**King Fahd University of Petroleum & Minerals**
Electrical Engineering Department
EE207: Signals and Systems (042)

# Major Exam II

May 9, 2005
06:30 PM-08:00PM
Building 7-119

Serial #

0
**-2 points for not writing your serial #**

Name:______________________________

ID:____________Key__________________

Sec. 1

| **Question** | **Mark** |
| --- | --- |
| 1 | /12 |
| 2 | /13 |
| 3 | /7 |
| 4 | /8 |
| Total | /40 |

**Instructions:**

1. This is a closed-books/notes exam.
2. The duration of this exam is one and half hours.
3. Read the questions carefully. Plan which question to start with.
4. Write explicitly the formulas that you use in your solution (e.g. by KVL … by KCL). No credit will be given if you do not show your formulas.
5. Work in your own.
6. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
7. Strictly no mobile phones are allowed.

**Good luck**

Dr. Ali Muqaibel

**Problem 1: (12 points)**

**Choose (Circle) the correct answer/answers:** **(4 points)**

**a.** If $x(t)$ is a real and even function of time then it is Fourier transform, $X(f)$ is **(real, imaginary, complex, even, odd)** function of frequency.

**b.** The spectrum of a non-periodic signal is **(continuous in frequency, discrete in frequency).**

**c.** The energy spectral density $G(f)$=

$$\left( |X(f)|, |X(f)|^2, \int_{-\infty}^{\infty} |X(f)|^2 df, \int_{-\infty}^{\infty} |X(f)| df \right)$$

**d.** We say that a rational function is proper if the degree of the numerator polynomial is **(less, equal, greater, less than or equal, greater than or equal)** than the degree of the denominator polynomial.

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**Select True or False (Correct answer +1, Wrong answer -0.5)** **(4 points)**

**a.** Ideal low pass filters cannot be implemented because they are non-causal. **(True, False)**
**b.** Many signals are not Fourier transformable. **(True, False)**
**c.** Fourier transform can be used to find the transient response. **(True, False)**
**d.** The rise time of a pulse is inversely proportional to its bandwidth. **(True, False)**

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A signal has Laplace transform **(4 points)**

$$X(s) = \frac{s+2}{s^2+4s+5}$$

Find the Laplace transforms, $Y(s)$, of the following signals. In each case, tell what Laplace transform theorems you used to find the signal.

**(a)** $y_1(t) = x(2t-1)u(2t-1)$ $= x(2(t-0.5))\, u(2(t-0.5))$

① $x(2t)\,u(2t)$ time scaling ⑩ $\frac{1}{2}\,\frac{s/2+2}{s^2/4+2s+5} = \frac{s+4}{s^2+8s+20}$

② time shift (delay) ⑤ $Y_1(s) = \frac{s+4}{s^2+8s+20}\, e^{-0.5s}$

**(b)** $y_2(t) = x(t) * x(t)$

$Y_2(s) = X(s)\,X(s) = \frac{(s+2)(s+2)}{(s^2+4s+5)(s^2+4s+5)}$ convolution

(C) $y_3(t) = e^{-3t}\,x(t)$

frequency - shift ④

$Y_3(s) = \frac{s+3+2}{(s+3)^2+4(s+3)+5} = \frac{s+5}{s^2+6s+9+4s+12+5}$

$= \frac{s+5}{s^2+10s+26}$

**Problem 2: (13 points)**

1. Given the function $f(t)$ shown in the Figure. Use the differentiation property to find the Fourier transform. Express the final answer in terms of trigonometric functions.

**(6 points)**

$$\frac{d^2f(t)}{dt^2} = \delta(t+3) - 3\delta(t+1) + 3\delta(t-1) - \delta(t-3)$$

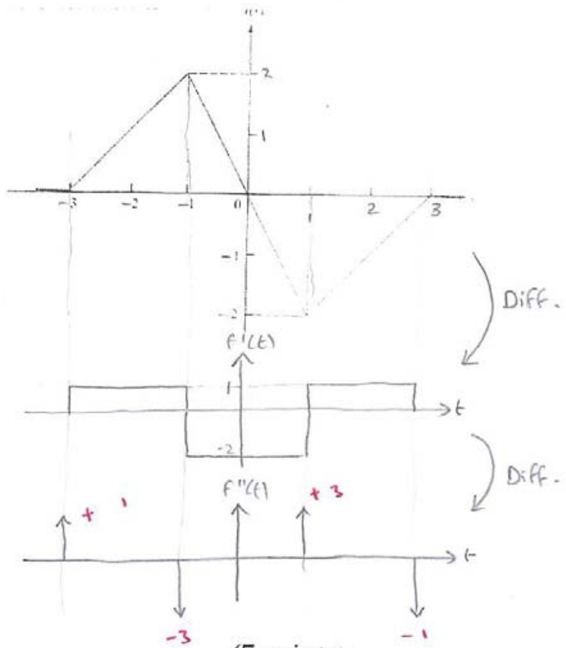
$$\mathcal{F}\left[\frac{d^2f(t)}{dt^2}\right] = e^{6j\pi f} - 3e^{2j\pi f} + 3e^{-2j\pi f} - e^{-6j\pi f}$$

