Serial #

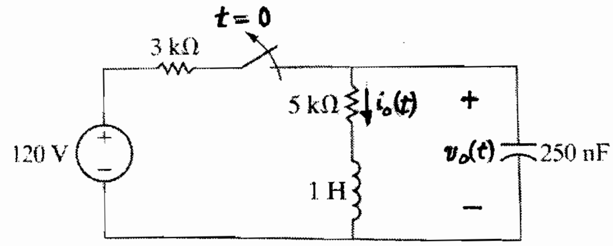
0

- 1 points for not writing your serial number

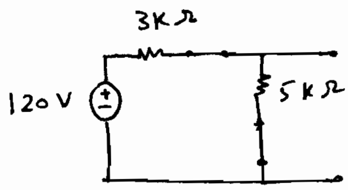
King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE205: Electric Circuit II (082)
Quiz 3: Natural and Step Responses of RLC Circuits

Name: KEY Ver.1 & 2 ← Two Quizes.

The switch in the circuit shown has been closed for a long time. The switch opens at t=0. Find $v_o(t)$ and $i_o(t)$ for $t \geq 0$.



$t<0$



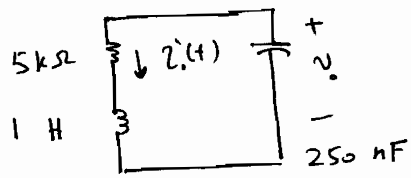
$i_o(0^-) = \frac{120}{(3+5)\,k\Omega} = \boxed{15\ mA}$

$v_o(0^-) = 120 \cdot \frac{5}{3+5} = \boxed{75\ V}$ (2)

$\frac{di}{dt}(0^+) = \frac{1}{L}\left(75 - 5k\,(15m)\right)$

$= 75 - 75 = \boxed{0\ A/sec}$

for $t \geq 0$



Series RLC Circuit

$i_c = C\frac{dv_o}{dt} = -i_o$

$\Rightarrow \frac{dv_o}{dt} = -\frac{i_o}{C}$ (2)

$\frac{dv_o}{dt}(0^+) = \frac{-i_o(0^+)}{C} = \frac{-15m}{250n} =$

$\boxed{= -60\,000\ V/sec.}$

By KVL

$-v_o + 5k\,i_o + L\frac{di}{dt} = 0$

$L\frac{di(0^+)}{dt} = v_o(0^+) - 5k\,i_o(0^+)$

$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{5\times10^{-4}} = 2000$ (1)

$\alpha = \frac{R}{2L} = \frac{5k}{2} = 2500$ (1)

$\alpha > \omega_o \Rightarrow$ over damped response.

$v_o = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ (1)

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2} = -2500 \pm \sqrt{6250000 - 4\times10^6}$ (1)

$= -2500 \pm 1500$

$s_1 = -4000, \quad s_2 = -1000$

from the initial condition

$75 = A_1 + A_2$ — (1)

$-60000 = -4000A_1 - 1000A_2$

$-60 = -4A_1 - A_2$ — (2)

add (1) & (2) (2)

$15 = -3A_1 \Rightarrow \boxed{A_1 = -5}$ & $\boxed{A_2 = 80}$

$\Rightarrow \boxed{v_o(t) = -5e^{-4000t} + 80e^{-1000t}\ V \quad t \geq 0}$

check i.c. $v_o(0) = -45 + 120 = 75$

$i_o = -C\frac{dv_o}{dt} = -250n\left[(-5)(-4000)e^{-4000t} + (80)(-1000)e^{-1000t}\right]$

$\boxed{i_o(t) = -5e^{-4000t} + 20e^{-1000t}\ mA \quad t \geq 0}$

Good Luck, **Dr. Ali Muqaibel**

Anoter solution "Same answer"
if we start with the current.

$$i_o'(t) = A_3 e^{-4000t} + A_4 e^{-1000t}$$

$$i'(0) = A_3 + A_4 = 15\,mA \quad \text{— (1)}$$

$$0 = -4000 A_3 - 1000 A_4 \Rightarrow$$

$$\boxed{A_4 = -4A_3} \quad \text{— (2)}$$

substitute (2) in (1)

$$A_3 - 4A_3 = 15\,mA$$

$$\Rightarrow \quad -3A_3 = 15\,mA$$

$$\Rightarrow \quad \boxed{A_3 = -5\,mA}$$

$$\boxed{A_4 = +20\,mA}$$

Same answer:

$$i_o(t) = -5 e^{-4000t} + 20 e^{-1000t} \quad mA$$

$$t \geq 0$$