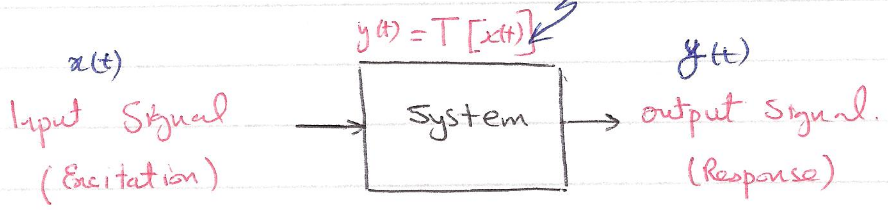
Chapter I Introduction.

Block diagram.



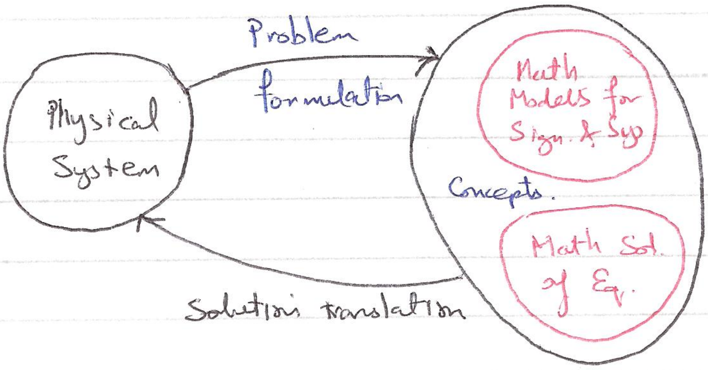
Def.

is a process for which cause-and-effect relations exist.

System: Combination & interconnection of several Components to perform a desired task.
(Modeled by mathematical Equations)

Signals: A function of time that (usually) represents a physical variable of interest associated with a system.
(Modeled by mathematical functions).

Modeling



Some prob. are solved only by Computers. GPS.

* Classifications of Systems based on time Continuity

What are Analog Digital Signals? DSP

* Amplitude classification.

①. Continuous-time signal (analog)
②. Discrete-time signal.
③. Hybrid (Sampled-data).

Examples

1.2 Examples of Continuous Time Physical Systems.

1.2 Electric Circuits, Op amps, Simple Pendulum, DC power Supply ...

1.3 SAMPLERS & Discrete time Physical Systems.
Analog to digital Converter, Numerical Integration, Picture in a Picture, Compact Disks (CD), Sampling in Telephone Sys, Data acquisition sys.

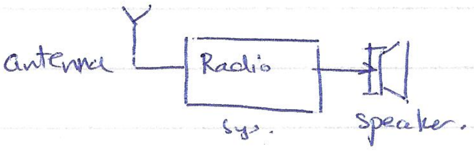
Examples.

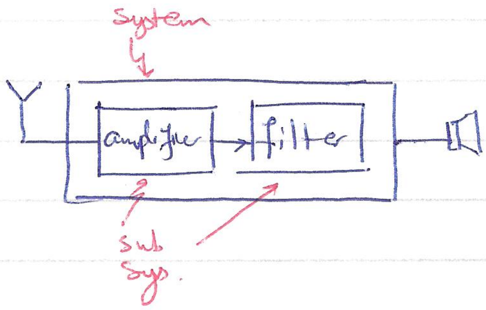
integrator. input $\rightarrow \boxed{\int dt} \rightarrow$ output.

Communication link Source $\rightarrow$ $\boxed{\text{Communication Channel.}}$ $\rightarrow$ sink

Human Nurve System.

Systems & Sub systems:



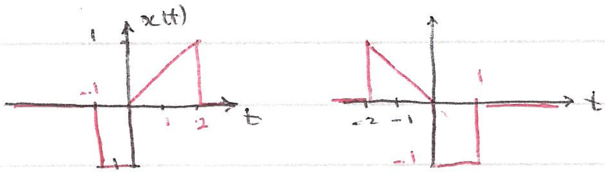


Understanding the systems help in
Design & Modeling

Ch 2

Continuous-Time Signals & ~~Mod~~ Systems

Transformation of Continuous-Time Signals



- Time Reversal

  $y(t) = x(-t)$

  Example : reverse MUSIC playing , Reverse tape.

