King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals & Systems (121)

Major Exam I

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Oct. 10, 2012
6:00-7:30 PM
Building 59

Name: ______________Key______________

ID#_________________0000_____________

Question	Mark
1	26
2	12
3	12
Total	50/50

Instructions:

1. This is a closed-books/notes exam.
2. The duration of this exam is one and half hours.
3. Read the questions carefully. Plan which question to start with.
4. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
5. Work in your own.
6. Strictly no mobile phones are allowed. Do not even look at them !

Good luck

Mark	sec	Timing	Instructor
	1	UT 8:30	Dr. M. Landolsi
	3	UT 10:00	Dr. Ali Al-Shaikhi
	4	SMW 10:00	Dr. Azzedine Zerguine
	5	SMW 11:00	Dr. Ali Muqaibel

Problem 1: **Choose the best answer. Fill in the table with CLEAR answer**

Question	1	2	3	4	5	6	7	8	9	10	11	12	13
Answer	e	b	d	c	a	d	e	e	d	b	b	c	c

Compared with x(t), y(t) has ① DC shift by +1.5 ② Amplitude scale & inversion −0.5 ③ time shift by two units (right) $t \leftarrow t-2$ ④ Time Compression $t \leftarrow 2t$

1. The two shown signals $y(t)$ and $x(t)$ are related by:

a) $y(t) = -0.5x(2t+1) + 1.5$ ✗ time shift in the wrong direction

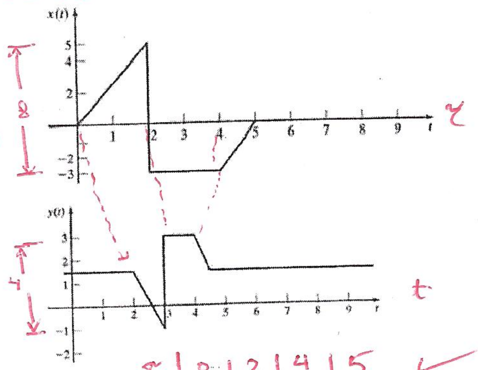
b) $y(t) = 2x(-t) - 1.5$ ✗ no time shift & wrong dc shift

c) $y(t) = -2x(2t) + 1.5$ ✗ " " "

d) $y(t) = -2x(0.5t + 1) + 1.5$ ✗ wrong time scale

e) $y(t) = -0.5x(2(t-2)) + 1.5$ ✓ $\tau = 2(t-2)$

$\boxed{t = \frac{\tau}{2} + 2}$



τ	0	2	4	5
t	2	3	4	4.5

✓

2. The value of $\int_{-\infty}^{+\infty}[e^{\propto t^2}\delta(t-10) + \sin(3\pi t)\,\delta(t)]dt =$

a) 0

b) $e^{100\propto}$

c) 1

d) $e^{100\propto} + 1$

e) $e^{\propto t^2} + \sin(3\pi t)$

By sifting property

$= e^{\alpha(10)^2} + \sin(3\pi(0)) = e^{\alpha 100}$

3. For the periodic sawtooth signal shown in the figure, the power is equal to

a) 2

b) 1

c) ½

d) 1/3

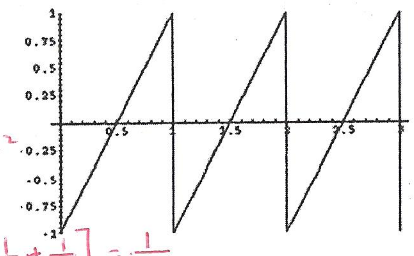
e) 0

Note that time delay does not change power

$P = \frac{1}{T}\int_{T_0} |x(t)|^2 dt$

slope = 2, T = 1

$P = \frac{1}{T}\int_{-\frac{1}{2}}^{\frac{1}{2}} (2t)^2 dt = 4\int_{-\frac{1}{2}}^{\frac{1}{2}} t^2 dt = \frac{4}{3} t^3 \Big|_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{4}{3}\left[\frac{1}{8} + \frac{1}{8}\right] = \frac{1}{3}$



4. A system is described by the following impulse response $h(t) = u(t+1) + \delta(t)$, the system is

non causal — $u(t+1)$ future

not bounded — area $u(t) \to \infty$, $\int_{-\infty}^{\infty} |h(t)|dt \to \infty$

