## King Fahd University of Petroleum \& Minerals

## Electrical Engineering Department

EE315: Probabilistic Methods in Electrical Engineering (112)

## Major Exam I

March 10, 2012
7:00-8:30 PM
Building 59-Rooms 2001-2004


Name: $\qquad$ ID\# $\qquad$

| Question | Mark |
| :---: | :---: |
| 1 | $/ 10$ |
| 2 | $/ 10$ |
| 3 | $/ 10$ |
| 4 | $/ 10$ |
| Total | $/ 40$ |

## Instructions:

1. This is a closed-books/notes exam.
2. The duration of this exam is one and half hours.
3. Read the questions carefully. Plan which question to start with.
4. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
5. Work in your own.
6. Strictly no mobile phones are allowed.
7. Table Attached

## Good luck

| Mark | sec | Timing | Instructor |
| :--- | ---: | ---: | :--- |
|  | 1 | SMW 9:00 | Dr. Ahmed Masoud |
|  | 2 | $\underline{\text { UT 10:00 }}$ | Dr. Ali Muqaibel (Coordinator) |
|  | 3 | $\underline{\text { UT 08:30 }}$ | Dr. Saad Al-Ubaidi |
|  | 4 | $\underline{\text { UT 10:00 }}$ | Dr. Saad Al-Ubaidi |

## Problem 1: (10 points)

Consider the following switching network shown. Let $A_{1}, A_{2}, A_{3}$, and $A_{4}$ denote the events that the associated switches are closed (connecting). Let $A_{a b}$ denote the event that there is a closed path between terminals $a$ and $b$. (i.e $A_{a b} c l o s e d$ )
a) Express $A_{a b}$ in terms of $A_{1}, A_{2}, A_{3}$, and $A_{4}$

b) If all switches are independent and the probability of being closed is 0.5 . That is $P\left(A_{1}\right)=P\left(A_{2}\right)=$ $P\left(A_{3}\right)=P\left(A_{4}\right)=0.5$. Find $P\left(A_{a b}\right)$. i.e $P($ path between $a$ and $b$ is closed (connecting))

## Complete the missing term(s)

For any three events $S_{1}, S_{1}$, and $S_{3}$ :

$$
P\left(S_{1} \cup S_{2} \cup S_{3}\right)=P\left(S_{1}\right)+P\left(S_{2}\right)+P\left(S_{3}\right)+\cdots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
$$

Consider the experiment of throwing two fair dice. What is the probability that the two faces are the same given that the sum is not than three?

## Problem 2: (10 points)

Consider a system that randomly assign a variable $X=\{1,2,3\}$ with uniform probability, to a variable $Y=\{1,2,3\}$. The conditional probability assignments are shown in the figure. Compute the following:

a) If the event $y=3$ was observed. What is the probability that it is coming from $x=3$.
(4 points)
b) Compute the expected value of $Y$.

## Problem 3:

Consider the network with nodes $A, B, C$, and $D$ shown below. A network is connected if a path exists connecting a node to all other nodes. A network is disconnected when two or more links are in failure. If the links are independent and the probability of a link failure is 0,1 , compute the probability of the network getting disconnected. (6 points)


The pdf of a continuous r.v. $X$ is given by

$$
f_{X}(x)=\left\{\begin{array}{cl}
1 / 3 & 0<x<1 \\
3 a & 1<x<2 \\
0 & \text { otherwise }
\end{array}\right.
$$

Find the value of $a$ for a valid pdf?
(2 points)

Sketch the CDF (show all numbers on the $x$-axis and the $y$-axis). No need for expression only sketch
( 2 points)

## Problem 4:

A noise signal (Random variable $X$ ) is passed through a system with the following input output relation.


X is a Gaussian Random variable, $f_{X}(x)=\frac{1}{\sqrt{2 \pi \sigma_{X}^{2}}} e^{-\frac{\left(x-a_{x}\right)^{2}}{2 \sigma_{X}^{2}}}$, compute $\mathrm{P}(y \geq 0)$, for the following cases:
a) $a_{x}=0$ and $\sigma_{X}=1$
b) $a_{x}=2$ and $\sigma_{X}=2$
c) $a_{x}=1$ and $\sigma_{X}=2$
d) $a_{x}=1$ and $\sigma_{X}=0$

Gaussian Table:

TABLE B-I
Values of $F(x)$ for $0 \leq x \leq 3.89$ in steps of 0.01

| $x$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | . 5000 | . 5040 | . 5080 | . 5120 | . 5160 | . 5199 | . 5239 | . 5279 | . 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | . 5753 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
| 0.3 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 | . 6480 | . 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| 0.5 | . 6915 | . 6950 | . 6985 | . 7019 | . 7054 | . 7088 | . 7123 | . 7157 | . 7190 | . 7224 |
| 0.6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7517 | . 7549 |
| 0.7 | . 7580 | . 7611 | . 7642 | . 7673 | . 7704 | . 7734 | . 7764 | . 7794 | . 7823 | . 7852 |
| 0.8 | . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
| 0.9 | . 8159 | 88186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
| 1.0 | . 8413 | . 8438 | . 8461 | .8485 ${ }^{\prime}$ | . 8508 | . 8531 | . 8554 | . 8577 | . 8599 | . 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | . 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
| 1.3 | .9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | . 9131 | . 9147 | . 9162 | . 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | . 9319 |
| 1.5 | . 9332 | . 9345 | . 9357 | . 9370 | . 9382 | . 9394 | . 9406 | . 9418 | . 9429 | . 9441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | . 9599 | . 9608 | . 9616 | . 9625 | .9633 |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9773 | . 9778 | . 9783 | . 9788 | . 9793 | . 9798 | . 9803 | . 9808 | . 9812 | . 9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
| 2.3 | . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 | . 9934 | . 9936 |
| 2.5 | . 9938 | . 9940 | . 9941 | . 9943 | . 9945 | . 9946 | . 9948 | . 9949 | . 9951 | . 9952 |
| 2.6 | .9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 9971 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9986 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | . 9991 | . 9992 | . 9992 | . 9992 | . 9992 | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| 3.3 | . 9995 | . 9995 | . 9996 | . 99996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 | . 9998 |
| 3.5 | . 9998 | . 9998 | . 9999 | . 9999 | . 9998 | . 9998 | . 9998 | . 9998 | . 9998 | . 9998 |
| 3.6 | . 9998 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 |
| 3.7 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 |
| 3.8 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | . 9999 | 1.0000 | 1.0000 | 1.0000 |