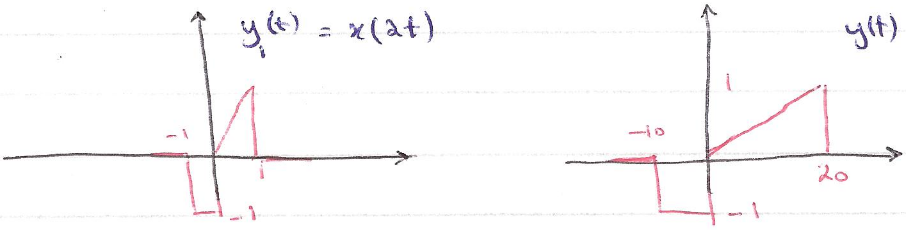
- Time Scaling

  $y(t) = x(at)$, a real constant

  $|a|>1$ time compressed (speed-up)

  $|a|<1$ " stretched (slowed-down)

  $y_1(t) = x(2t)$ $\quad$ $y(t) = x(0.1t)$



  Example : 45 rpm at 33 rpm , low battery cassette.

- Time Shifting $\quad y(t) = x(t-t_0)$

  . sketch $y_1(t) = x(t-2)$, $y_2(t) = x(t+1)$

- Combined time transformation.

  $y(t) = x\left(1 - \frac{t}{2}\right)$

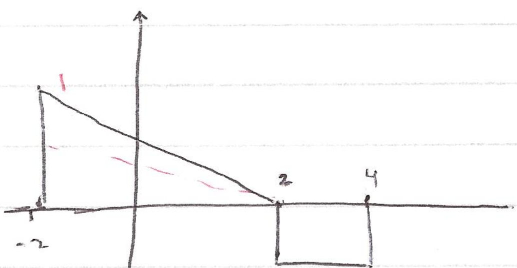
original

$\tau = 1 - \frac{t}{2}$

$-t = 2\tau - 2$

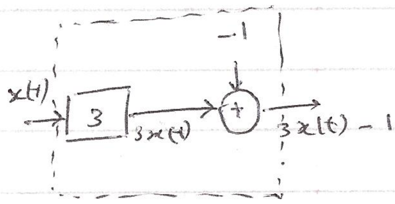
new $t = 2 - 2\tau$

τ	-1	0	2
t	4	2	-2



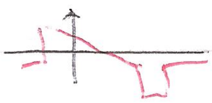
Amplitude Transformation:

$y(t) = A\,x(t) + B$ $\quad$ Explain

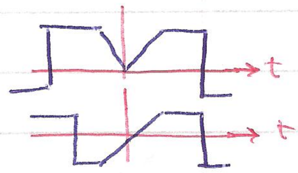


Combined amplitude & time Transformation

sketch $y(t) = 3\,x\left(t - \frac{t}{2}\right) - 1$



Signal Characteristics



- Even & odd signals. $x_e(t) = x_e(-t)$

  $x_o(t) = -x_o(-t)$

any signal can be expressed as the sum of even part and odd part.

$$x(t) = x_e(t) + x_o(t) \qquad \text{— ①}$$

replace $t \leftarrow -t$

$$x(-t) = x_e(-t) + x_o(-t)$$
$$= x_e(t) - x_o(t) \qquad \text{— ②}$$

add or subtract ① & ②

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$
$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

- Average value $\quad A_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, dt$

The average of the signal is contained in its even part

$\therefore$ odd part is zero.

