

Name: KEY

In all parts show your steps

- 3 1. Consider a (7,4) code defined by the generator polynomial, $g(X)=1+X^2+X^3$.
Is $c(x)=1+X+X^4+X^5$ a valid codeword at the output of this systematic encoder.
- 3 2. Sketch the general block diagram for a Migget decoder
- 4 3. Given that $X^9+1 = (X+1)(X^2+X+1)(X^6+X^3+1)$. We would like to design a (9,k) code which has the following property: Correct single error. Find k, and the generator matrix $g(x)$.

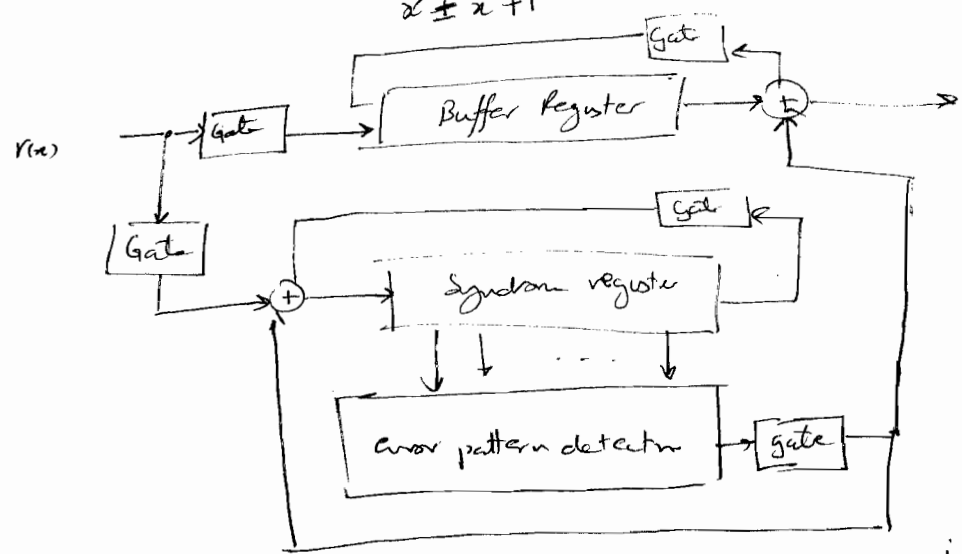
1.
$$\text{Rem} \left[\frac{C(x)}{g(x)} \right] = \frac{1+x+x^4+x^5}{x^3=x^2+1} = \frac{1+x+x^2+x^3+x^4+x^5}{x^3=x^2+1} = \frac{1+x+x^2+x^3+x^4+x^5}{x^3-x^2-1}$$

$= x^3 + x = x^2 + x + 1$

$x^3 \pm x^2 + 1$

(not a valid code word because syndrome $\neq 0$)

2.



3. Start by finding roots conjugates $(\alpha^i)^2 \text{ mod } n$
- $l=1 \Rightarrow \alpha^0$
- $\alpha^1, \alpha^2, \alpha^4, \alpha^8, \alpha^7, \alpha^5 \leftarrow$ six roots $l=2$
- $\alpha^3, \alpha^6 \quad l=2$
- \Rightarrow if we choose the factor of power 6 $\Rightarrow l=2 \Rightarrow d_{\min} = l+1 = 3$
- for single error correcting code $d_{\min} = 3 \Rightarrow g(x) = (x^6+x^3+1)$
- $\Rightarrow r = 6 \Rightarrow k = 3 \quad (9, 3) \text{ Code.}$