$$= (j2\pi f)^2 X(f)$$

$$\Rightarrow X(f) = \frac{e^{6j\pi f} - e^{-6j\pi f}}{(j2\pi f)^2} - \frac{3e^{2j\pi f} - 3e^{-2j\pi f}}{(j2\pi f)^2}$$

$$= \frac{-1}{2j(\pi f)^2}\left[\sin(6\pi f) - 3\sin(2\pi f)\right]$$

$$\equiv \frac{2j}{\omega^2}(3\sin\omega - \sin 3\omega)$$




**2. Given** $X(s) = \dfrac{9s}{(s+2)^2(s+8)}$

**(7 points)**

**a.** Find the initial value of $x(t)$ (Theorem 8) **(2 points)**

$$\text{Initial value} = \lim_{s\to\infty} sX(s) = \lim_{s\to\infty} \frac{9s^2}{(s+2)^2(s+8)} = 0$$

**b.** Find the Inverse Laplace transform for $X(s)$ **( 4 points)**

$$\frac{9s}{(s+2)^2(s+8)} = \frac{k_1}{s+8} + \frac{k_2}{s+2} + \frac{k_3}{(s+2)^2}$$

$$k_1 = \frac{9s}{(s+2)^2}\bigg|_{s=-8} = -2 \qquad k_3 = \frac{9s}{s+8}\bigg|_{s=-2} = -3$$

PARTIAL FRACTIONS

$$k_2 = \frac{1}{1!}\frac{d}{ds}\frac{9s}{s+8}\bigg|_{s=-2} = \frac{72}{(s+8)^2}\bigg|_{s=-2} = 2 \qquad \left(\frac{-2}{s+8} + \frac{2}{s+2} - \frac{3}{(s+2)^2}\right)$$

$$\Rightarrow x(t) = \left(-2e^{-8t} + 2e^{-2t} - 3te^{-2t}\right)u(t)$$

**c.** Justify your answer to part **a** by finding the initial value in the time domain **(1 point)**

(Theorem 8)

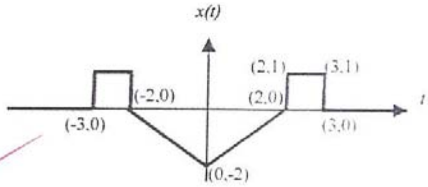
$$\text{Initial value} = \lim_{t\to 0^-} x(t) = \lim_{t\to 0^-}\left[\left(-2e^{-8t} + 2e^{-2t} - 3te^{-2t}\right)u(t)\right]$$

$$= \lim_{t\to 0^-}\left[\left(-2e^{-0} + 2e^{-0} - 0\,e^{-0}\right)u(0^-)\right] = (-2+2)u(0^-)$$

$$= (0)u(0^-) = 0$$

**Problem 3:** **(7 points)**

**a.** Write $x(t)$ in terms of $\Pi(t)$ and $\Lambda(t)$ **(2 points)**




$$x(t) = -2\Lambda\left(\frac{t}{2}\right) + \pi\left(t-\frac{5}{2}\right) + \pi\left(t+\frac{5}{2}\right)$$

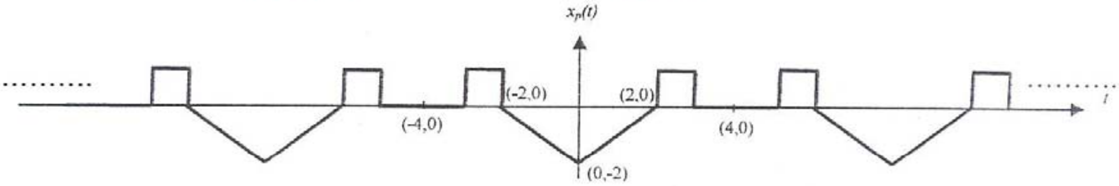
**b.** Find $X(f)$ which is the Fourier transform of $x(t)$ **(2 points)**

