k = I_k + \sum_{\substack{n=0\\ n\ne k}}^{\infty} I_n x_{k-n} + v_k$

Eye-diagram

Amt of ISI & noise in a DC sys can be measure $\bar{c}$ E-P.



①

Design of B-L signals for no ISI

d Nyquist Criterion:

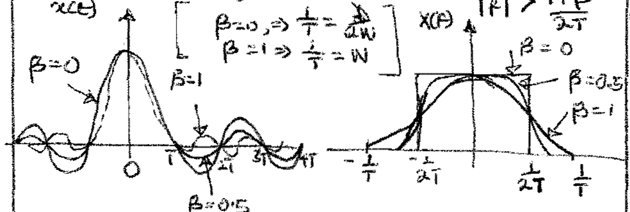
We make d ff assumptions:

1.) d B-Ltd xnnel has ideal freq-response i.e $C(f) = 1\ \forall\ |f| \le W$.
2.) Base on ① d pulse $x(t)$ has a special-xtic $X(f) = |G(f)|^2$, where $x(t) = \int_{-W}^{W} X(f) e^{j2\pi ft} df$.

Nyquist Condn for zero ISI

Thrm: d necessary & sufficient condn for $x(t)$ to satisfy
$$x(nT) = \begin{cases} 1 & n = 0 \\ 0 & n \ne 0 \end{cases} \quad \text{—— ①}$$
is dat its Fourier-T $X(f)$ satisfy
$$\sum_{m=-\infty}^{\infty} X(f + m/T) = T \quad \text{—— ②}$$

Raised Cosine
$$X_{rc}(f) = \begin{cases} T & 0 \le |f| \le \frac{1-\beta}{2T} \\ \frac{T}{2}\left[1+\cos\left[\frac{\pi T}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right]\right] & \frac{1-\beta}{2T} \le |f| \le \frac{1+\beta}{2T} \\ 0 & |f| > \frac{1+\beta}{2T} \end{cases}$$



Excess BW $W = \frac{R}{2}(1+\beta)$.

Design of B-Ld signals $\bar{c}$ Controlled ISI

Duo-binary:

$x(nT) = \begin{cases} 1, & n = 0, 1 \\ 0 & \text{otherwise} \end{cases} \Rightarrow b_n = \begin{cases} T, & n = 0, -1 \\ 0 & \text{otherwise} \end{cases}$ $b_n = T x(nT)$

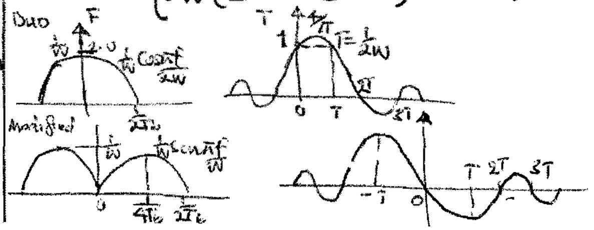
$B(f) = T + Te^{-j2\pi fT} = \sum_{n=-\infty}^{\infty} X\left(f + \frac{m}{T}\right)$

For $T = \frac{1}{2W} \Rightarrow X(f) = \begin{cases} \frac{1}{W} e^{-j\pi f/2W} \cos\frac{\pi f}{2W} & |f| \le W \\ 0 \end{cases}$

∴ $x(t) = \mathrm{Sinc}(2\pi Wt) + \mathrm{Sinc}[2\pi(Wt - \frac{1}{2})]$

Modified duobinary

$$x\left(\frac{n}{2W}\right) = x(nT) = \begin{cases} 1 & n = -1 \\ -1 & n = 1 \\ 0 & \text{otherwise} \end{cases}$$
$$X(f) = \left\{\frac{1}{2W}\left(e^{j\pi f/W} - e^{-j\pi f/W}\right)\right. \quad |f| \le W$$



②

Data detection for Controlled ISI

2-methods

1. Symbol-by-Symbol

$y_m = B_m + v_m$, $B_m = I_m + I_{m-1}$

$P_m = D_m \ominus P_{m-1} \pmod 2$, $I_m = 2P_m - 1$

$D_m = \frac{1}{2}B_m + 1$, if $B_m = \pm 2$, $D_m = 0$, $B_m = 0$, $D_m = 1$.