✗ a) Causal, Bounded Input Bounded Output (BIBO) stable, has memory (dynamic)

b) Noncausal, Bounded Input Bounded Output (BIBO) stable, memoryless (static)

c) Noncausal, not Bounded Input Bounded Output (BIBO) stable, has memory (dynamic)

✗ d) Causal, not Bounded Input Bounded Output (BIBO) stable, memoryless (static)

✗ e) Causal, not Bounded Input Bounded Output (BIBO) stable, has memory (dynamic)

5. A causal linear Time-invariant system has the impulse response, $h(t) = \frac{1}{3}e^{-\frac{t}{3}}u(t)$. If the input signal is $2u(t-5)$, then the output is

a) $2\left(1 - e^{\frac{-(t-5)}{3}}\right)u(t-5)$

b) $\frac{2}{9}\left(1 - e^{\frac{-(t-5)}{3}}\right)u(t-5)$

c) $\frac{2}{3}e^{-\frac{(t-5)}{3}}u(t-5)$

d) $\frac{1}{3}e^{-\frac{t}{3}}u(t).\,2u(t-5)$

e) This input cannot be applied because the system is causal

step response i.e response to u(t)

$s(t) = \int_{-\infty}^{t} h(\tau)d\tau = \frac{1}{3}\int_{-\infty}^{t} e^{-\tau/3}u(\tau)d\tau$

$= -\left[e^{-\tau/3}\right]_0^t = \left(1 - e^{-t/3}\right)u(t)$

Now, the response to $2u(t-5)$ is scale by 2 & shift time

2

6. **Which of the following statements is true?**

a) For a linear time invariant system, the step response, $s(t)$, is related to the impulse response, $h(t)$, by $s(t)=\int_{-\infty}^{\infty} h(t)dt$. — should be t ; area! not function of time

b) Even functions are symmetric about the x-axis. × (y-axis)

c) The following system $y(t)=5+x(t)+x(t^2)$ is memoryless. × it has memory ; $y(t=2)=5+x(t=2)+x(t=4)$

d) In the complex exponential Fourier representation, the complex coefficient C_0 is always real number. (average value) ✓

e) $\delta(t)\cos(2\pi t)=1$ × $=\delta(t)$

7. **A system made of four subsystems connected as shown in the Figure. Every system is expressed with its impulse response. The overall impulse response is given by:**

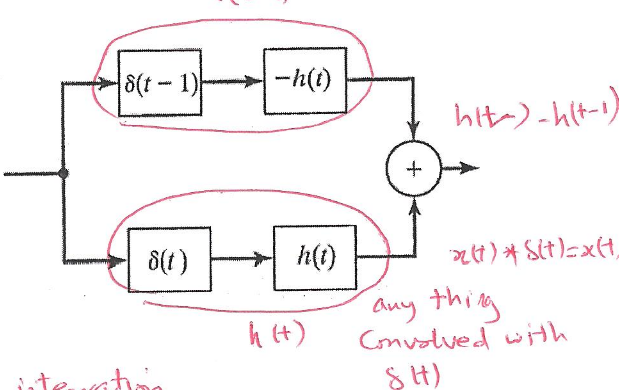
a) $-\delta(t-1)h(t)+\delta(t)h(t)$

b) $-h(t)+h(t)$

c) $\delta(t-1)+\delta(t)$

d) $\delta(t)-\delta(t-1)$

e) $h(t)-h(t-1)$



8. **The value of** $\int_{-1}^{5} e^{-4t^2}\cos(t^2)\delta(t-10)dt=$ — outside the integration limits.

a) $e^{-400}\cos(100)\delta(t-10)$

b) $e^{-400}\cos(100)$

c) $\delta(t-10)$

d) $e^{-400}\sin(100)$

e) 0

9. **The system defined by the input output relationship ………………….is a causal system.**

