KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

Electrical Engineering Department

EE-205 Electric Circuits II

Spring 2009/2010(092) Second Major Exam

Duration : 90 min.

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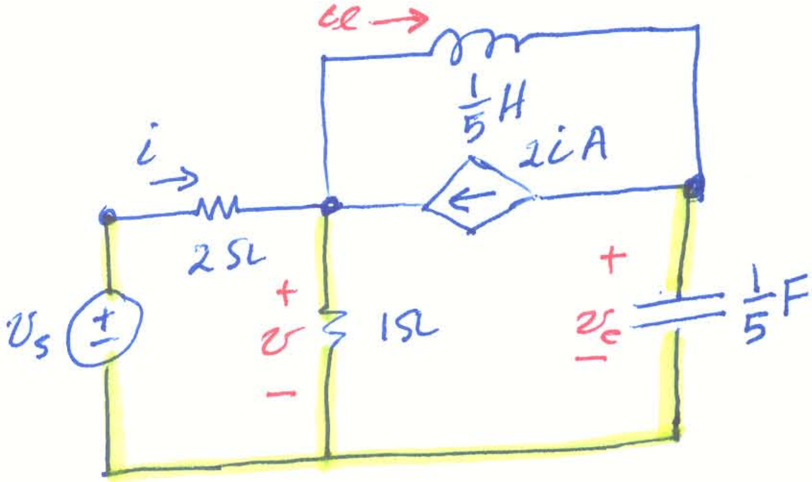
Question	Grade
1 (10 points)	10
2 (10 points)	10
3 (10 points)	10
Total (30 points)	30

Notes :
1) Read the question very carefully.
2) Use a sketch to help you understand the question.
3) Write neatly.

Q1(a)

For the circuit shown, write the matrix state equation



KVL for L (FL)

$$\frac{1}{5}\frac{di_l}{dt} + v_c - v = 0 \quad (1)$$

KCL for c (FCS)

$$\frac{1}{5}\frac{dv_c}{dt} + 2i - i_l = 0 \quad (2)$$

KVL for 2Ω (FL)

$$2i + v - v_s = 0 \quad (3) \Rightarrow v = v_s - 2i$$

State variables are v_c and i_l

KCL for 1Ω (FCS)

$$v + i_l - i - 2i = 0 \quad (4)$$

(3) in (4)

$$v_s - 2i + i_l - 3i = 0$$

$$-5i = -v_s - i_l$$

$$i = \frac{v_s + i_l}{5} \quad (a)$$

(a) in (3)

$$v = v_s - 2\left(\frac{v_s + i_l}{5}\right)$$

$$v = v_s - \frac{2v_s}{5} - \frac{2i_l}{5}$$

$$v = \frac{3v_s}{5} - \frac{2i_l}{5} \quad (b)$$

(a) and (b) in (1) and (2)

$$\frac{di_l}{dt} = -2i_l - 5v_c + 3v_s \quad \mathrm{I}$$

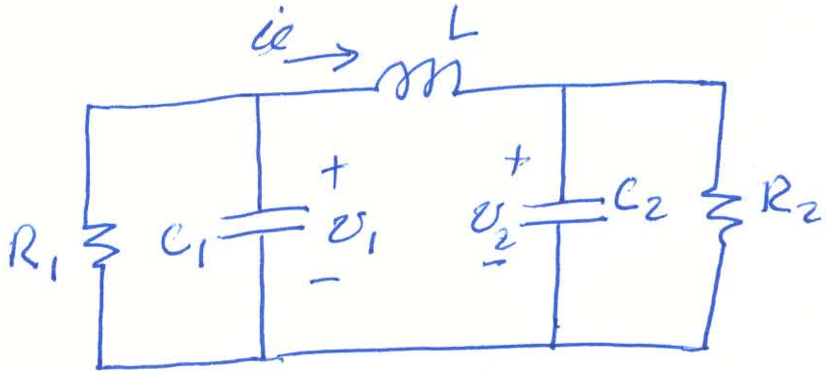
$$\frac{dv_c}{dt} = 3i_l - 2v_s \quad \mathrm{II}$$

$$\frac{d}{dt}\begin{bmatrix} i_l \\ v_c \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} i_l \\ v_c \end{bmatrix} + \begin{bmatrix} 3 \\ -2 \end{bmatrix} v_s$$

Q1 (b) For the circuit shown, the state equations of the circuit is given below. Given that $R_1 = R_2 = 2\Omega$ and $C_1 = \frac{1}{2}F$, $C_2 = \frac{1}{2}F$ and $L = 1H$, assume that $i_\ell(0) = 1A$, $v_1(0) = 2V$ and $v_2 = 0V$.