$I_m = -1$ if $P_m = 0$, $I_m = 1$ if $P_m = 1$.

General $I_m = 2P_m - (M-1)$

$D_m = \frac{1}{2}B_m + (M-1) \pmod M$.

In case of M-duobinary

$x\left(\frac{n}{2W}\right) = -1$ for $n = 1$, $x\left(\frac{n}{2W}\right) = 1$ for $n = -1$

$B_m = I_m - I_{m-2}$, $I_m = 2P_m - (M-1)$,

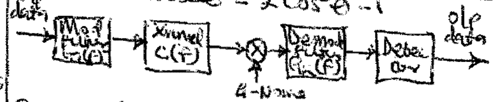
$P_m = D_m \oplus P_{m-2} \pmod M$, $D_m = \frac{1}{2}B_m$

2. MLSD

$DM_m = DM_{m-1} + (y_m - B_m)^2$, $DM_0(1) = 0$.

Signal design for xnnels $\bar{c}$ distortion

NB: $\cos 2\theta = 2\cos^2\theta - 1$



Case 1: Precompensate @ TX.

$|G_T(f)| = \frac{\sqrt{X_{rc}(f)}}{|C(f)|}$, $|f| \le W$

$|G_R(f)| = \sqrt{X_{rc}(f)}$, $|f| \le W$

Avg TX power $P_{av} = \frac{d^2}{T}\int_{-W}^{W} \frac{X_{rc}(f)}{|C(f)|^2} df$

$d^2 = P_{av} T \left[\int_{-W}^{W} \frac{X_{rc}(f)}{|C(f)|^2} df\right]^{-1}$

SNR $\frac{d^2}{\sigma_v^2} = \frac{2P_{av}T}{N_0}\left[\int_{-W}^{W} \frac{X_{rc}(f)}{|C(f)|^2} df\right]^{-1}$, $\sigma_v^2 = \frac{N_0}{2}$

Case 2: Equal

$|G_T(f)| = |G_R(f)| = \frac{\sqrt{X_{rc}(f)}}{|C(f)|^{1/2}}$, $|f| \le W$

$\mathrm{SNR} = \frac{d^2}{\sigma_v^2} = \frac{2P_{av}}{N_0} T \left[\int_{-W}^{W} \frac{X_{rc}(f)}{|C(f)|} df\right]^{-2}$

In dB, d loss is $10\log \mathrm{SNR}$

Prob of error

Zero ISI: $P_m = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6(\log_2 M)E_g}{(M^2-1)N}}\right)$ *

Upper Bound for D & M-binary

$$P_m < 2\left(1 - \frac{1}{M^2}\right) Q\left(\sqrt{\left(\frac{\pi}{4}\right)^2 \frac{6}{(M^2-1)} \frac{E_{av}}{N_0}}\right) \ *$$

③ ④

Equalizers

Under ISI & AWGN: optimum Rx = Matched F. optimum detector = MLSD.

PDC = $D_0 = \frac{1}{|f_0|}\sum_{n=1} |f_n|$. After use filter

ZFE

$q_m = \sum_{n=-K}^{K} C_n x_{m-n}$, $XC = q$, $X = \begin{bmatrix} x_0 & x_{-1} & \cdots & x_{-2K} \\ x_1 & x_0 & & \vdots \\ \vdots & & \ddots & \\ x_{2K} & \cdots & & x_0 \end{bmatrix}$

$C = [C_{-K} \cdots C_0 \cdots C_K]^T$, $q = [0 \cdots 1 \cdots 0]$.

Variance $\sigma_u^2 = \sigma_v^2 \sum_{n=-K}^{K} C_n^2$.