a) $y(t)=x(-t)$ × $y(t=-1)=x(t=1)$ future input

b) $y(t)=x(t^2)$ × $y(t=2)=x(t=4)$ future input. any $|t|>1$

c) $y(t)=x(t^{\frac{1}{2}})$ × $y(t=0.04)=x(t=0.2)$ future input any $t<1$

d) $y(t)=x(t)+t-1$

e) $y(t)=x(t+5)-5$ × future input

10 **The signal shown can be represented by:**

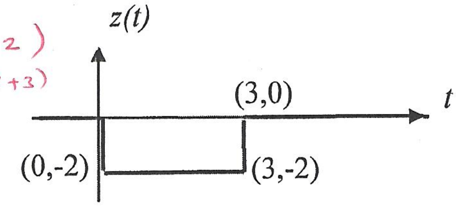
a) $-u(t)+u(t-3)$ × ← amplitude is -1 (not -2)

b) $-2u(t)\times u(-t+3)$ ← $u(-t+3)$

c) $-3u(t)\times u(-t-2)$

d) $-3u(-t)\times rect(-t-2)$

e) $-z(t)$ × signal cannot be negative of itself. (function) in general.



11. **Let $s(t) = e^{t-1}u(t+1)$ be the step response of a linear time invariant system. The unit impulse response, $h(t)$, for this system is:**

$h(t) = \frac{d}{dt} s(t)$ product of two terms.

a) $h(t) = e^{t-1}u(t+1)$ ← first term.
b) $h(t) = e^{t-1}u(t+1) + e^{-2}\delta(t+1)$
c) $h(t) = e^{t-1}\delta(t+1)$
d) $h(t) = e^{t-1}u(t+1) + \delta(t+1)$
e) $h(t)$ cannot be found from $s(t)$

$e^{t-1}u(t+1) + e^{t-1}\delta(t+1)$; $t-1 \to -(-1)$

now using delta properties "sifting"

$e^{t-1}u(t+1) + e^{-2}\delta(t+1)$

12. **Evaluate $\cos\left(\frac{\pi t}{5}\right) * \delta(t) * \delta(t-3)$=**

a) $\cos\left(\frac{\pi t}{5}\right)$
b) $\cos\left(\frac{\pi}{5}\right)$
c) $\cos\left[\frac{\pi}{5}(t-3)\right]$
d) $\delta(t)$
e) $\cos\left(\frac{3\pi}{5}\right)$

using associative property

$\cos\left(\frac{\pi t}{5}\right) * (\delta(t) * \delta(t-3))$

$= \cos\left(\frac{\pi t}{5}\right) * \delta(t-3)$

$= \cos\left(\frac{\pi(t-3)}{5}\right)$

13. **Given that $x(t) = 6e^{-6t}u(t)$, the energy of $x(t)$ is:**

a) 0.5
b) 1.0
c) 3.0
d) 6.0
e) 4.0

$$\text{Energy} = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

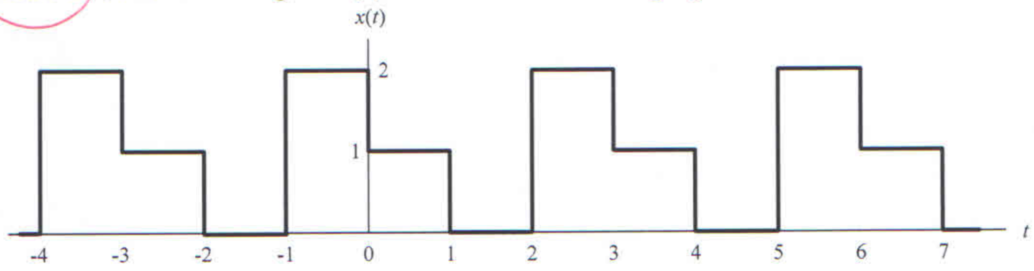
$$= \int_{-\infty}^{\infty} 36 e^{-12t} u(t)\, dt = 36 \int_{0}^{\infty} e^{-12t} dt$$

$$= -\frac{36}{12} e^{-12t}\Big|_0^{\infty} = -3\,[0-1] = 3$$

Problem 2: [12pts]

Part 1: Consider the signal $x(t)$ shown in the following figure.

