King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE207: Signals & Systems (121)

Major Exam II

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Dec. 1, 2012
6:30-8:00PM
Building 59

Name: KEY

ID#

Question	Mark
1	/16
2	/14
3	/10
4	/10
Total	/50

Instructions:

1. This is a closed-books/notes exam.
2. The duration of this exam is one and half hours.
3. Read the questions carefully. Plan which question to start with.
4. LABEL ALL SIGNIFICANT VALUES ON BOTH AXES OF ANY SKETCH
5. Work on your own.
6. Strictly no mobile phones are allowed. Do not even look at them !

Good luck

Mark	sec	Timing	Instructor
	1	UT8:30	Dr. M. Landolsi
	3	UT 10:00	Dr. Ali Al-Shaikhi
	4	SMW10:00	Dr. Azzedine Zerguine
	5	SMW 11:00	Dr. Ali Muqaibel

Problem 1: [16pts] **Choose the best answer. Fill in the table with CLEAR answers**

Question	1	2	3	4	5 *	6	7	8
Answer	d	c	b	b	a	b	c	b

1. If a signal $x(t)$ has Fourier transform $X(\omega)=\frac{1}{(1+j\omega)^2}$, the Fourier transform of $x\left(\frac{t-2}{4}\right)$ is:

   a. $\frac{e^{-j2\omega}}{4(1+j\omega/4)^2}$

   b. $\frac{4e^{-j\omega/2}}{(1+j4\omega)^2}$

   c. $\frac{4e^{-j2\omega}}{(1+j\omega)^2}$

   d. $\frac{4e^{-j2\omega}}{(1+j4\omega)^2}$

   e. $\frac{4e^{-j8\omega}}{(1+j4\omega)^2}$

Using Time Transformation property

$f(at-t_o) \leftrightarrow \frac{1}{|a|}F\left(\frac{\omega}{a}\right)e^{-j\omega t_o/a}$

$a=\frac{1}{4}$, $t_o=\frac{2}{4}=\frac{1}{2}$

$x\left(\frac{t-2}{4}\right) \leftrightarrow 4X(4\omega)e^{-j\omega 2}$

$\leftrightarrow \frac{4}{(1+j4\omega)^2}e^{-j2\omega}$

2. If you know that $e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2}$, $a>0$ then the FT of $x(t)=\frac{1}{a^2+t^2}$ is:

   a. $X(\omega)=\frac{1}{a^2+\omega^2}$

   b. $X(\omega)=\frac{2a}{a^2+\omega^2}$

   c. $X(\omega)=\frac{\pi}{a}e^{-a|\omega|}$

   d. $X(\omega)=2\pi e^{-a|-\omega|}$

   e. $X(\omega)=2\pi e^{-a|\omega|}$

By duality.

$F(t) \leftrightarrow 2\pi f(-\omega)$

$\frac{2a}{a^2+t^2} \leftrightarrow 2\pi e^{-a|-\omega|}$

divide both sides by $2a$

$\frac{1}{a^2+t^2} \leftrightarrow \frac{\cancel{2}\pi}{\cancel{2}a}e^{-a|\omega|}$

3. The FT of $u(-t)$

   a. $\pi\delta(\omega)+\frac{1}{j\omega}$ ×

   b. $\pi\delta(\omega)-\frac{1}{j\omega}$

   c. $-\pi\delta(\omega)-\frac{1}{j\omega}$

   d. $\pi\delta(-\omega)+\frac{1}{j\omega}$ ×

   e. $-\pi\delta(-\omega)+\frac{1}{j\omega}$ ×

using time scaling property

$f(at) \leftrightarrow \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$

$a=-1$

$f(-t) \leftrightarrow F(-\omega)$

from the table $u(t) \leftrightarrow \pi\delta(\omega)+\frac{1}{j\omega}$

$\Rightarrow u(-t) \leftrightarrow \pi\delta(-\omega)-\frac{1}{j\omega}$

$\equiv \pi\delta(\omega)-\frac{1}{j\omega}$

δ: is even function

4. Given the signal $x(t) = 2\text{sinc}(2t)$, the Fourier transform of its derivative dx/dt is:

   a. $\pi\delta(\omega/2)$
   b. $j\pi\omega\text{rect}(\omega/4)$
   c. $2\cos(2\omega t)$
   d. $2\pi\omega\text{rect}(4\omega)$
   e. None of the above

$\frac{\beta}{\pi}\,\text{Sinc}(\beta t) \leftrightarrow \text{rect}\left(\frac{w}{2\beta}\right)$

$\beta\,\text{Sinc}(\beta t) \leftrightarrow \pi\,\text{rect}\left(\frac{w}{2\beta}\right)$, $\beta = 2$

For the derivative we multiply by jw

$2\,\text{Sinc}(2t) \leftrightarrow jw\pi\,\text{rect}\left(\frac{w}{4}\right)$

5. The total energy of the signal $x(t) = 2\text{sinc}(2t)$ is: *(hint: use Parseval's Theorem)*

   a. 2π
   b. 0
   c. 4
   d. Infinite
   e. Not defined

Energy is area under square of the curve.