MSE-E

$\Gamma C = \xi$, $\Gamma = \begin{bmatrix} x_0+N_0 & x_{-1} & x_{-2} \cdots \\ x_1 & x_0+N_0 & x_{-1} \\ \vdots & \vdots & x_0+N_0 \end{bmatrix}$, $C = \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix}$

$\xi = [f_1^* \ f_0^* \ f_{-1}^*]^T$

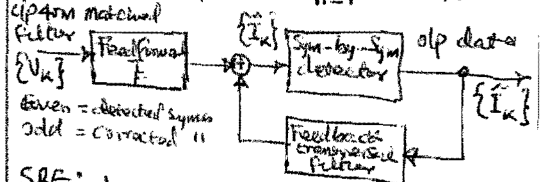
Also $\xi = \sum_{n=-K}^{K} C_n \Gamma_{kn}$, 3-tap $X(z) = (f_0 + f_1 z^{-1})(f_0 + f_1 z)$

SNR $\gamma_\infty = \frac{1 - J_{min}}{J_{min}}$, $J_{min} = \frac{N_0}{N_0+1}$

Variance $\sigma_n^2 = N_0 \sum_{j=-K}^{K} C_j^2$

DFE: a non-linear eqz dat employs previous decision to eliminate d ISI caused by previously detected symbols on d current symbol to be detected. What makes is NLE d detector.

$y_m = V_0 I_m + \sum_{n=-K_1}^{-1} V_n I_{m-n} + \sum_{n=1}^{K_2} V_n I_{m-n}$



Given = detected syms
odd = corrected "

SRE: d eqz taps are spaced @ $\frac{1}{T}$ (Symb rate)

FSE: is based on sampling d incoming symb @ least as fast as Nyquist rate.

Tap spacing $= \frac{T}{1+\beta}$ or $\frac{MT}{N}$ $N > M$.

d objective of FSE is to avoid aliasing.

SRE: is optimum if eqz is preceded after matched to d xnnel distorted xmitted pulse.

FSE: optimum FSE is equ to d optimum linear RX consisting of d matched filter followed by a symbol rate eqz.

FSE show better performance & less sensitivity to timing phase.

Adaptive eqzn (Time varying xnnel)

$BC = d \Rightarrow C_{opt} = B^{-1}d$

Step Size Δ: if it's too big, we miss d min. If it is too small, it would take us a lot of time to hit d min.

LMS: $0 < \Delta < 2/\lambda_{max}$, λ_K = eigenvalue

$|1 - \Delta\lambda_{max}|$. NB: MSE = Cost, MMSE = Criteria

LS = algorithm requires matrix inversion.

MLS = " avoids " "

Spread Spectrum Signal (5)

2-types of mod are considered:

1. **PSK**: appropriate in applcs where phase coherence b/w d xmitted signal & d Rxed signal can be maintained over a time interval dat is relatively long compared to d reciprocal of d xmitted signal BW.

2. **FSK**: If it cannot be maintained due to time-varying effects on d Cmmn links.

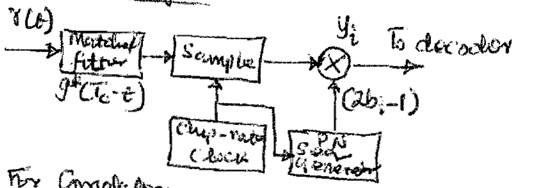
Types of SS

1 DCSS $B_e = W/R = \frac{1/T_c}{1/T_b} = \frac{T_b}{T_c} = L_c$

$R_c = K/n = \frac{1}{T_c}$.

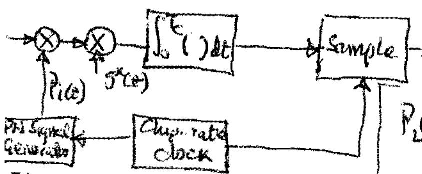
ope. demod: A filter matched to d wave form or as a Correlator. (Gaussian)

For Matched filter



For Correlator

1. All optimum if z(t) is Gaussian



$P_2(m) = Q\left[\frac{\text{mean}}{\sqrt{\text{varn}}}\right]$

Error rate performance of SS decoder ↑ un-coded

For BPSK $P_2(m) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$, $P_2(m) = Q\left(\sqrt{\frac{2W/R}{J_{av}/P_{av}}}\right)$