i) Write the state equations in their numerical form using the numerical technique - (Euler's Method)

ii) Find the approximate values of i_ℓ, v_1 and v_2 at $t = 0.1s$, $t = 0.2s$ and $t = 0.3s$



$$\frac{d}{dt}\begin{bmatrix} i_\ell \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} & -\frac{1}{L} \\ -\frac{1}{C_1} & -\frac{1}{R_1C_1} & 0 \\ \frac{1}{C_2} & 0 & \frac{1}{R_2C_2} \end{bmatrix}\begin{bmatrix} i_\ell \\ v_1 \\ v_2 \end{bmatrix}$$

for the given values

$$\frac{d}{dt}\begin{bmatrix} i_\ell \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -2 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_\ell \\ v_1 \\ v_2 \end{bmatrix}$$

K	t	$i_\ell(t)$	$v_1(t)$	$v_2(t)$		
0	0.1					
1	0.2					
2	0.3					

Q1 (b) Sol.

i) $i_\ell([K+1]\Delta t) \approx i_\ell(K\Delta t) + [v_1(K\Delta t) - v_2(K\Delta t)]\Delta t$

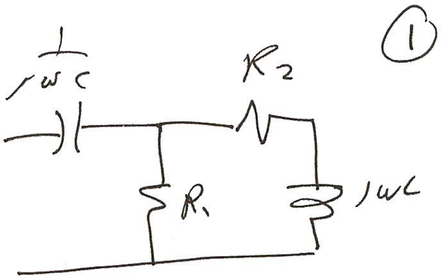
$v_1([K+1]\Delta t) \approx v_1(K\Delta t) + [-2\, i_\ell(K\Delta t) - v_1(K\Delta t)]\Delta t$

$v_2([K+1]\Delta t) \approx v_2(K\Delta t) + [2\, i_\ell(K\Delta t) + v_2(K\Delta t)]\Delta t$

ii)

K	t (s)	$i_\ell(t)$	$v_1(t)$	$v_2(t)$
0	0.1	1.2	1.6	0.2
1	0.2	1.34	1.2	0.46
2	0.3	1.414	0.812	0.774

Find ~~$H(j\omega)$~~ ω_r for the Circuit shown



$$Z_{in} = \frac{(R_2 + j\omega L)R_1}{R_1 + R_2 + j\omega L} + \frac{1}{j\omega C}$$

$$= \frac{R_1(R_2 + j\omega L)(R_1 + R_2 - j\omega L)}{(R_1+R_2)^2 + \omega^2 L^2} + \frac{1}{j\omega C}$$

$$= \frac{R_1}{(R_1+R_2)^2 + \omega^2 L^2}\left[j\omega L(R_1+R_2) - j\omega L R_2\right] + \frac{1}{j\omega C}$$

$$= \frac{j\omega L R_1^2}{(R_1+R_2)^2 + \omega^2 L^2} = +\frac{j}{\omega C}$$

$$\therefore \quad \omega^2 L C R_1^2 = \omega^2 L^2 + (R_1+R_2)^2$$

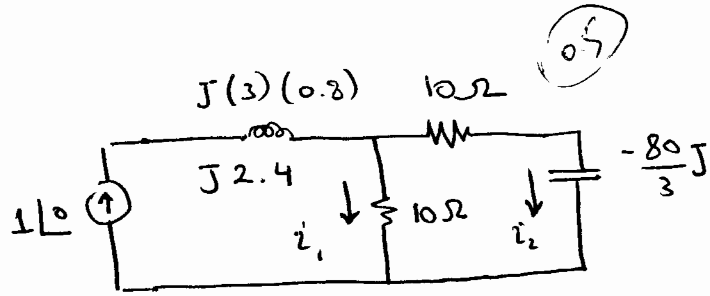
$$\omega^2 (LCR_1^2 - L^2) = (R_1+R_2)^2$$

$$\omega_r = \frac{R_1 + R_2}{\sqrt{LCR_1^2 - L^2}} = \frac{1}{\sqrt{LC}} \cdot \left(\frac{R_1+R_2}{\sqrt{R_1^2 - L/C}}\right)$$

if $R_1 \to \infty \quad \to \omega_r \simeq \frac{1}{\sqrt{LC}}$

$w_r = 3$ rad/sec

apply a test current source



$$Q = \frac{2\pi \left[W_L(t) + W_c(t)\right]_{max}}{P_R T}$$ 0.5

$i_{source}(t) = \cos 3t$

$W_L(t) = \frac{1}{2} L \, i^2_{source}(t)$ 0.5

$= \frac{1}{2}(0.8)\, i^2_{source}(t)$

$= 0.4 \cos^2 3t$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2}{3}\pi$$ 0.5