$$X(f) = \left(-2 \times 2\,\text{sinc}^2(2f)\right) + \left(\text{sinc}\, f \times e^{-5j\pi f}\right) + \left(\text{sinc} f \times e^{+5j\pi f}\right)$$

$$= -4\,\text{sinc}^2(2f) + 2\,\text{sinc}(f)\cos(5\pi f)$$

**c.** Find $X_P(f)$ which is the Fourier transform of $x_p(t)$. Note that $x_p(t)$ is the periodic extension of $x(t)$.
**(7 points)**

$$\sum_{m=-\infty}^{\infty} p(t-mT_s) \leftrightarrow \sum_{n=-\infty}^{\infty} f_s\, P(nf_s)\,\delta(f-nf_s)$$




$$X_p(f) = f_s \sum_{n=-\infty}^{\infty} X(nf_s)\,\delta(f-nf_s) \quad \text{where } f_s = 1/8,\ T_s = 8$$

$$= \frac{1}{4}\sum_{n=-\infty}^{\infty} \left(\text{sinc}(n/8)\cos\left(\frac{5}{8}n\pi\right) - 2\,\text{sinc}^2(n/4)\right)\delta(f-n/8)$$

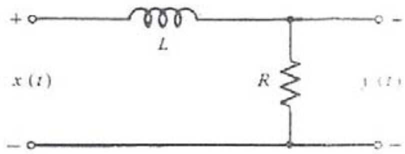
**d.** What are the differences between the spectra of $x(t)$ and $x_p(t)$ **(1 point)**

**1)** spectra of $x(t)$ is continuous, not for $x_p(t)$

**2)** spectra of $x_p(t)$ may be imaginary. $\times$

① spectrum of $x(t)$ continuous while for $x_p(t)$ it is discrete in freq.

② Magnitude spectrum of $x_p(t)$ is scaled by $f_s$ — magnitude spectrum of $x(t)$.




**Problem 4:** **(8 points)**

Let $R$=1 Ω and $L$=2 H

**a.** Write the differential equation relating $y(t)$ to $x(t)$ for the circuit shown in the figure. (2 points)

Apply KVL: $-x(t) + L\frac{di(t)}{dt} + y(t) = 0$ $\left(i(t) = \frac{y(t)}{R}\right)$ by ohm's low

$\Rightarrow x(t) = \frac{L}{R}\frac{dy(t)}{dt} + y(t)$ $\boxed{x(t) = 2\frac{dy(t)}{dt} + y(t)}$

**b.** Rewrite the equation in the s-domain (Laplace transform) (1 point)

$$X(s) = 2sY(s) - 2y(0^-) + Y(s)$$

$$X(s) = Y(s)[2s+1] - 2y(0^-)$$

**c.** Assuming the initial values are zeros, solve for the transfer function $H(s) = \frac{Y(s)}{X(s)}$ (1 point)

intial values are zeros $\Rightarrow -2y(0^-) = 0$

$$\Rightarrow X(s) = Y(s)[2s+1] \quad \Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{1}{2s+1}$$

**d.** If the input is $x(t) = 5e^{-2t}u(t)$, find the output, $y(t)$ **(4 points)**

$$x(t) = 5e^{-2t}u(t) \Longrightarrow X(s) = \frac{5}{s+2}$$

$$Y(s) = H(s)X(s) = \frac{1/2}{s+1/2} \cdot \frac{5}{s+2} = \frac{5/2}{(s+1/2)(s+2)}$$

$$y(t) = \mathcal{L}^{-1}[Y(s)]$$

$$\frac{5/2}{(s+1/2)(s+2)} = \frac{k_1}{s+1/2} + \frac{k_2}{s+2}$$

$$k_1 = \left.\frac{5/2}{s+2}\right|_{s=-1/2} = \frac{5}{3}$$

$$k_2 = \left.\frac{5/2}{s+1/2}\right|_{s=-2} = -\frac{5}{3}$$

PARTIAL FRACTIONS

$$\left(\frac{5/3}{s+1/2} - \frac{5/3}{s+2}\right)$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{5/3}{s+1/2} - \frac{5/3}{s+2}\right] = \left(\frac{5}{3}e^{-\frac{1}{2}t} - \frac{5}{3}e^{-2t}\right)u(t)$$

$$= \frac{5}{3}\left(e^{-\frac{1}{2}t} - e^{-2t}\right)u(t)$$

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**TABLE 4-1**
Fourier Transform Theorems[a]