Coded: steps

- decision will be made based on d FF:
- $D = CM_1 - CM_m = 4E_c w_m - 2\sum_{j=1}^{n} C_{mj}(2b_j - 1)V_j$
- Error will be made if $D < 0$

Mean $= 4E_c w_m$, $\sigma_m^2 = 4W_m E[V_j^2]$, $\int_{-\infty}^{\infty} |G(f)|^2 S_{zz}(f)\,df$

For Broad band jamming

$S_{zz}(f) = J_0 \Rightarrow E[V_j^2] = J_0 \int_{-W/2}^{W/2} |G(f)|^2 df = J_0\, 2E_c$

$\sigma_m^2 = 8E_c J_0 w_m$.

If $D < 0 \Rightarrow P_2(m) = Q\left(\sqrt{\frac{2E_c}{J_0} w_m}\right) = Q\left(\sqrt{\frac{2E_b}{J_0} R_c w_m}\right)$

where $E_b = \frac{n}{k} E_c$, using d union bound

$\Rightarrow P_M \le \sum_{m=2}^{M} Q\left(\sqrt{2\frac{G_p}{J_{av}/P_{av}} R_c w_m}\right)$ $\quad \frac{E_b}{J_0} = \frac{P_{av}/R}{J_{av}/W}$

For Partial Jamming: $S_{zz}(f) = \left\{\frac{J_{av}}{W_1} = \frac{J_0 W}{W_1} \quad |f| \le \frac{W_1}{2}\right.$

For Pulse Jamming:

$P_2(\alpha) = (1-\alpha) Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \alpha Q\left(\sqrt{\frac{2E_b}{N_0 + \frac{J_0}{\alpha}}}\right)$, $N_0 \ll J_0$

α-optimal

$\alpha^* = \begin{cases} \frac{0.71}{E_b/J_0} & \text{for } \frac{E_b}{J_0} > 0.71 \\ 1 & \text{''} \le 0.71 \end{cases}$ $\quad P_2 = \begin{cases} \frac{0.083}{E_b/J_0} & \text{for } \frac{E_b}{J_0} > 0.71 \\ Q\left(\sqrt{\frac{2E_b}{J_0}}\right) & \text{''} \le 0.71 \end{cases}$

Processing gain (6)

same $= 2J_0$.

$E_b = P_{av} T_b = P_{av}/R$, $J_0 = \frac{J_{av}}{W}$, $\frac{E_b}{J_0} = \frac{P_{av}/R}{J_{av}/W} = \frac{W/R}{J_{av}/P_{av}}$

Generation of PN seq

Props:

1. Auto-C, $R(j) = \begin{cases} n & j = 0 \\ 0 & 1 \le j \le n-1 \end{cases}$, $= \begin{cases} n & j = 0 \\ -1 \end{cases}$ for ML shift
2. Cross-C, $R(k,l) = \sum_{n=1}^{n} (2b_{ik} - 1)(2b_{il} - 1)$, $0 \le j \le n-1$

Max-L-Shift-R-seq

- period $n = 2^m - 1$, $\frac{R_{max}}{R(0)} = -\frac{1}{n}$.

Gold Seq: C-C values $\{-1, -t(m), t(m)-2\}$ $\quad n = 2^m - 1$

where $t(m) = \begin{cases} 2^{\frac{(m+1)}{2}} + 1 & n = \text{odd} \\ 2^{\frac{(m+2)}{2}} + 1 & n = \text{even}. \end{cases}$ $\quad M = n+2$

Welch Bound lower bound $R_{max} \ge n\sqrt{\frac{M-1}{Mn-1}}$ for large values of M & n.

Kasami-codes optimal in terms of C-C

C-C values $\{-1, -(2^{\frac{m}{2}}+1), 2^{\frac{m}{2}} - 1\}$ — Welch Bound is a lower bound on d min possible Cross Correln.