even + even = even

odd + odd = odd

even + odd = neither ? in general

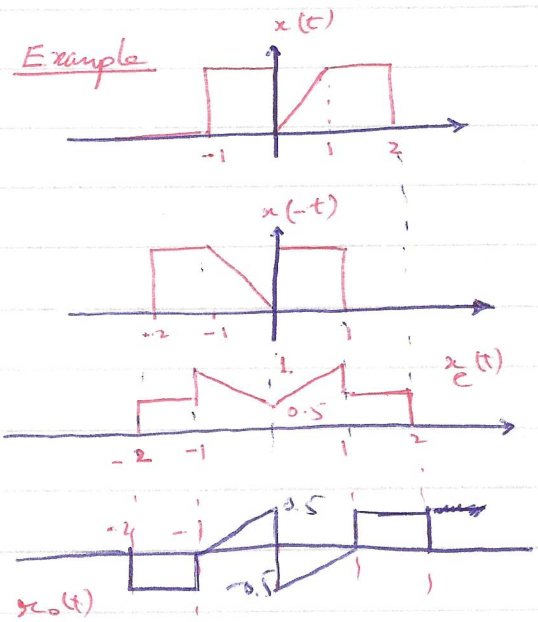
even * even = even.

odd * odd = even.

even * odd = odd

Example

Fig 2.8



add to verify

(point by point!)

Periodic Signals (vs. aperiodic)

$x(t) = x(t+T)$, $T > 0$ ← min. value of T — fundamental period T_0

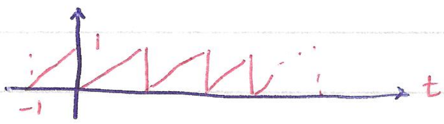
$x(t) = x(t+nT)$ n, integer

f_0 fundamental frequency $f_0 = \frac{1}{T_0}$ Hz

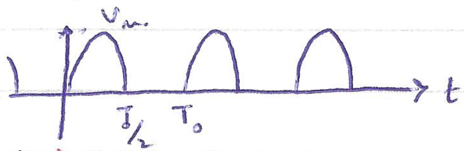
$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$ rad/s

Examples

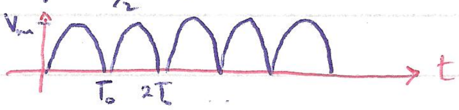
Sawtooth waveform.



Half-wave rectified



full wave rectified



Sinusoidal

DC (limiting case of sin)

$\omega \to 0$, $T \to \infty$

Example which one is periodic $x(t) = e^{\sin t}$ ✓ or $x(t) = t\,e^{\sin t}$ ✗

* The sum of continuous-time periodic signals is periodic if and only if the ratios of the periods of the individual signals are ratios of integers.

if ratio between all periods (freq) is rational.
The period is the least common multiple of the periods.
or freq is the greatest = factor of freq.

Example I: $x_1(t) = \cos(3.5\,t)$ $T_{01} = \frac{2\pi}{\omega} = \frac{2\pi}{3.5}$

$x_2(t) = \sin(2\,t)$ $T_{02} = \frac{2\pi}{2}$

$x_3(t) = 2\cos\left(\frac{7t}{6}\right)$ $T_{03} = \frac{2\pi}{7/6}$

$$\frac{T_{01}}{T_{02}} = \frac{2}{3.5} = \frac{4}{7} \quad , \quad \frac{T_{01}}{T_{03}} = \frac{7/6}{3.5} = \frac{7}{21}$$

