Angle (Exponential) Modulation

Contents

Introduction

Definition of angle Modulation (PM, FM)

In the previous chapter, we studied the different AM technique in which the amplitude of some carrier signal is modified according to the message signal. The frequency and phase of the carrier of the carrier signal in all AM modulation techniques were constant. In this chapter, we will study a different method for transmitting information by changing the phase or frequency (changing the angle) of the carrier signal and keeping its amplitude constant.

Historical Notes

- FM was introduced to reduce sideband noise (noise α bandwidth)
- $\boldsymbol{\omega(t) = \omega_c + km(t)}$, $\omega_c - km_p \leq \omega \leq \omega_c + km_p$, the center frequency is ω_c and the bandwidth is $2km_p$.
- By having a small k we can have a very small bandwidth.
- By experiment: FM bandwidth ≥ AM bandwidth What is wrong?

Concept of instantaneous Frequency

A sinusoidal can be represented as by its amplitude and angle $x(t) = A\cos(\theta(t))$. A sinusoid with fixed frequency and phase, is given by

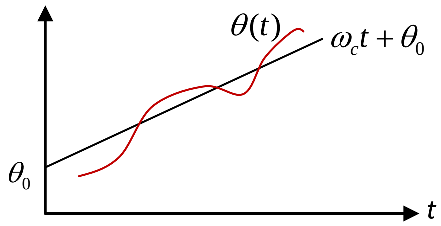
$$x(t) = \cos(\omega_c t + \theta_0)$$

The angular frequency is equal to ω_c since it is a constant with respect to *t*, and the phase of the cosine is the constant θ_0. The angle of the cosine $\theta(t) = \omega_c t + \theta_0$ is a linear relationship with respect to *t* (a straight line with slope of ω_c and y–intercept of θ_0). However, for other sinusoidal functions, the frequency may itself be a function of time, and therefore, we should not think in terms of the constant frequency of the sinusoid but in terms of the INSTANTANEOUS frequency of the sinusoid since it is not constant for all *t*. Consider for example the following sinusoid

$$y(t) = \cos[\theta(t)],$$

where $\theta(t)$ is a function of time. The frequency of $y(t)$ in this case depends on the function of $\theta(t)$ and may itself be a function of time. The instantaneous frequency of $y(t)$ given above is defined as

$$\omega_i(t) = \frac{d\theta(t)}{dt}.$$



As a checkup for this definition, we know that the instantaneous frequency of $x(t)$ is equal to its frequency at all times (since the instantaneous frequency for that function is constant) and is equal to ω_c. Clearly this satisfies the definition of the instantaneous frequency since $\theta(t) = \omega_c t + \theta_0$ and therefore $\omega_i(t) = \omega_c$.

If we know the instantaneous frequency of some sinusoid from $-\infty$ to some time *t*, we can find the angle of that sinusoid at time *t* using

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha) d\alpha.$$

Changing the angle $\theta(t)$ of some sinusoid is the bases for the two types of angle modulation: Phase and Frequency modulation techniques.

Phase Modulation (PM)

In this type of modulation, the phase of the carrier signal is directly changed by the message signal. The phase modulated signal will have the form

$$\boxed{g_{PM}(t) = A \cdot \cos\left[\omega_c t + k_p m(t)\right]},$$

where *A* is a constant, ω_c is the carrier frequency, $m(t)$ is the message signal, and k_p is a parameter that specifies how much change in the angle occurs for every unit of change of $m(t)$. The phase and instantaneous frequency of this signal are

$$\theta_{PM}(t) = \omega_c t + k_p m(t),$$
$$\omega_i(t) = \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p \dot{m}(t).$$

So, the frequency of a PM signal is proportional to the derivative of the message signal.

Frequency Modulation (FM)

This type of modulation changes the frequency of the carrier (not the phase as in PM) directly with the message signal. The FM modulated signal is

$$g_{FM}(t) = A \cdot \cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha\right],$$

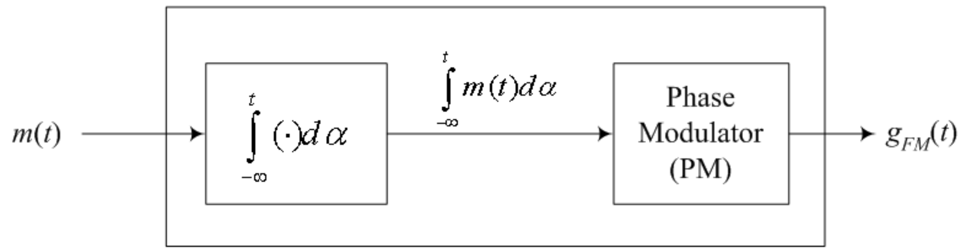
where k_f is a parameter that specifies how much change in the frequency occurs for every unit change of $m(t)$. The phase and instantaneous frequency of this FM are

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)d\alpha,$$
$$\omega_i(t) = \omega_c + k_f \frac{d}{dt}\left[\int_{-\infty}^{t} m(\alpha)d\alpha\right] = \omega_c + k_f m(t).$$

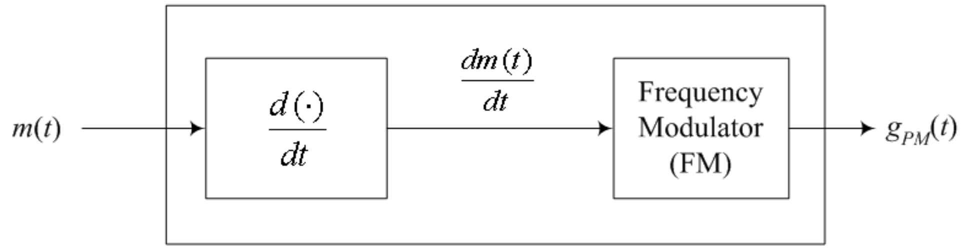
Relation between PM and FM

PM and FM are tightly related to each other. We see from the phase and frequency relations for PM and FM given above that replacing $m(t)$ in the PM signal with $\int_{-\infty}^{t} m(\alpha)d\alpha$ gives an FM signal and replacing $m(t)$ in the FM signal with $\frac{dm(t)}{dt}$ gives a PM signal. This is illustrated in the following block diagrams.

Frequency Modulator (FM)



Phase Modulator (PM)



- PM and FM are indistinguishable.
- PM and FM are two examples of the more general angle modulation.
- Bandwidth of FM=$2k_f m_p$, where m_p is the peak of $m(t)$.
- Bandwidth of PM=$2k_p \dot{m}_p$, where m_p is the peak of $\dot{m}_p$.
- Historically, we start with FM, in most cases PM performs better, optimum performance is in between. (This note is just for your information. The explanation is beyond the scope of this course).

Check examples 5.1 & 5.2 solved in the book. (Digital PSK)

Range of Values for k_p

To avoid ambiguity in demodulation the value of kp should be limited to a certain range. Consider the following example:

$$g_{pm}(t) = A\cos\left(\omega_c t + \frac{3\pi}{2} m(t)\right)$$

Notice that
$g_{pm}(t) = A\sin(\omega_c t)$ when $m(t)$=1 or -1/3. Remember that $\theta + 2\pi = \theta$

If there is discontinuity in $m(t)$, then $K_p m(t)$ should be restricted to the range $(-\pi, \pi)$. If $m(t)$ is continuous no restriction on k_p because the change is gradual. Real m(t) are band limited and k_p has no restriction.

K_p is usually small

Power of Angle Modulated Signals

PM or FM the power is $A^2/2$, regardless of k_p or k_f.