2 Freq hopping: Types:

1. SFH: if $R_H \le R_S$, $R_c = R_S$, & $\Delta f = R_S$
2. FFH: if $R_H > R_S$, $R_c = R_H$ & $\Delta f = R_H$.

Processing gain: $G_p = \frac{W}{R} = \frac{2^L \Delta f}{1/T_b} = \frac{2^L \cdot \frac{1}{T_c}}{\frac{1}{T_b}} = 2^L K$ (FFH)

For SFH $= G_p = 2^L$.

\# of hops/sym $= K$, \# of carrier freqs $= 2^L$.

L = length of d binary.

FH & DS

1. **Processing gain**: Since tech limits d speed of d clock (A) FS can achieve several GHz of BW larger compared to DS for a given clock rate. (B) For d same gain, d clock rate can run @ a much slower rate dan DS.
2. **Synch**: (a) Both DS & FS require synch within a fractn of a chip. (b) DS is more demanding. (C) In FH, d chip interval is d time spent in xmitting a signal w/ d same freq.
3. **Near-far Prblm** (a) Severe in DS (b) It is less in FH b/c it's an avoidance syst rada dan averaging syst. In FH interference occurs if 2-signals occupy d same slot simultaneously (Power does not matter much).
4. **Error Coding**: (a) Required by both DS & FH (b) More needed by FH b/c 2-signals occupy d same freq slot could destroy d data.

Performance

Broadband $P_2 = \frac{1}{2} e^{-\gamma_b/2}$, $\gamma_b = \frac{E_b}{J_0}$; (FSK, $N_0 \ll J_0$)

Partial: $P_2 = \frac{1-\alpha}{2} e^{-\frac{E_b}{2N_0}} + \frac{\alpha}{2} e^{-E_b/2(N_0 + J_0/\alpha)} \approx \frac{\alpha}{2} e^{-\frac{\alpha E_b}{2J_0}}$

opt. α @ $\frac{dP}{d\alpha} \Rightarrow \alpha^* \begin{cases} \frac{2}{E_b/J_0} & \frac{E_b}{J_0} > 2 \\ 1 & \text{''} \le 2 \end{cases}$ $\quad 0 \le \alpha \le 1$.

$P_2 = \begin{cases} \frac{e^{-1}}{E_b/J_0} & \frac{E_b}{J_0} > 2 \\ \frac{1}{2} e^{-\frac{E_b}{2J_0}} & \text{''} \le 2. \end{cases}$

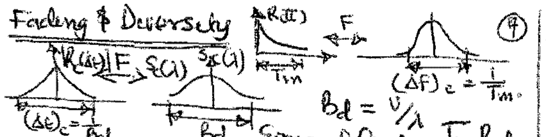
$\therefore P = Q\left(\sqrt{\frac{2W/R}{N_u - 1}}\right)$

No of Users

$\frac{P_{av}}{J_{av}} = \frac{P_{av}}{(N_u - 1)P_{av}} = \frac{1}{N_u - 1}$

with Coding $P_M = (M-1) Q\left(\sqrt{\frac{2W/R}{N_u - 1} R_c d_{min}}\right)$

Fading & Diversity (7)



$B_d = v/\lambda$ Spread factor $= T_m B_d$

Freq.-Selective: When an infmn bearing signal is xmitted thru d xnnel, if $(\Delta F)_c <$ BW of d xmitted signal.

Freq non-Selective $(\Delta F)_c >$ BW.

Diversity: useful assuming errors occur in reception when d attenuation is large i.e. d xnnel is in deep fade.

Types: **Freq diversity**: when sepn b/w carrier freqs $> (\Delta f)$

Time D: when sepn b/w xmission intervals $> (\Delta t)_c$.

Space D: multiple antennas spaced by 10λ.