all rational simplify $\frac{4}{7}$ & $\frac{1}{3}$ ↙ $7/21$

least common multiple $= 7 * 3 = 21$ $T_0 = 21 * T_{01}$

of the dominidar. $= 21 * \frac{2\pi}{3.5} = 12\pi$

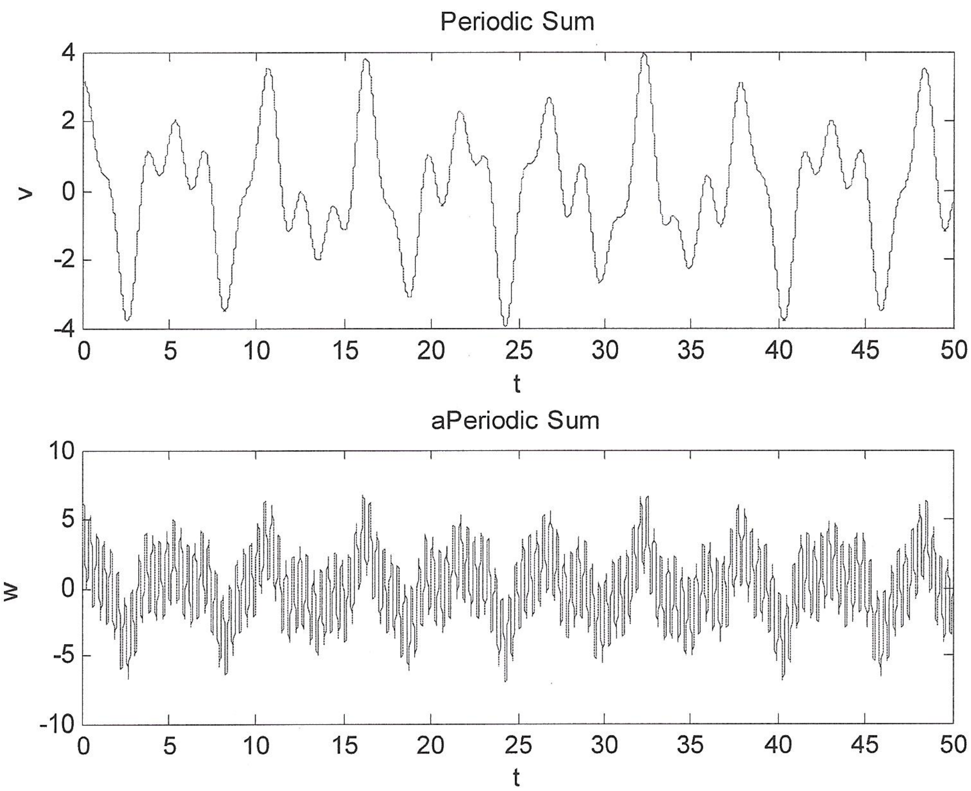
→ if we add $x_4(t) = 3\sin(5\pi t)$

$w(t) = v(t) + x_4(t)$ irrational ⇒ not periodic.

see Matlab code.

```
% Dr. Ali Hussein Muqaibel
% Matlab Example for the Sum of periodic signals
% Example 2.7 Philips Book 4ed p 38
close all
clear all
t=0:0.01:50;
x1=cos(3.5*t);
x2=sin(2*t);
x3=2*cos(7*t/6);
x4=3*sin(5*pi*t);
v=x1+x2+x3;
subplot(2,1,1)
plot(t,v)
axis=([0 50 -4 4])
title('Periodic Sum')
xlabel('t')
ylabel('v')

subplot(2,1,2)
w=v+x4;
plot(t,w)
axis=([0 50 -4 4])
title('aPeriodic Sum')
xlabel('t')
ylabel('w')
```



Common Signals in Engineering

Most of the systems we deal with are modeled with ordinary differential Equation (ODE) with Constant Coeffecients.

A signal that appears often in these systems is one whose time rate of change is directly propotional to the signal itself.

$$\frac{d\,x(t)}{dt} = a\,x(t)$$

Sinusoidal & Exponential function — $x(t) = c\,e^{at}$

- Complex signals do not appear in real life but the solution can be greatly simplified by assuming complex excitation & taking the real or imaginary part of the final answer.

Euler's relation $\quad e^{j\theta} = \cos\theta + j\sin\theta \quad — (1)$

$\theta \leftarrow -\theta$

$$e^{-j\theta} = \cos\theta - j\sin\theta \quad — (2)$$

add or subtracte (1) & (2)

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

in polar form $\quad e^{j\theta} = 1\,\angle\theta$

$$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta} = 1 \quad , \quad \text{ang}\ e^{j\theta} = \tan^{-1}\frac{\sin\theta}{\cos\theta} = \theta$$

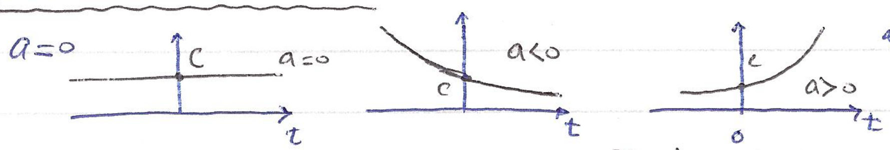
Continuoe 2.3 Common Signals in Engineering. $x(t) = Ce^{at}$ unitless

a is 1/seconds

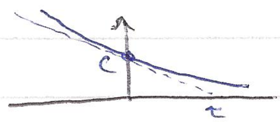
$x(t) = Ce^{-t/\tau}$ $\tau > 0$

τ time constant.