8pts



a) Find the period T_0 and the fundamental frequency ω_0 for the signal $x(t)$.

b) Compute the complex Fourier coefficients C_k.

c) Find the trigonometric Fourier series coefficients A_0, A_1, and B_3.

a) $T_0 = 3$ 0.5pt $\quad \omega_0 = 2\pi/3$ 0.5pt

b) $C_k = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{-jk\omega_0 t}\, dt$ 1pt

$$= \frac{1}{3}\int_{-1}^{0} 2e^{-jk\omega_0 t}\,dt + \frac{1}{3}\int_{0}^{1} 1.e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{3}\frac{1}{jk\omega_0}\left[2e^{jk\omega_0} - 2 + 1 - e^{-jk\omega_0}\right]$$ 2pts

this is enough for full credit

$$= \frac{1}{2\pi jk}\left[2e^{+jk2\pi/3} - 1 - e^{-jk2\pi/3}\right], \quad k \neq 0$$

and $C_0 = \frac{1}{3}\left\{\int_{-1}^{0} 2\,dt + \int_{0}^{1} dt\right\} = 1$ ← 1pt.

Note: Can further simplify C_k to $\frac{1}{\pi k}\left[\frac{e^{jk\frac{2\pi}{3}} - e^{-jk\frac{2\pi}{3}}}{2j} + \frac{e^{jk\frac{\pi}{3}}}{\pi k}\frac{(e^{jk\frac{\pi}{3}} - e^{-jk\frac{\pi}{3}})}{2j}\right]$

$$= \frac{2}{3}\,\mathrm{sinc}\left(k\frac{2\pi}{3}\right) + 1/3\; e^{jk\frac{\pi}{3}} \times \mathrm{sinc}\left(k\frac{\pi}{3}\right)$$

c) $A_0 =$ C0=1 1pt.

$A_1 = 2\,\mathrm{Re}\{C_1\} = 0.827$ 1pt

$B_3 = -2\,\mathrm{Im}\{C_3\} = 0$ 1pt.

Just different integration limits $0 \to 3$

$$T_0 = 3 \quad , \quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{3}$$

$$C_K = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-JK\omega_0 t}\, dt = \frac{1}{3}\left[\int_0^1 e^{-JK\frac{2\pi}{3}t}\, dt + \int_2^3 2\, e^{-JK\frac{2\pi}{3}t}\, dt\right]$$

$$= \frac{1}{\cancel{3}\left(-JK\frac{2\pi}{\cancel{3}}\right)}\left[e^{-JK\frac{2\pi}{3}t}\Big|_0^1 + 2\, e^{-JK\frac{2\pi}{3}t}\Big|_2^3 \right]$$

$$= \frac{-1}{JK2\pi}\left[e^{-JK\frac{2\pi}{3}} - 1 + 2\, e^{-JK2\pi} - 2\, e^{-JK\frac{4\pi}{3}} \right]$$

$K \neq 0$

note that $e^{-JK2\pi} \equiv 1 \angle -K2\pi = 1$

multiples of 2π

$$\Rightarrow \quad C_K = \frac{-1}{JK2\pi}\left[e^{-JK\frac{2\pi}{3}} + 1 - 2\, e^{-JK\frac{4\pi}{3}} \right] \quad , \quad K \neq 0$$

Note that we can chose different integration limits this's will result in the same answer, Multiples of two pi (2π) will apper in the angle.

For Example

$$C_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt = \frac{1}{3}\left[\int_0^1 dt + \int_2^3 2\, dt\right] = \frac{1}{3}\left[1+2\right] = 1$$

$A_0 = C_0 = 1$

$A_1 = 2\,\mathrm{Re}\{C_1\} = 0.827$

$B_3 = -2\,\mathrm{Im}\{C_3\} = 0$

other possible solution of C_K

$$\frac{2}{3}\,\mathrm{Sinc}\left(\frac{K2\pi}{3}\right) + \frac{1}{3}\, e^{jK\frac{\pi}{3}}\, \mathrm{Sinc}\left(K\frac{\pi}{3}\right)$$

or.

$$\frac{1}{2\pi jK}\left[2\, e^{jK\frac{2\pi}{3}} - 1 - e^{-jK\frac{2\pi}{3}} \right] \quad , \quad K \neq 0$$

Problem 2:

4 pts **Part 2:** For a signal with complex Fourier series coefficients:

$C_0 = 0$ and $C_k = \frac{-3j}{k\pi}\left[\cos\left(\frac{k\pi}{2}\right) - 1\right], k > 0.$

