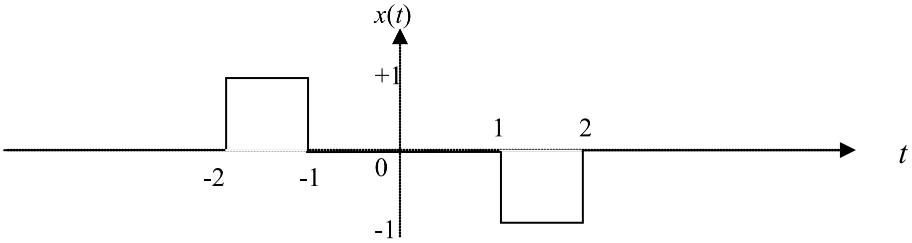
Serial #

- 1 points for not writing your serial number

King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals and Systems (111)
Quiz 4: Fourier Transform

Name: Sec.

1. For the following signal find the Fourier Transform using two different methods and make sure that the answers are the same:



a. By definition: $X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$. **(4 points)**

$$X(f) = \int_{-2}^{-1} (1)\, e^{-j2\pi ft}\, dt + \int_{1}^{2} (-1)\, e^{-j2\pi ft}\, dt$$

$$= \frac{-1}{j2\pi f}\left[e^{j2\pi f} - e^{j4\pi f}\right] + \frac{1}{j2\pi f}\left[e^{-j4\pi f} - e^{-j2\pi f}\right]$$

$$= \frac{1}{j\pi f}\left[\frac{e^{j4\pi f} + e^{-j4\pi f}}{2}\right] - \frac{1}{j\pi f}\left[\frac{e^{j2\pi f} + e^{j2\pi f}}{2}\right] = \frac{j}{\pi f}\left[\cos 2\pi f - \cos 4\pi f\right]$$

b. Taking the derivative until you get impulses. Then transform to frequency domain, then use the integral theorem to find F.T of $x(t)$. **(4 points)**

$$\dot{x}(t) = \delta(t+2) - \delta(t+1) - \delta(t-1) + \delta(t+2)$$

$$\dot{X}(f) = e^{j4\pi f} - e^{j2\pi f} - e^{-j2\pi f} + e^{-j4\pi f}$$

$$= 2\cos(4\pi f) - 2\cos(2\pi f)$$

$$\Rightarrow X(f) = \frac{1}{j2\pi f}\dot{X}(f) = \frac{j}{\pi f}\left[\cos 2\pi f - \cos 4\pi f\right]$$

c. Show that the two answers are equivalent. **(1 point)**

Both answers are having the same form

d. Is the F.T pure real, pure imaginary or complex? Why ? **(1 point)**

It is pure imaginary because x(t) is odd signal

Good Luck, **Dr. Ali Muqaibel**

TABLE 4-1
Fourier Transform Theorems[a]

Name of Theorem		
1. Superposition (a_1 and a_2 arbitrary constants)	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(f) + a_2X_2(f)$
2. Time delay	$x(t - t_0)$	$X(f)e^{-j2\pi ft_0}$
3a. Scale change	$x(at)$	$\|a\|^{-1}X\left(\frac{f}{a}\right)$
b. Time reversal	$x(-t)$	$X(-f) = X*(f)$
4. Duality	$X(t)$	$x(-f)$
5a. Frequency translation	$x(t)e^{j\omega_0 t}$	$X(f - f_0)$
b. Modulation	$x(t)\cos\omega_0 t$	$\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$
6. Differentiation	$\frac{d^n x(t)}{dt^n}$	$(j2\pi f)^n X(f)$
7. Integration	$\int_{-\infty}^{t} x(t')\,dt'$	$(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$
8. Convolution	$\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt' = \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$	$X_1(f)X_2(f)$
9. Multiplication	$x_1(t)x_2(t)$	$\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df' = \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df'$

[a] $\omega_0 = 2\pi f_0$; $x(t)$ is assumed to be real in 3b.

TABLE 4-2
Fourier Transform Pairs

Pair Number	$x(t)$	$X(f)$	Comments on Derivation
1.	$\Pi\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}\,\tau f$	Direct evaluation
2.	$2W\,\mathrm{sinc}\,2Wt$	$\Pi\left(\frac{f}{2W}\right)$	Duality with pair 1, Example 4-7
3.	$\Lambda\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}^2\,\tau f$	Convolution using pair 1
4.	$\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{\alpha + j2\pi f}$	Direct evaluation
5.	$t\exp(-\alpha t)u(t),\ \alpha > 0$	$\frac{1}{(\alpha + j2\pi f)^2}$	Differentiation of pair 4 with respect to α
6.	$\exp(-\alpha\|t\|),\ \alpha > 0$	$\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$	Direct evaluation
7.	$e^{-\pi(t/\tau)^2}$	$\tau e^{-\pi(f\tau)^2}$	Direct evaluation
8.	$\delta(t)$	1	Example 4-9
9.	1	$\delta(f)$	Duality with pair 7
10.	$\delta(t - t_0)$	$\exp(-j2\pi f t_0)$	Shift and pair 7
11.	$\exp(j2\pi f_0 t)$	$\delta(f - f_0)$	Duality with pair 9
12.	$\cos 2\pi f_0 t$	$\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$	Exponential representation of cos and sin and pair 10
13.	$\sin 2\pi f_0 t$	$\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$	
14.	$u(t)$	$(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$	Integration and pair 7
15.	$\mathrm{sgn}\,t$	$(j\pi f)^{-1}$	Pair 8 and pair 13 with superposition
16.	$\frac{1}{\pi t}$	$-j\,\mathrm{sgn}(f)$	Duality with pair 14
17.	$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\lambda)}{t - \lambda}\,d\lambda$	$-j\,\mathrm{sgn}(f)X(f)$	Convolution and pair 15
18.	$\sum_{m=-\infty}^{\infty}\delta(t - mT_s)$	$f_s\sum_{m=-\infty}^{\infty}\delta(f - mf_s),\ f_s = T_s^{-1}$	Example 4-10

Good Luck, **Dr. Ali Muqaibel**