* C and a are real.



drug in human body



$$\left.\frac{d\,x(t)}{dt}\right|_{t=0} = \frac{-C}{\tau}\left. e^{-t/\tau}\right|_{t=0} = \frac{-C}{\tau}$$

after one time constant $0.368C$

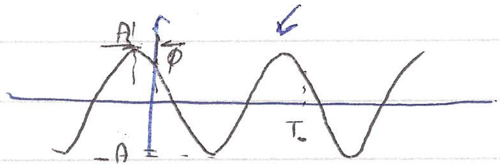
$4\tau \longrightarrow < 2\%$

$5\tau \longrightarrow < 1\%$

* C complex a Imaginary

$$x(t) = Ce^{at}, \quad C = Ae^{j\phi} = A\angle\phi, \quad a = j\omega_0$$

$$x(t) = A e^{j\phi} e^{j\omega_0 t} = A e^{j(\omega_0 t + \phi)} = A\cos(\omega_0 t + \phi) + jA\sin(\omega_0 t + \phi)$$



* Both C and a Complex.

$$x(t) = Ce^{at}, \quad C = Ae^{j\phi}; \quad a = \sigma_0 + j\omega_0$$

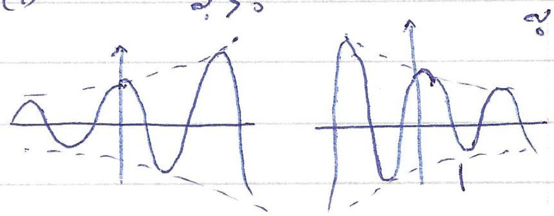
$$x(t) = A e^{j\phi} e^{(\sigma_0 + j\omega_0)t} = A e^{\sigma_0 t} e^{j(\omega_0 t + \phi)}$$

$$= A e^{\sigma_0 t}\cos(\omega_0 t + \phi) + jAe^{\sigma_0 t}\sin(\omega_0 t + \phi)$$

$$= x_r(t) + j\,x_i(t)$$

$x_r(t) = \mathrm{Re}[x(t)]$

$x_i(t) = \mathrm{Im}[x(t)]$



$$-Ae^{\sigma_0 t} \leq x_i(t) \leq Ae^{\sigma_0 t}$$

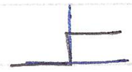
$$\text{"} \leq x_r(t) \leq \text{"}$$

2.4 Singularity Functions

↑ related to impulse function

①. <u>Unit Step function</u>

$$u(t) = \begin{cases} 1 & t>0 \\ 0 & t<0 \end{cases}$$



sketch $u(t-5)$, $u(t-t_0)$

$t=0$?! not defined.

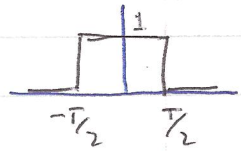
note : • $[u(t-t_0)]^k = u(t-t_0)$

• useful switching function. sketh $\cos\omega t \, u(t)$

②. <u>Rect</u>

$$\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & -T/2 < t < T/2 \\ 0 & \text{otherwise} \end{cases}$$

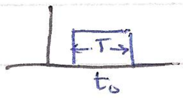
rect $\left(\frac{t}{T}\right)$



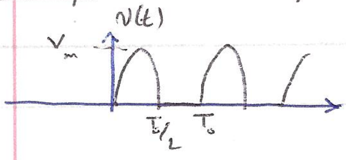
$$\text{rect}\left(\frac{t}{T}\right) = \begin{cases} u(t+T/2) - u(t-T/2) \\ u(T/2-t) - u(-T/2-t) \\ u(t+T/2)\, u(T/2-t) \end{cases}$$