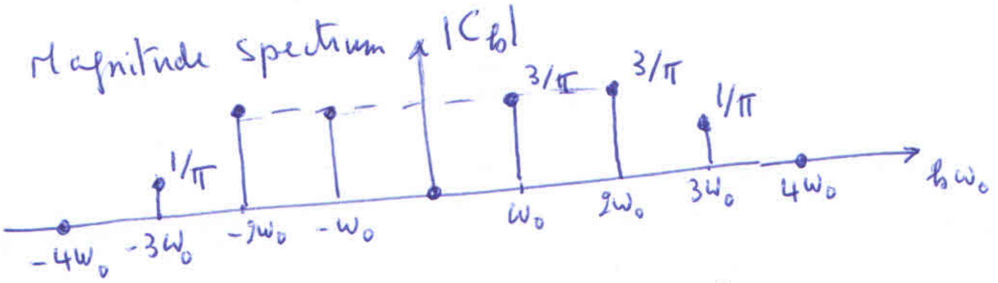
a) Plot the two-sided magnitude spectrum (up to the 4th harmonic).

b) Plot the two-sided phase spectrum (up to the 4th harmonic).

a) $C_0 = 0, \quad C_1 = -\frac{3j}{\pi}\left(\cos\frac{\pi}{2} - 1\right) = \frac{3j}{\pi}, \quad C_2 = -\frac{3j}{2\pi}\left(\cos\pi - 1\right) = +\frac{3j}{\pi}$

$C_3 = -\frac{3j}{3\pi}\left(\cos\frac{3\pi}{2} - 1\right) = \frac{\cancel{3}j}{\pi}, \quad C_4 = 0.$

Magnitude spectrum



2 pts

1.5 pt → values

0.5 → even

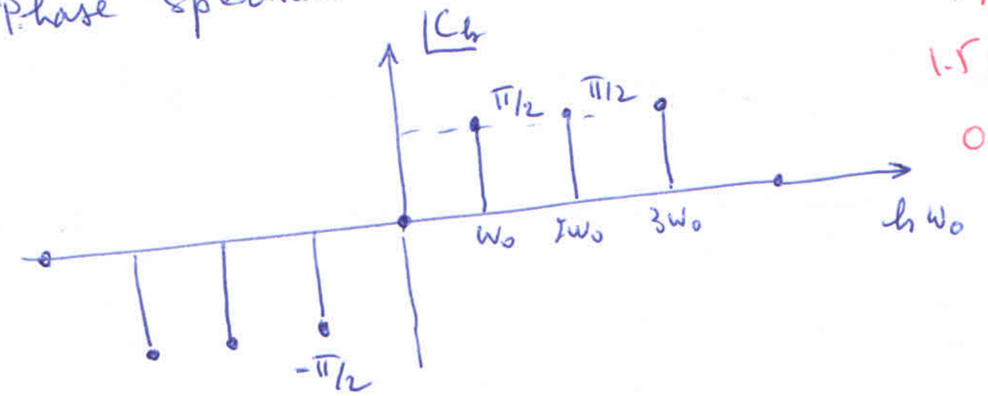
$|C_{-k}| = |C_k|$ (even.)

b) Phase spectrum

2 pts

1.5 pt → values

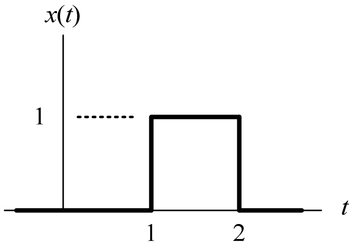
0.5 pt → odd

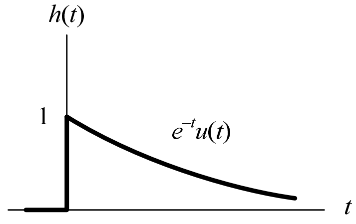


$\angle C_{-k} = -\angle C_k$

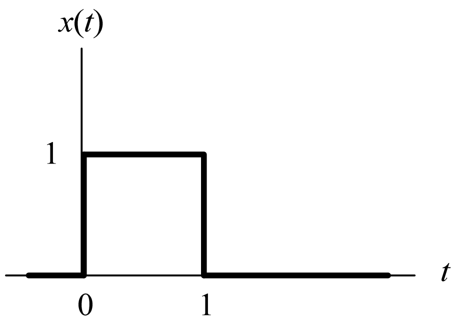
<u>Problem 3</u>:

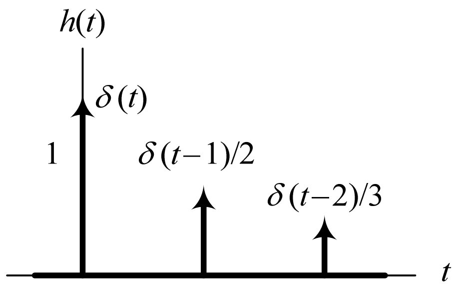
Part 1. Find $y(t) = x(t) * h(t)$ for the two signals shown below:



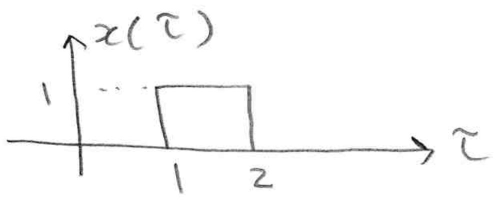


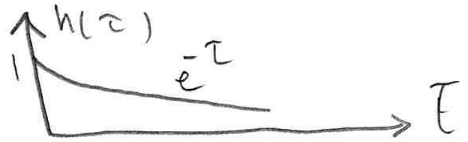
Part 2. Find $y(t) = x(t) * h(t)$ for the two signals shown below:





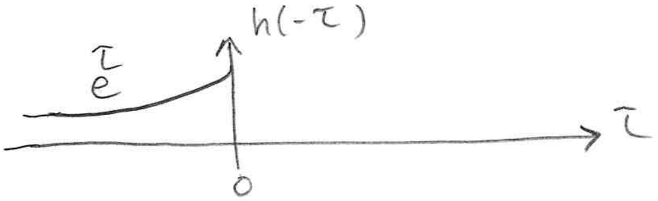
Problem 3 :. Part (1)