In Time domain

F-Selective: If $T_m > T_s$. **Flat**: $T_m < T_s$

T_m = Symbol time, T_s =

In F-domain **F-Selective**: $f_0 < \frac{1}{T_s} = W$, Flat $f_0 > W$

Fast fading: $T_0 < T_s$, **Slow fading**: $T_0 > T_s$

T_0 = Coherence time, T_s = Time durn

Quiz 1

Modified duobinary: $x\left(\frac{n}{2W}\right) = \begin{cases} 1 & n = -1 \\ -1 & n = 1 \\ 0 & \text{o/w} \end{cases}$

D_m	1	1	1	0	1	0	0	1
I_m	+1	+1	+1	-1	+1	-1	-1	+1
B_m	1	0	-2	0	0	-2	+2	1

Uniform $P_{(x)} = \frac{1}{b-a}$, $E[X] = \frac{b+a}{2}$

Assg 2 $P_M = 1 - (1 - P_{\sqrt{M}})^2$ ~~$P_{\sqrt{M}}$ = two root of ...~~

$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left[\sqrt{\frac{3E_{av}}{(M-1)N_0}}\right]$

Assg 4 $E_b = P_R T$, $\Rightarrow P_R = [N_0] \cdot R \frac{E_b}{N_0}$

Peak distorn $D = \sum_{m=-2}^{2} |f_m| = 1.6$

FH → Chip rate $= K R_b = 30 \times 10^5$, clock rate $= LKR_b$

IS-95 Forward (1.25 MHz) → Each base station has 64 codes → 1 Code 4 Synch → 61-users → Sync (No interference)

Reverse → Asynch → Power Limited → Frame size (20 ms) 576.

FD over RV → No Power Constraint → Synch Commy

Eg 7 $P_e = \frac{1}{4}\left[Q\left(\frac{0.7028}{0.5\sqrt{2}}\right) + 2Q\left(\frac{1}{0.5\sqrt{2}}\right) + Q\left(\frac{1.0971}{0.5\sqrt{2}}\right)\right]$

if $C = [0.243, 1, 6, -0.243]$.

$P_e = \sum_{i=2}^{3} \binom{3}{i} P_c^i (1 - P_c)^{3-i}$, $\quad P_c = \frac{1}{2} e^{-\frac{E_c}{2N_0}}$ (FSK)

Spread factor $= T_m B_d$ (if < 1, under spread)

Slow fading: when d signal interval T < d coherence time of d xnnel ($T < (\Delta t)_c$)

Prob of error $P_e = \sum_{k=\frac{d+1}{2}}^{d} \binom{d}{k} P^k (1-P)^{n-k}$, $P = \frac{1}{2 + \bar{\gamma}_c}$

For Soft decision $P_{b} \approx \left(\frac{1}{4\bar{\gamma}_c}\right)^L \binom{2L-1}{L}$.

L = order

$P_e(a) = Q\left(\sqrt{\frac{2Ea^2}{N_0}}\right)$

(8)

$S(t) = \cos[2\pi f_1 t] + \cos[2\pi f_2 t]$.

$r(t) = \alpha_1 S(t - \tau_0) + \alpha_2 S(t - \tau_0 - T_m)$

To obtain order in diversity, $\frac{W}{1/T}$ ①, $\frac{W}{(\Delta f)_c} = L$ ②

∴ no of carriers = ①/②

More

Sym-by-Sym VS MLSD

S → Simple & used practically in many systs employing duobinary & M-D

→ Not optimal due to inherent memory in d Rxed signal of a syst using partial response signalling.

MLSD → is optimal but introduces a considerable delay. dis memory what introduces delay.

Antenna Spacing in Space diversity

Two antennas are used to improve d quality of d Rxed signal, placed an equal distance to an uneven multiple of a quarter of λ as such avoid interrupts caused by fading — or 1 dey can be spaced

Equal gain Combining VS MRC:

MRC: each matched filter o/p is × by d conjugate of d xnnel gain.

Error $P_2(\gamma_b) = Q(\sqrt{2\gamma_b})$, $\gamma_b = \sum_{k=1}^{L} \gamma_k$

Equal: $P = \left(\sqrt{\frac{2E(\sum a_m)^2}{N_0}}\right)$, Does not have closed form

Each Segment.

$P_{loss} = 50\,\text{km} \times 1\,\text{dB/km} = 50\,\text{dBW}$.

$P_{trans} = P_R + P_{loss}$

Modified duobinary

$b_m = I_m - I_{m-2}$