• used to Extract part of the signal. $x(t) = \cos t \, \text{rect}\left[(t-\pi)/2\pi\right]$

$\text{rect}((t-t_0)/T)$



<u>Example</u> 2.9 Equation for the half wave ~~ref~~ rectified signal.



$$v_1(t) = V_m \sin(\omega_0 t)\, \text{rect}\left[\frac{t - T_0/4}{T_0/2}\right]$$

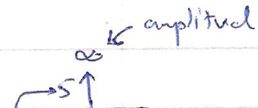
$$v(t) = \sum_{k=0}^{\infty} v_1(t - kT_0)$$

③ Unit Impulse function (Dirac delta function)

$\delta(t-t_0) = 0 \qquad t \neq t_0$

$\int_{-\infty}^{\infty} \delta(t-t_0)\, dt = 1$

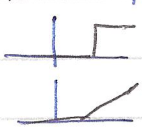
not realistic ; no instantanuous transfer of energy



multiply Constant.

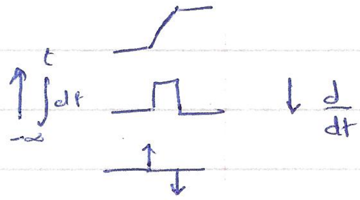
④ Unit ramp function $r(t)$

$$f(t) = \int_0^t u(\tau - t_0)\, d\tau = \int_{t_0}^{t} d\tau = \tau \Big|_{t_0}^{t} = (t-t_0)\, u(t-t_0)$$



Properties of the Unit Impulse Function

Table 2.3 p 52.



① $\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$

② $\int_{-\infty}^{\infty} f(t-t_0)\,\delta(t)\,dt = f(-t_0)$

③ $f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$

sifting or sampling properties.

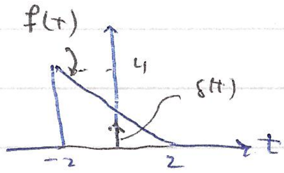
if the integration is outside $= 0$ $\int_a^b$ $a \le t_0 \le b$

④ $\delta(t-t_0) = \frac{d}{dt} u(t-t_0)$

⑤ $u(t-t_0) = \int_{-\infty}^{t} \delta(\tau - t_0)\,d\tau$

⑥ $\int_{-\infty}^{\infty} \delta(at-t_0)\,dt = \frac{1}{|a|} \underbrace{\int_{-\infty}^{\infty} \delta\left(t-\frac{t_0}{a}\right) dt}_{1\ ?}$

⑦ $\delta(-t) = \delta(t)$



see Example 2.10

$\int_{-\infty} f(t)\,\delta(t)\,dt = f(0) = 2$

$\int_{-\infty}^{\infty} f(t-1)\,\delta(t)\,dt = f(-1) = 3$

$\int_{-\infty}^{\infty} f(t-1)\,\delta(t-1)\,dt = f(0) = 2$

$\int_{-\infty}^{\infty} f(t)\,\delta(4t)\,dt = \frac{1}{4}\int_{-\infty} f(t)\,\delta(t)\,dt = \frac{f(0)}{4} = \frac{1}{2}$

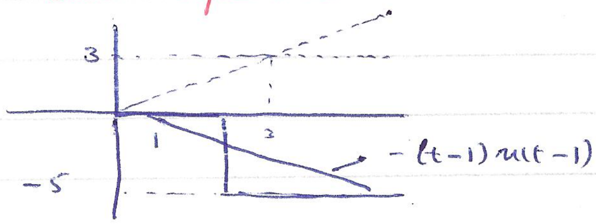
$\int_{+5}^{\infty} f(t)\,\delta(t-1)\,dt = 0$?

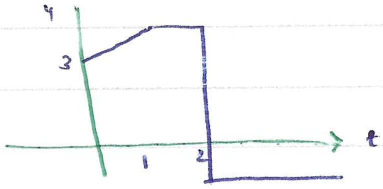
* Singularity functions: set of functions obtained by $\int dt$ or $\frac{d}{dt}$ of $\delta(t)$.

