بسم الله الرحمن الرحيم

King Fahd University of Petroleum & Minerals

Department of Electrical Engineering
EE 207 Signals and Systems
First Semester (111)

Exam I
Wednesday, 19 October 2011
7:00 pm – 8:30 pm

Name: ________________________________

ID: ________________________________

Instructors:

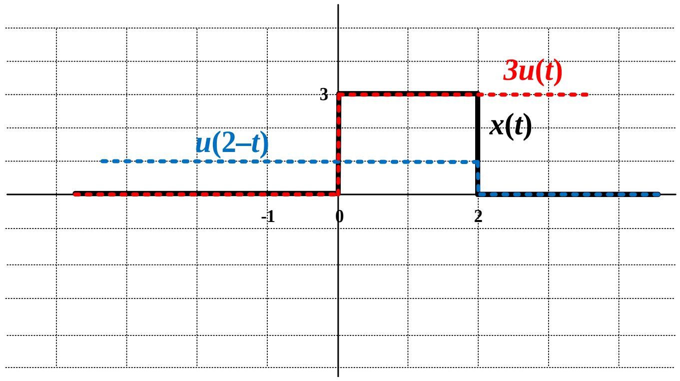
Dr. Khurram Qureshi	**(Sections 1 & 5)**
Dr. Husain Al-Jamed	**(Section 2)**
Dr. Ali Muqaibel	**(Section 3)**
Dr. Wajih Abu-Al-Saud	**(Section 4)**

Problem	Score	Out of
1		56
2		20
3		24
Total		100

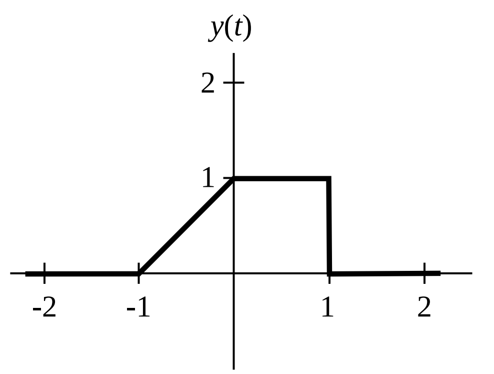
Good luck!

Problem 1:

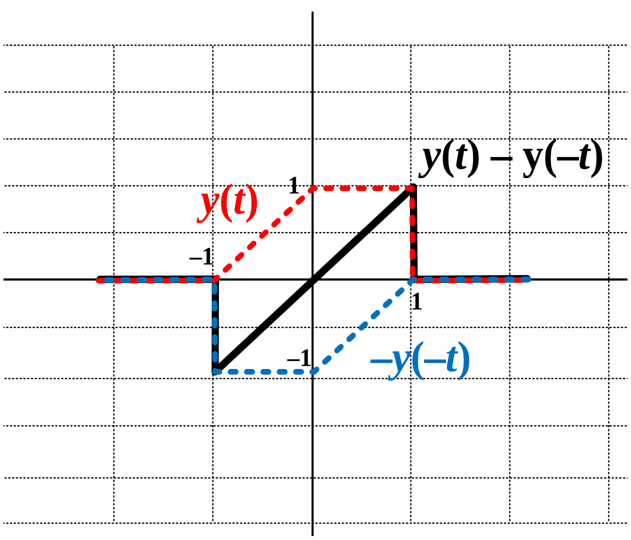
a) Sketch the signal $x(t) = 3u(t) \cdot u(2-t)$ showing all important values on both axes of the sketch.



b) Given the function $y(t)$ shown to the right:



Sketch the function $y(t) - y(-t)$ showing all important values on both axes of the sketch.



