King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE430: Information Theory and Coding (072)
Quiz 3

Serial # 0

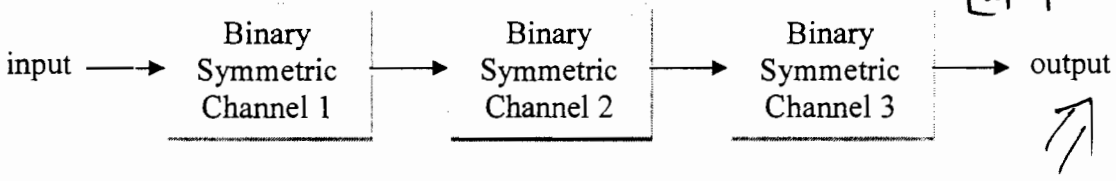
- 1 points for not writing your serial number

Name: KEY

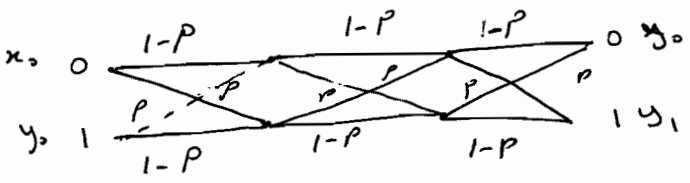
Three binary symmetric channels are connected in cascade, as shown in the figure below. Assuming that all channels have the same crossover probability, ***p***,

1. Find the transition matrix for the overall channel in term of ***p***.
2. If ***p*=0.25**, find the capacity of the cascaded connection (Overall channel)
3. Compare the capacity of the overall cannel to the capacity of channel 1, *comment*.

$$\begin{bmatrix} 1-3p-2p^2 & 2p^2-p^3 \\ 2p^2-p^3 & 1-3p-2p^2 \end{bmatrix}$$



(1)



or $\begin{bmatrix} 1-p & p \\ p & 1-p \end{bmatrix}^3$

$P(0|0) = (1-p)^3 + (1-p)p^2 + (1-p)p^2 + (1-p)p^2$

$= (1-p)^3 + 3(1-p)p^2$ ← Two errors cancel each other. The two errors can occur in three scenarios (ch1 & ch2, ch1 & 3, ch2 & ch3)

$P(1|0) = 3p(1-p)^2 + p^3$

$= 1 - P(0|0)$

$P(0|0)$

$$\begin{bmatrix} (1-p)^3 + 3(1-p)p^2 & p^3 + 3p(1-p)^2 \\ p^3 + 3p(1-p)^2 & (1-p)^3 + 3(1-p)p^2 \end{bmatrix}$$

(2) for $p = 0.25$

$$\begin{bmatrix} 0.5625 & 0.4375 \\ 0.4375 & 0.5625 \end{bmatrix}$$ 6

$C = 1 - H(P) = 1 - H(0.4375) = 0.0113$ almost useless

for single channel.

$C = 1 - H(0.25) = 0.1887$ 2

still bad but much better than the over all channel. 1