2.5 Mathematical Functions for Signals

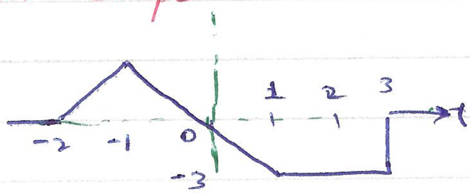
$$f(t) = 3u(t) + tu(t) - [t-1]u(t-1) - 5u(t-2)$$

① Scan Example 2.25 (2.11 Fig.)



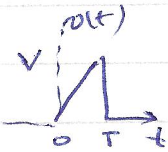


② Example 2.12 Fig. 2.27 In class Example



$$x(t) = 3[t+2]u(t+2)$$
$$-6[t+1]u(t+1) + 3[t-1]u(t-1) + 3u(t-3)$$

③ Example



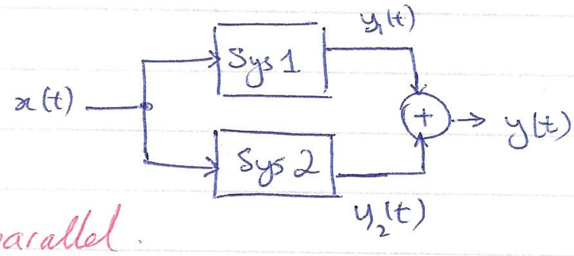
$$v(t) = \frac{V}{T}t\,[u(t) - u(t-T)] = \frac{V}{T}t\,\text{rect}\left[\left(t-\tfrac{T}{2}\right)/T\right]$$

? 

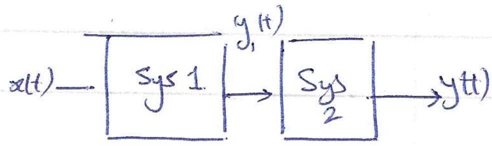
2.6 Continuous-Time Systems.

Systems are represented by block diagrams.

Interconnecting Systems



parallel.
Connection

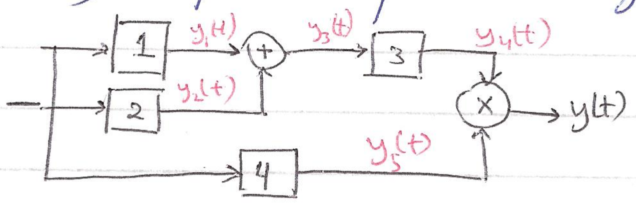


Series or Cascade
Connection

- Developing accurate models for physical systems can be one of the most difficult & time-consuming tasks for Engineers.

- The implicit assumption is made that the ch/s of all systems are unaffected by the presence of the other systems. (x)

Example 1

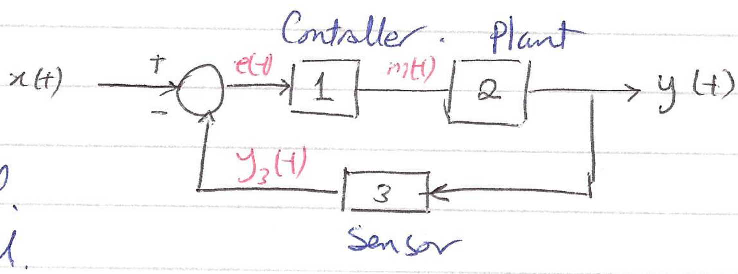


$y_1(t) = T_1[x(t)]$

$y_3(t) = T_1[x(t)] + T_2[X(t)]$

$y(t) = T_4[x(t)]\ T_3\left[T_1[x(t)] + T_2[X(t)]\right]$

Feedback



feedback - control.
Automatic control.

- Explain the basic operation of the feedback system?