$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(w)|^2\, dw = \frac{\pi^2}{2\pi}\int_{-\infty}^{\infty} \text{rect}^2\left(\frac{w}{4}\right) dw$

$= \frac{\pi^2(4)}{2\pi} = \boxed{2\pi}$

-2 2

6. When comparing Laplace transform with Fourier transform, which of the following is **not** true:
   a. More functions are Laplace transformable than Fourier transformable.
   b. Fourier transform is more appropriate for circuits with switches and initial conditions.
   c. The transfer function $H(\omega)$ can be found from $H(s)$ by setting $s = j\omega$.
   d. Fourier transform is a special case of Laplace transform.
   e. Fourier uses the angular frequency, ω, while Laplace uses the complex frequency,s.

7. The Laplace transform of $f(t) = (1 - 4e^{-2t})\delta(t)$ $= -3\delta(t)$ by sifting property
   a. 0
   b. $-3\delta(t)$
   c. -3
   d. $1 - \frac{4}{s+2}$
   e. $\frac{1}{s} - \frac{4}{s+2}$

$\delta(t) \leftrightarrow 1$

$\Rightarrow -3\delta(t) \leftrightarrow -3$

8. The inverse Laplace Transform of $X(s) = \frac{8}{s^2+9s+20}$ is given by:

   a. $4e^{-4t}u(t-4) + 5e^{-5t}u(t-5)$
   b. $8e^{-4t}u(t) - 8e^{-5t}u(t)$ ✓
   c. $4e^{4t}u(t) - 5e^{5t}u(t)$
   d. $8e^{-4t}\cos(4t)u(t) - 8e^{-5t}\sin(5t)u(t)$
   e. None of the above

$X(s) = \frac{8}{(s+5)(s+4)} = \frac{A_1}{s+5} + \frac{A_2}{s+4}$

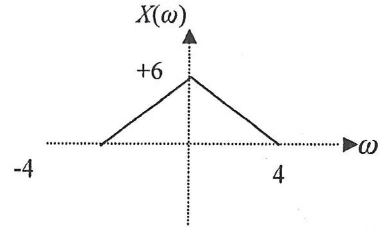
$A_1 = \left.\frac{8}{s+4}\right|_{s=-5} = -8$, $A_2 = \left.\frac{8}{s+5}\right|_{s=-4} = 8$

$X(s) = \frac{-8}{s+5} + \frac{8}{s+4} \Rightarrow x(t) = (-8e^{-5t} + 8e^{-4t})\,u(t)$

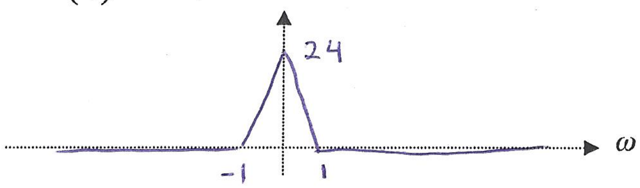
Problem 2: [14pts]

Part A [7pts]:

A signal $x(t)$, has the spectrum shown in the figure, Sketch the spectrum of the following signals. *Show all important values on both amplitude and frequency axes.*



I. $x\left(\frac{t}{4}\right)$



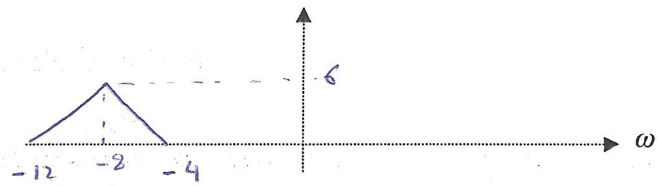
$f(at) \leftrightarrow \frac{1}{|a|} F\left(\frac{\omega}{a}\right)$

$f\left(\frac{t}{4}\right) \leftrightarrow 4 \cdot F(4\omega)$

amplitude scale by 4

Compress by 4.