c) The step response of a linear time-invarient system is given by

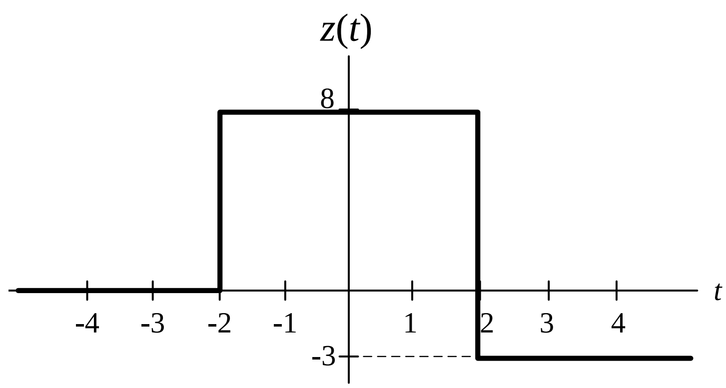
$$a(t) = \left[2 + 3e^{-4(t+1)}\right] \cdot u\left(t+1\right)$$

i) Is the system causal or non-causal?
ii) Find the impulse response of the system in the simplest possible form.

$$h(t) = \frac{da(t)}{dt} = \frac{d}{dt}\left\{\left[2 + 3e^{-4(t+1)}\right] \cdot u\left(t+1\right)\right\}$$
$$= \left[-12e^{-4(t+1)}\right] \cdot u\left(t+1\right) + \left[2 + 3e^{-4(t+1)}\right] \cdot \delta\left(t+1\right) \Rightarrow$$
$$= \left[-12e^{-4(t+1)}\right] \cdot u\left(t+1\right) + 5 \cdot \delta\left(t+1\right)$$

Since $h(t)$ is not zero for some values of $t < 0$ ➔ System is Non-Causal

d) Consider the signal $z(t)$ shown to the right:



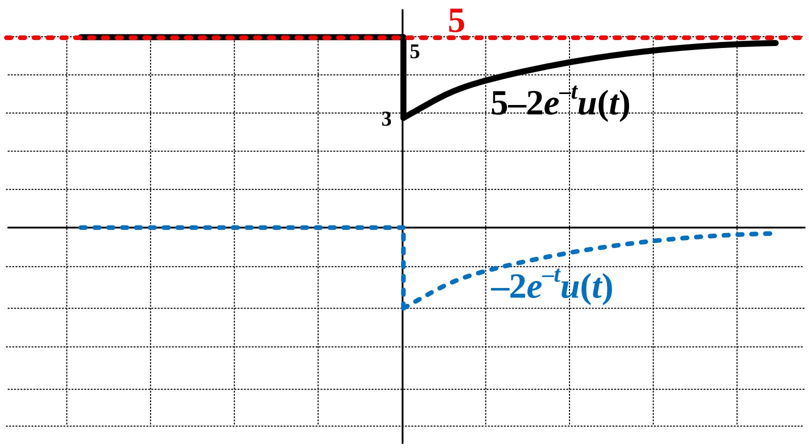
i) Find both the **total energy** of $z(t)$ and its **average power**.
ii) Is $z(t)$ an energy siganl, power signal, or neither energy nor power signal?

$$E = \lim_{T\to\infty} \int_{-T}^{T} z^2(t)dt$$
$$= \lim_{T\to\infty}\left[\int_{-2}^{2}(8)^2 dt + \int_{2}^{T}(-3)^2 dt\right]$$
$$= \lim_{T\to\infty}\left[64t\Big|_{-2}^{2} + 9t\Big|_{2}^{T}\right]$$
$$= \lim_{T\to\infty}\left[64(2+2) + 9(T-2)\right]$$
$$= \lim_{T\to\infty}(9T) = \infty$$