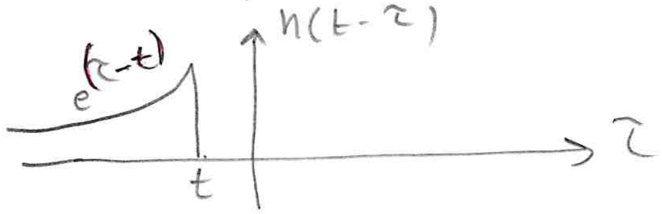


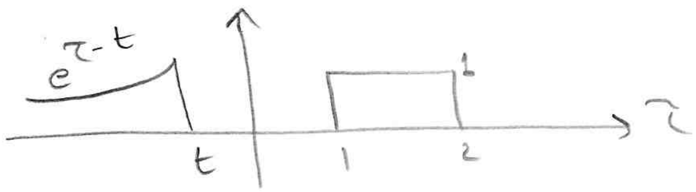


$y(t) = x(t) * h(t)$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$







for $t<1$

$y(t) = 0$

for $1<t<2$

$$y(t) = \int_1^t e^{\tau-t}(1)\, d\tau$$

$$= e^{-t}\,(e^{\tau})_1^t = e^{-t}(e^{t}-e^{1})$$

$$= 1-e^{1-t}$$

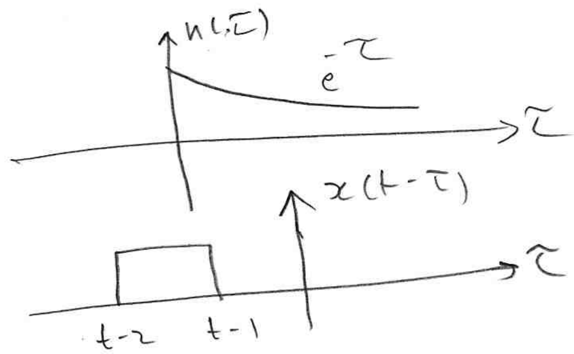
for $t>2$

$$y(t) = \int_1^2 e^{\tau-t}\, d\tau$$

$$= e^{-t}\,(e^{\tau})_1^2 = e^{-t}(e^{2}-e^{1})$$

$$= e^{-t+2}-e^{-t+1} = e^{2-t}-e^{1-t}$$

other way $y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$



if $t-1<0 \Rightarrow t<1$

$y(t) = 0$

if $0<t-1<1 \Rightarrow 1<t<2$

$$y(t) = \int_0^{t-1} e^{-\tau}\, d\tau$$

$$= -(e^{-\tau})_0^{t-1}$$

$$= -(e^{-t+1}-1) = 1-e^{1-t}$$

If $t-1>1 \Rightarrow t>2$

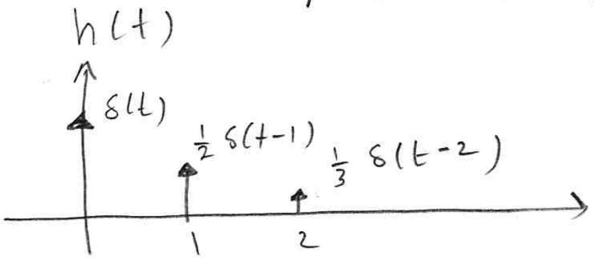
$$y(t) = \int_{t-2}^{t-1} e^{-\tau}\, d\tau$$

$$= -(e^{-\tau})_{t-2}^{t-1}$$

$$= -[e^{1-t}-e^{2-t}]$$

$$= e^{2-t}-e^{1-t} = e^{-t}(e^{2}-e^{1})$$

problem 3 - part 2



$$h(t) = \delta(t) + \frac{1}{2}\,\delta(t-1) + \frac{1}{3}\,\delta(t-2)$$

$$\therefore\ y(t) = x(t) * h(t)$$

$$= x(t) * \left[\delta(t) + \frac{1}{2}\,\delta(t-1) + \frac{1}{3}\,\delta(t-2)\right]$$

$$= x(t) * \delta(t) + \frac{1}{2}\,x(t) * \delta(t-1) + \frac{1}{3}\,x(t) * \delta(t-2)$$

$$y(t) = x(t) + \frac{1}{2}\,x(t-1) + \frac{1}{3}\,x(t-2)$$

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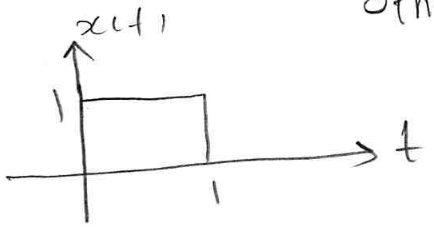
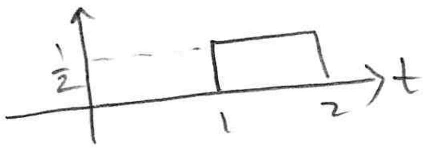
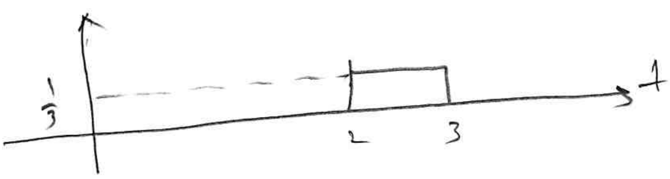
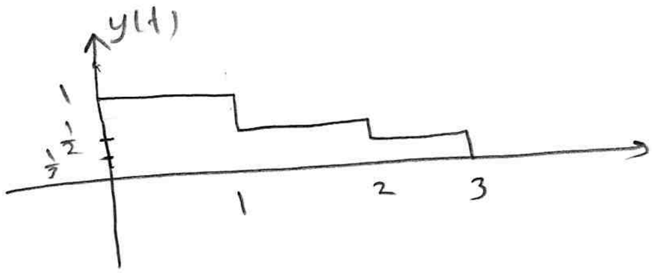
other way









$$y(t) = \int h(\tau)\,x(t-\tau)\,d\tau$$

if $t < 0$

$y(t) = 0$

if $0 < t < 1$

$$y(t) = \int \delta(\tau)\,x(t-\tau)\,d\tau$$

$$= x(t) = 1$$

if $1 < t < 2$

$$y(t) = \int \frac{1}{2}\,\delta(\tau-1)\,x(t-\tau)\,d\tau$$

$$= \frac{1}{2}\,x(t-1) = \frac{1}{2}$$

if $2 < t < 3$

$$y(t) = \frac{1}{3}\,x(t-2) = \frac{1}{3}$$

If $t>3$ $y(t)=0$

$$\therefore\ y(t) = x(t) + \frac{1}{2}\,x(t-1) + \frac{1}{3}\,x(t-2)$$