| Name of Theorem | | |
|---|---|---|
| 1. Superposition ($a_1$ and $a_2$ arbitrary constants) | $a_1x_1(t) + a_2x_2(t)$ | $a_1X_1(f) + a_2X_2(f)$ |
| 2. Time delay | $x(t - t_0)$ | $X(f)e^{-j2\pi f t_0}$ |
| 3a. Scale change | $x(at)$ | $\lvert a\rvert^{-1}X\left(\frac{f}{a}\right)$ |
| b. Time reversal | $x(-t)$ | $X(-f) = X^*(f)$ |
| 4. Duality | $X(t)$ | $x(-f)$ |
| 5a. Frequency translation | $x(t)e^{j\omega_0 t}$ | $X(f - f_0)$ |
| b. Modulation | $x(t)\cos\omega_0 t$ | $\frac{1}{2}X(f - f_0) + \frac{1}{2}X(f + f_0)$ |
| 6. Differentiation | $\frac{d^n x(t)}{dt^n}$ | $(j2\pi f)^n X(f)$ |
| 7. Integration | $\int_{-\infty}^{t} x(t')\,dt'$ | $(j2\pi f)^{-1}X(f) + \frac{1}{2}X(0)\delta(f)$ |
| 8. Convolution | $\int_{-\infty}^{\infty} x_1(t - t')x_2(t')\,dt'$ $= \int_{-\infty}^{\infty} x_1(t')x_2(t - t')\,dt'$ | $X_1(f)X_2(f)$ |
| 9. Multiplication | $x_1(t)x_2(t)$ | $\int_{-\infty}^{\infty} X_1(f - f')X_2(f')\,df'$ $= \int_{-\infty}^{\infty} X_1(f')X_2(f - f')\,df'$ |

[a] $\omega_0 = 2\pi f_0$; $x(t)$ is assumed to be real in 3b.

**TABLE 4-2**
Fourier Transform Pairs

| Pair Number | $x(t)$ | $X(f)$ | Comments on Derivation |
|---|---|---|---|
| 1. | $\Pi\left(\frac{t}{\tau}\right)$ | $\tau\,\mathrm{sinc}\,\tau f$ | Direct evaluation |
| 2. | $2W\,\mathrm{sinc}\,2Wt$ | $\Pi\left(\frac{f}{2W}\right)$ | Duality with pair 1, Example 4-7 |
| 3. | $\Lambda\left(\frac{t}{\tau}\right)$ | $\tau\,\mathrm{sinc}^2\,\tau f$ | Convolution using pair 1 |
| 4. | $\exp(-\alpha t)u(t),\ \alpha > 0$ | $\frac{1}{\alpha + j2\pi f}$ | Direct evaluation |
| 5. | $t\exp(-\alpha t)u(t),\ \alpha > 0$ | $\frac{1}{(\alpha + j2\pi f)^2}$ | Differentiation of pair 4 with respect to $\alpha$ |
| 6. | $\exp(-\alpha\lvert t\rvert),\ \alpha > 0$ | $\frac{2\alpha}{\alpha^2 + (2\pi f)^2}$ | Direct evaluation |
| 7. | $e^{-\pi(t/\tau)^2}$ | $\tau e^{-\pi(f\tau)^2}$ | Direct evaluation |
| 8. | $\delta(t)$ | $1$ | Example 4-9 |
| 9. | $1$ | $\delta(f)$ | Duality with pair 7 |
| 10. | $\delta(t - t_0)$ | $\exp(-j2\pi f t_0)$ | Shift and pair 7 |
| 11. | $\exp(j2\pi f_0 t)$ | $\delta(f - f_0)$ | Duality with pair 9 |
| 12. | $\cos 2\pi f_0 t$ | $\frac{1}{2}\delta(f - f_0) + \frac{1}{2}\delta(f + f_0)$ | Exponential representation of cos and sin and pair 10 |
| 13. | $\sin 2\pi f_0 t$ | $\frac{1}{2j}\delta(f - f_0) - \frac{1}{2j}\delta(f + f_0)$ | |
| 14. | $u(t)$ | $(j2\pi f)^{-1} + \frac{1}{2}\delta(f)$ | Integration and pair 7 |
| 15. | $\mathrm{sgn}\,t$ | $(j\pi f)^{-1}$ | Pair 8 and pair 13 with superposition |
| 16. | $\frac{1}{\pi t}$ | $-j\,\mathrm{sgn}(f)$ | Duality with pair 14 |
| 17. | $\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\lambda)}{t - \lambda}\,d\lambda$ | $-j\,\mathrm{sgn}(f)X(f)$ | Convolution and pair 15 |
| 18. | $\sum_{m=-\infty}^{\infty} \delta(t - mT_s)$ | $f_s \sum_{m=-\infty}^{\infty} \delta(f - mf_s)$, $f_s = T_s^{-1}$ | Example 4-10 |