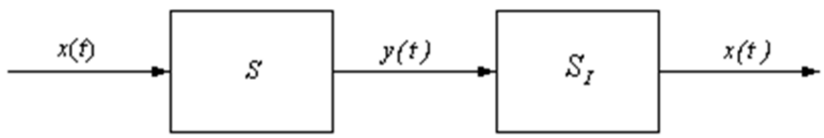
Properties of Continuous Time Systems

Systems with memory (Dynamic system)

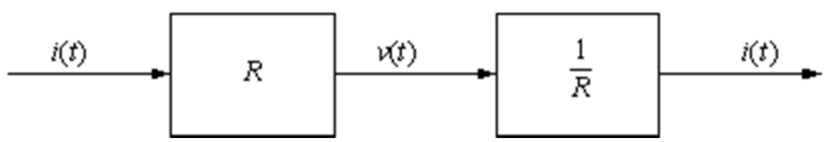
Systems whose output $y(t_0)$ at time t_0 depends on values of the input other than just $x(t_0)$ have memory. Otherwise, the system is **MEMORYLESS (Static system).**

Inverse of a System

A system is **invertible** if you can determine the input uniquely from the output, i.e. there is a one-to-one relationship between the input and output. In this case, we would write that the inverse of the system S is S_I:



A resistor is invertible because you can recover the current from the voltage: $x(t) = i(t)$, $y(t) = v(t)$, $x(t) = y(t)/R$.



$y(t) = x^5(t)$ is an invertible system since it is one-to-one.

Examples of some systems that are not invertible:

$y(t) = x(t)u(t) \rightarrow$ zeros out much of the input
$y(t) = x^2(t) \rightarrow$ don't know sign
$y(t) = \cos[x(t)] \rightarrow$ add 2π to $x(t)$

Causality

Output $y(t)$ depends only on past and present inputs and **not on the future.**

All physical real-time systems are causal because we can not anticipate the future.

If a system is memoryless, it is also causal. However, being causal does not necessarily imply that a system is memoryless

Stability

We will consider Bounded Input - Bounded Output (BIBO) Stability

We say that a system is BIBO stable if an input $x(t)$ that is bounded (finite) for all time produces an output $y(t)$ that is also bounded or finite for all time.

Mathematically, we write if $|x(t)| \leq B_1 \rightarrow |y(t)| \leq B_2$, where B_1 and B_2 are finite constants and $y(t)$ is the output.

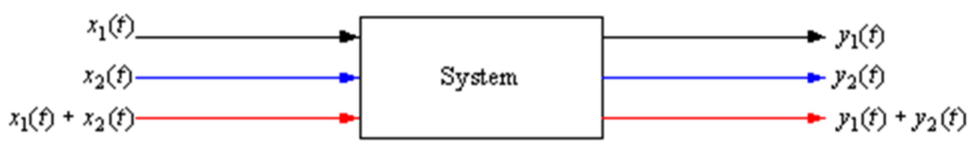
Time-Invariance

Given a system that is *time-invariant*, if the input signal is shifted in time, all that will happen is the output signal will be shifted by the same amount in time.

Linearity

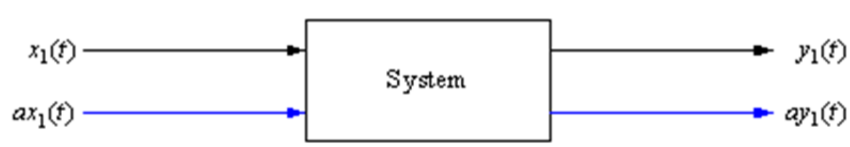
For a system to be linear, it must satisfy both the additivity and homogeneity properties:

1. Additivity



If $S[x_1(t)] = y_1(t)$ and $S[x_2(t)] = y_2(t) \rightarrow S[x_1(t) + x_2(t)] = y_1(t) + y_2(t)$ means that a system satisfies the additivity property.

2. Homogeneity or Scaling



$S[x(t)] = y(t) \rightarrow S[ax(t)] = ay(t)$ means that a system satisfies the scaling or homogeneity property.

Combine Additivity and Homogeneity to get the SUPERPOSITION CONDITION:

If $S[x_1(t)] = y_1(t)$ and $S[x_2(t)] = y_2(t)$
then $S[ax_1(t) + bx_2(t)] = ay_1(t) + by_2(t)$

Example : Determine the properties of the following system $y(t) = \sin(2t)\, x(t)$.

Memoryless, not-invertible, causal, stable, time varying, linear (test for linearity)