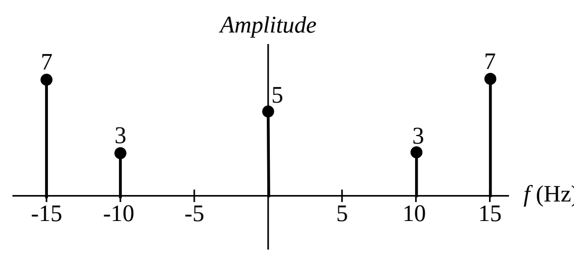
$$P=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}z^2(t)dt$$
$$=\lim_{T\to\infty}\frac{1}{2T}\left[\int_{-2}^{2}(8)^2\,dt+\int_{2}^{T}(-3)^2\,dt\right]$$
$$=\lim_{T\to\infty}\frac{1}{2T}\left[64t\Big|_{-2}^{2}+9t\Big|_{2}^{T}\right]$$
$$=\lim_{T\to\infty}\frac{1}{2T}\left[64(2+2)+9(T-2)\right]$$
$$=\lim_{T\to\infty}\left(\frac{9T}{2T}\right)=\frac{9}{2}$$

$z(t)$ is a POWER signal.

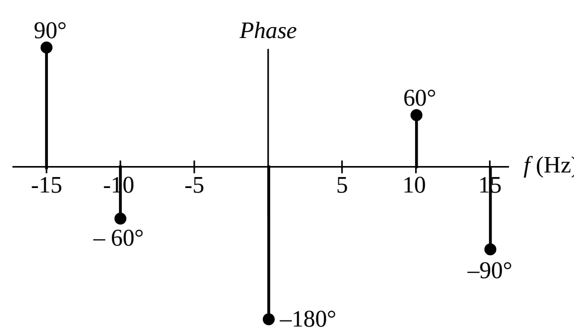
e) Sketch the signal $f(t)=5-2e^{-t}u(t)$ showing all important values on both axes of the sketch.



f) The double-sided amplitude and phase spectra of a siganl $x(t)$ are shown below. Write an expression for $x(t)$.



$$x(t)=-5+6\cos(2\pi\cdot10\cdot t+60°)+14\cos(2\pi\cdot15\cdot t-90°)$$
$$=-5+6\cos(2\pi\cdot10\cdot t+60°)+14\sin(2\pi\cdot15\cdot t)$$



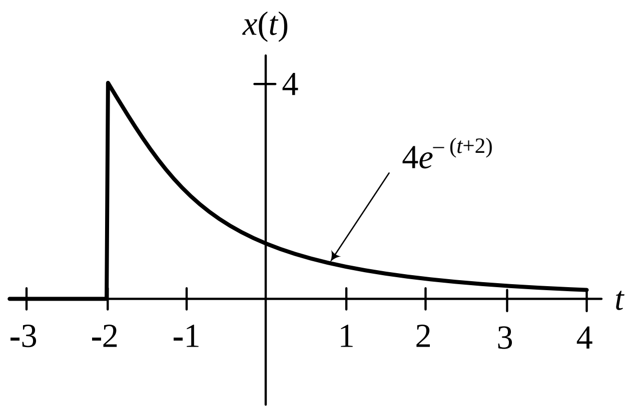
g) The signal $x(t)$ is given as: $x(t) = 5\int_{-2}^{3} \sin(\pi t)\delta\left(t - \frac{1}{2}\right)dt + \int_{-5}^{5} t^2 \dot{\delta}(t+1)dt$, where $\delta(t)$ is the impulse function and $\dot{\delta}(t)$ is the first deravitive of the impulse function.

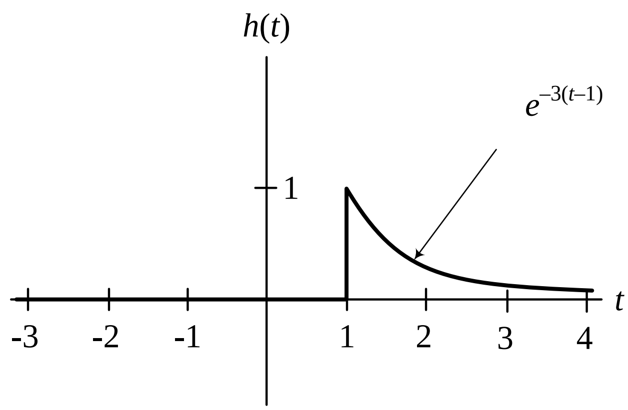
The numerical value of $x(t)$ will be (circle the correct answer):