**TABLE 5-2**
Laplace Transform Theorems

| Name | Operation in Time Domain | Operation in Frequency Domain |
|---|---|---|
| 1. Linearity | $a_1x_1(t) + a_2x_2(t)$ | $a_1X_1(s) + a_2X_2(s)$ |
| 2. Differentiation | $\frac{d^nx(t)}{dt^n}$ | $s^nX(s) - s^{n-1}x(0^-) - \cdots - x^{(n-1)}(0^-)$ |
| 3. Integration | $\int_{-\infty}^{t} x(\lambda)\,d\lambda$ | $\frac{X(s)}{s} + \frac{x^{(-1)}(0^-)}{s}$ |
| 4. $s$-shift | $x(t)\exp(-\alpha t)$ | $X(s+\alpha)$ |
| 5. Delay | $x(t-t_0)u(t-t_0)$ | $X(s)\exp(-st_0)$ |
| 6. Convolution | $x_1(t) * x_2(t) = \int_0^\infty x_1(\lambda)x_2(t-\lambda)\,d\lambda$ | $X_1(s)X_2(s)$ |
| 7. Product | $x_1(t)x_2(t)$ | $\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X_1(s-\lambda)X_2(\lambda)\,d\lambda$ |
| 8. Initial value (provided limits exist) | $\lim_{t\to 0^+} x(t)$ | $\lim_{s\to\infty} sX(s)$ |
| 9. Final value (provided limits exist) | $\lim_{t\to\infty} x(t)$ | $\lim_{s\to 0} sX(s)$ |
| 10. Time scaling | $x(at), \quad a > 0$ | $a^{-1}X\left(\frac{s}{a}\right)$ |

**TABLE 5-3**
Extended Table of Single-Sided Laplace Transforms

| Signal | Laplace Transform | Comments on Derivation |
|---|---|---|
| 1. $\delta^{(n)}(t)$ | $s^n$ | Direct evaluation with aid of (1-66) |
| 2. 1 or $u(t)$ | $\frac{1}{s}$ | Direct evaluation |
| 3. $\frac{t^n\exp(-\alpha t)u(t)}{n!}$ | $\frac{1}{(s+\alpha)^{n+1}}$ | Differentiation applied to pair 3, Table 5-1 |
| 4. $\cos\omega_0 t\, u(t)$ | $\frac{s}{s^2+\omega_0^2}$ | Example 5-1 |
| 5. $\sin\omega_0 t\, u(t)$ | $\frac{\omega_0}{s^2+\omega_0^2}$ | Example 5-1 |
| 6. $\exp(-\alpha t)\cos\omega_0 t\, u(t)$ | $\frac{s+\alpha}{(s+\alpha)^2+\omega_0^2}$ | $s$-shift and pair 4 |
| 7. $\exp(-\alpha t)\sin\omega_0 t\, u(t)$ | $\frac{\omega_0}{(s+\alpha)^2+\omega_0^2}$ | $s$-shift and pair 5 |
| 8. Square wave: $u(t) - 2u\left(t-\frac{T_0}{2}\right) + 2u(t-T_0) - \cdots$ | $\frac{1}{s}\frac{1-e^{-sT_0/2}}{1+e^{-sT_0/2}}$ | Example 5-5 |
| 9. $(\sin\omega_0 t - \omega_0 t\cos\omega_0 t)u(t)$ | $\frac{2\omega_0^3}{(s^2+\omega_0^2)^2}$ | Example 5-12, pair 5, and convolution |
| 10. $(\omega_0 t\sin\omega_0 t)u(t)$ | $\frac{2\omega_0^2 s}{(s^2+\omega_0^2)^2}$ | Pair 4 and convolution |
| 11. $\omega_0 t\exp(-\alpha t)\sin\omega_0 t\, u(t)$ | $\frac{2\omega_0^2(s+\alpha)}{[(s+\alpha)^2+\omega_0^2]^2}$ | $s$-shift and pair 10 |
| 12. $\exp(-\alpha t)(\sin\omega_0 t - \omega_0 t\cos\omega_0 t)u(t)$ | $\frac{2\omega_0^3}{[(s+\alpha)^2+\omega_0^2]^2}$ | $s$-shift and pair 9 |

Good Luck, **Dr. Ali Muqaibel**