II. $x(t)e^{-j8t}$

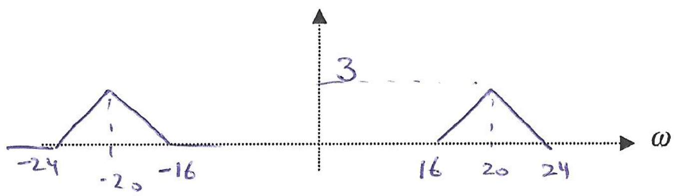


Frequency shift

$f(t)\, e^{j\omega_0 t} \leftrightarrow F(\omega-\omega_0)$

$x(t)\, e^{-j8t} \leftrightarrow F(\omega+8)$

III. $x(t)\cos(20t)$

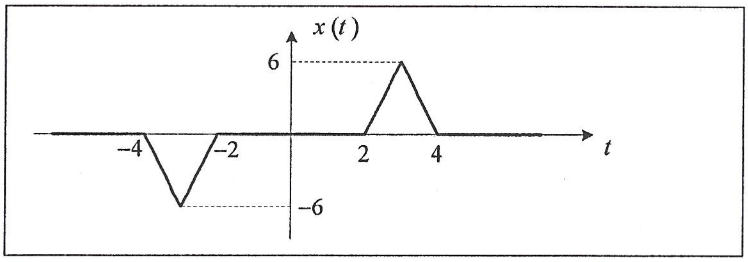


$\cos(20t) = \frac{1}{2}\left[e^{j20t} + e^{-j20t}\right]$

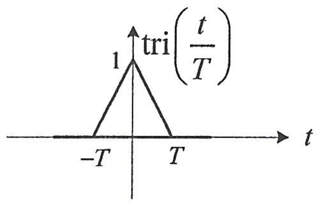
two frequency shift & amplitude adjust by $\frac{1}{2}$

Part B [7pts]:

Find the Fourier transform of the signal $x(t)$ shown in the figure. Express your answer in simplest form in terms of sinc functions



Hint: use the FT table with the definition of the triangular function given below.



$$x(t) = -6\,\mathrm{tri}\left(\frac{t+3}{1}\right) + 6\,\mathrm{tri}\left(\frac{t-3}{1}\right)$$

$$\mathrm{tri}\left(\frac{t}{T}\right) \longleftrightarrow T\,\mathrm{sinc}^2\left(\frac{T\omega}{2}\right)$$

$$\mathrm{tri}(t) \longleftrightarrow \mathrm{sinc}^2\left(\frac{\omega}{2}\right)$$

Time Transformation.

$$f(at - t_0) \longleftrightarrow \frac{1}{|a|} F\left(\frac{\omega}{a}\right) e^{-j\omega t_0/a}$$

$$f(1t+3) \longleftrightarrow F(\omega)\, e^{j\omega 3}$$

$$f(1t-3) \longleftrightarrow F(\omega)\, e^{-j\omega 3}$$

$X(\omega)$

$$-6\,\mathrm{sinc}^2\left(\frac{\omega}{2}\right) e^{j\omega 3} + 6\,\mathrm{sinc}^2\left(\frac{\omega}{2}\right) e^{-j\omega 3}$$

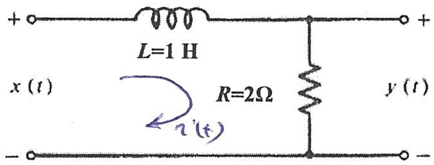
$$-12j\left[\mathrm{sinc}^2\left(\frac{\omega}{2}\right)\right]\left[\frac{e^{j\omega 3} - e^{-j\omega 3}}{2j}\right]$$

$$\boxed{X(\omega) = -12 \cdot j\left[\mathrm{sinc}^2\left(\frac{\omega}{2}\right)\right] \sin(3\omega)}$$

$$\boxed{X(\omega = -j\,36\omega\;\mathrm{sinc}^2\left(\frac{\omega}{2}\right)\;\mathrm{sinc}(3\omega)}$$

$$= \frac{36\omega}{j}\;\mathrm{sinc}^2\left(\frac{\omega}{2}\right)\;\mathrm{sinc}(3\omega)$$

Problem 3: [10pts]



a. Write the differential equation relating $y(t)$ to $x(t)$ for the circuit shown in the figure.