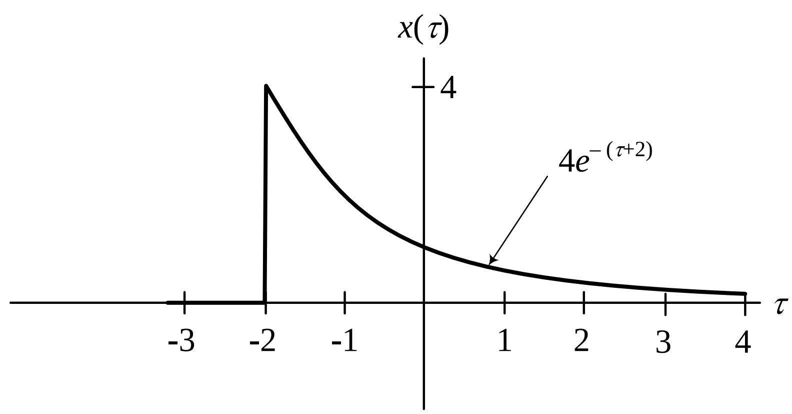
i) –2 ii) 0 iii) 3 iv) 5 v) 7

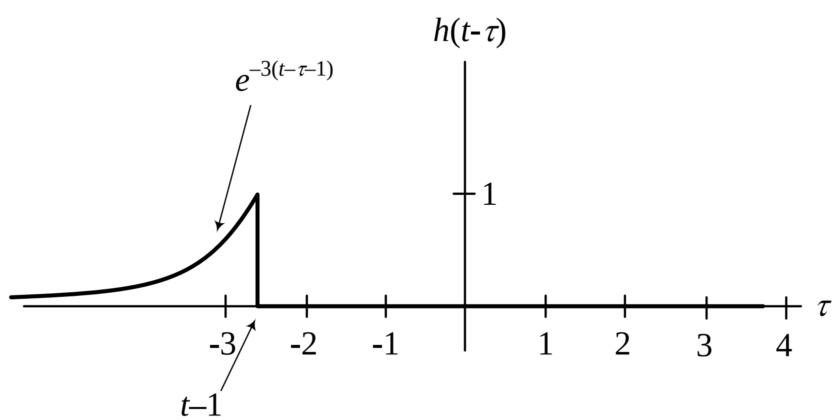
Problem 2:

Given $x(t)$ and $h(t)$ shown below, find the convolution $y(t) = x(t) * h(t)$ for all values of t in interval form.









For $t - 1 < -2$ ➔ $t < -1$

$$y(t) = 0$$

For $t - 1 < -2$ ➔ $t < -1$

$$
\begin{aligned}
y(t) &= \int_{-2}^{t-1} \left(4e^{-(\tau+2)}\right) \cdot \left(e^{-3(t-\tau-1)}\right) d\tau \\
&= 4\int_{-2}^{t-1} \left(e^{-(\tau+2)} \cdot e^{-3(t-\tau-1)}\right) d\tau \\
&= 4\int_{-2}^{t-1} \left(e^{-\tau} \cdot e^{-2} \cdot e^{-3t} \cdot e^{+3\tau} \cdot e^{+3}\right) d\tau \\
&= 4e^{-3t+1} \int_{-2}^{t-1} e^{2\tau} d\tau \\
&= \frac{4e^{-3t+1}}{2} e^{2\tau} \Big|_{-2}^{t-1} \\
&= 2e^{-3t+1}\left(e^{2(t-1)} - e^{-4}\right) \\
&= 2e^{-3t+1} \cdot e^{2t-2} - 2e^{-3t+1} \cdot e^{-4} \\
&= 2e^{-t-1} - 2e^{-3t-3}
\end{aligned}
$$

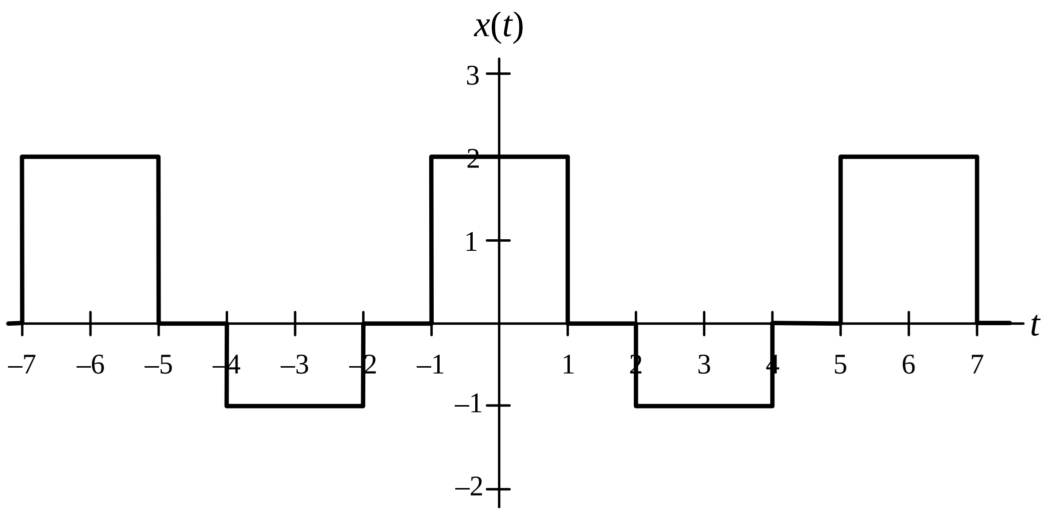
So,

$$
y(t) = \begin{cases} 0, & t < -1 \\ 2e^{-t-1} - 2e^{-3t-3}, & t > -1 \end{cases}
$$

Problem 3:

For the periodic signal $x(t)$ shown below, find the following trigonometric Fourier series coefficients:

i) a_0
ii) a_4
iii) b_3



$T_0 = 6$ seconds ➔ $\omega_0 = 2\pi / 6$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$$

$$= \frac{1}{T_0}\int_{-1}^{5} x(t)dt$$

$$= \frac{1}{6}\left[\int_{-1}^{1} 2dt + \int_{2}^{4} -1dt\right]$$

$$= \frac{2t}{6}\Big|_{-1}^{1} - \frac{t}{6}\Big|_{2}^{4}$$

$$= \frac{2(1-(-1))}{6} - \frac{(4-2)}{6}$$

$$= \frac{4}{6} - \frac{2}{6}$$

$$= \frac{1}{3}$$

$$a_4 = \frac{2}{T_0}\int_{T_0} x(t)\cdot\cos(4\omega_0 t)dt$$

$$= \frac{2}{T_0}\int_{-1}^{5} x(t)\cdot\cos(4\omega_0 t)dt$$

$$= \frac{2}{6}\left[\int_{-1}^{1} 2\cos\left(\frac{4\pi}{3}t\right)dt + \int_{2}^{4} -\cos\left(\frac{4\pi}{3}t\right)dt\right]$$

$$= \frac{4\cdot 3}{6\cdot 4\pi}\sin\left(\frac{4\pi}{3}t\right)\Bigg|_{-1}^{1} - \frac{2\cdot 3}{6\cdot 4\pi}\sin\left(\frac{4\pi}{3}t\right)\Bigg|_{2}^{4}$$

$$= \frac{1}{2\pi}\left[\sin\left(\frac{4\pi}{3}\right)+\sin\left(\frac{4\pi}{3}\right)\right] - \frac{1}{4\pi}\left[\sin\left(\frac{16\pi}{3}\right)-\sin\left(\frac{8\pi}{3}\right)\right]$$

$$= -0.137832$$

$b_3 = 0$ Because $x(t)$ is an even function.