$$x(t) = L\,\frac{d\,i(t)}{dt} + R\,i(t) = \frac{d\,i(t)}{dt} + 2\,i(t) \quad (1)$$

$$y(t) = 2\,i(t) \quad (2) \Rightarrow i(t) = \frac{y(t)}{2} \qquad \text{Substitute (2) in (1)}$$

$$x(t) = \frac{1}{2}\frac{d\,y(t)}{dt} + y(t) \quad \Rightarrow \quad \frac{dy(t)}{dt} + 2y(t) = 2x(t)$$

b. Rewrite the equation in the s-domain (Laplace transform)

$$s\,Y(s) - y(0^+) + 2\,Y(s) = 2\,X(s)$$

c. Assuming the initial values are zeros, find the transfer function $H(s) = \dfrac{Y(s)}{X(s)}$

$$y(0^+) = 0$$

$$sY(s) + 2\,Y(s) = 2\,X(s)$$

$$Y(s)\,(s+2) = 2\,X(s) \Rightarrow \frac{Y(s)}{X(s)} = \boxed{\frac{2}{s+2} = H(s)}$$

d. If the input is $x(t) = 5e^{-2t}u(t)$, find the output, $y(t)$

Using Laplace transform.

$$X(s) = \frac{5}{s+2}$$

$$Y(s) = X(s)\,H(s) = \frac{5}{s+2}\;\frac{2}{s+2} = \frac{10}{(s+2)^2}$$

Directly from the table.

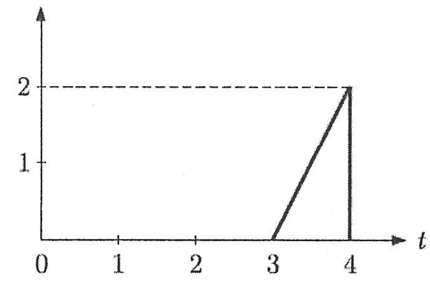
$$t\,e^{-at} \longleftrightarrow \frac{1}{(s+a)^2}$$

$$\Rightarrow \quad t\,e^{-2t} \longleftrightarrow \frac{1}{(s+2)^2}$$

$$10\,t\,e^{-2t} \longleftrightarrow \frac{10}{(s+2)^2} \qquad \Rightarrow \boxed{y(t) = 10\,t\,e^{-2t}\,u(t)}$$

Problem 4: [10pts]

a) Find the Laplace transform of the signal $f(t)$ shown in the figure (Hint: express $f(t)$ with ramp and step functions) **[5pts]**



$$f(t) = 2(t-3)u(t-3) - 2(t-4)u(t-4)$$
$$- 2u(t-4)$$
$$= 2r(t-3) - 2r(t-4) - 2u(t-4)$$

Laplace Transform with shift property

$$\mathcal{L}[f(t-t_0)u(t-t_0)] = e^{-t_0 s}F(s)$$

$$r(t) \leftrightarrow \frac{1}{s^2}$$

$$u(t) \leftrightarrow \frac{1}{s}$$

$$F(s) = \frac{2e^{-3s}}{s^2} - \frac{2e^{-4s}}{s^2} - \frac{2e^{-4s}}{s}$$

$$= \frac{2}{s}\left[\frac{e^{-3s}}{s} - \frac{2e^{-4s}}{s} - e^{-4s}\right]$$

b) Use a Laplace Transform approach tofind the convolution of $x(t) = tu(t)$ with $h(t) = -e^{2t}u(t)$. **[5pts]**

$$X(s) = \frac{1}{s^2} \quad \text{from the table.} \quad tu(t) \leftrightarrow \frac{1}{s^2}$$

$$H(s) = \frac{-1}{s\ 2} \quad \text{from the table} \quad e^{-at}u(t) \leftrightarrow \frac{1}{s+a}$$

$$Y(s) = X(s)\,H(s) = \frac{-1}{s^2(s-2)}$$

$$Y(s) = \frac{A}{s-2} + \frac{B_1}{s^2} + \frac{B_2}{s}$$

$$A = \left.\frac{-1}{s^2}\right|_{s=2} = -\frac{1}{4}, \quad B_1 = \left.-\frac{1}{s-2}\right|_{s=0} = \frac{1}{2}$$

$$B_2 = \frac{d}{ds}\left[-(s-2)^{-1}\right]\Big|_{s=0} = (s-2)^{-2}\Big|_{s=0} = (-2)^{-2} = \frac{1}{(-2)^2} = \frac{1}{4}$$

$$Y(s) = \frac{-1/4}{s-2} + \frac{1/2}{s^2} + \frac{1/4}{s}$$

to check $Y(s) = \dfrac{\cancel{-\frac{1}{4}s^2} + \cancel{\frac{1}{2}s} - 1 + \cancel{\frac{1}{4}s^2} - \cancel{\frac{s}{2}}}{s^2(s-2)}$ ✓

$$\boxed{y(t) = \left[-\frac{1}{4}e^{2t} + \frac{1}{2}t + \frac{1}{4}\right]u(t)}$$