By CDR

$$\vec{I}_1 = 1\angle 0 \; \frac{10 - \frac{80}{3}J}{20 - \frac{80}{3}J}$$

$= 0.82 - J0.24 = 0.8544 \angle -16.314^\circ$

0.5

$P_{R_1} = \frac{1}{2}(10)|I_1|^2 = \frac{1}{2} R_1 |I_1|^2 = 3.65$

$R_{R_2} = \frac{1}{2}(10)|I_2|^2 = 0.45$ 0.5

$$\vec{I}_2 = 1\angle 0 \; \frac{10}{20 - \frac{80}{3}J}$$ 1

or $\vec{I}_2 = 1\angle 0 - \vec{I}_1$

$\vec{I}_2 = 0.18 + J0.2 = 0.3\angle 53.13^\circ$

$\vec{V}_c = \vec{I}_2 \; \frac{-80}{3} J = 8 \angle -36.87^\circ$ 1

$$Q = \frac{\cancel{2\pi}\left[0.4\cos^2 3t + W_c(t)\right]_{max}}{\frac{2}{3}\cancel{\pi}\,(3.65 + 0.45)}$$

$= 0.7317 \left[W_L(t) + W_c(t)\right]_{max}$

$= (0.7317)(0.4)\left[\cos^2 3t + \cos^2(3t - 36.87^\circ)\right]$

$= 0.2927 \left[\quad 1.8 \quad\right]$

$= 0.53$ 0.5

$W_c(t) = \frac{1}{2} C \, V_c^2(t)$

$= \frac{1}{2}\,\frac{1}{80}\, V_c^2(t)$

$= \frac{1}{160}(8)^2 \cos^2(3t - 36.87^\circ)$

$= 0.4 \cos^2(3t - 36.87^\circ)$

0.5

How to find the maximum ?

$\cos^2 3t + \cos^2(3t - 36.87^\circ)$

$\frac{1}{2}[1 + \cos 6t] + \frac{1}{2}[1 + \cos(6t + 73.74^\circ)]$ 1

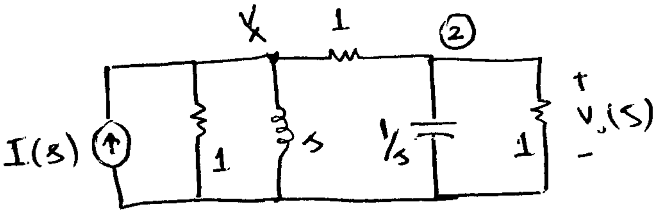
$1 + \frac{\cos 6t + \cos(6t + 73.74^\circ)}{2}$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= 0.28\cos 6t + 0.96 \sin 6t$

$1 + \frac{1.28\cos 6t + 0.96\sin 6t}{2}$

max $\quad 1 + \frac{1}{2}\sqrt{(1.28)^2 + (0.96)^2} = 1.8$



assign the voltage V_x as shown

By KCL @ node V_x

$$I = \frac{V_x}{1} + \frac{V_x}{s} + \frac{V_x - V_o}{1}$$

$$I + V_o = V_x + \frac{V_x}{s} + V_x = \left(2 + \frac{1}{s}\right) V_x = \left(\frac{2s+1}{s}\right) V_x$$

$$V_x = (I + V_o)\left(\frac{s}{2s+1}\right) \quad \text{——} \ 1$$

By KCL @ node 2

$$\frac{V_x - V_o}{1} = \frac{V_o}{1/s} + \frac{V_o}{1}$$

$$V_x - V_o = sV_o + V_o \Rightarrow V_x = (s+2)V_o \quad \text{——} \ 2$$

By Equating 1 to 2 to Eliminate V_x

$$(s+2)V_o = I\frac{s}{(2s+1)} + V_o\frac{s}{(2s+1)}$$

$$\left(s+2 - \frac{s}{2s+1}\right)V_o = I\frac{s}{2s+1}$$, multiply both sides by $(2s+1)$

$$\left[(2s+1)(s+2) - s\right]V_o = sI$$

$$H(s) = \frac{V}{I} = \frac{s}{(2s+1)(s+2) - s} = \frac{s}{2s^2+s+4s+2-s} = \boxed{\frac{s}{2s^2+4s+2}}$$

I) Given the following Transfer function

under damped

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{2(s+1)}{s^2+s+2} = \frac{2(s+1)}{s^2 + s + 2}$$

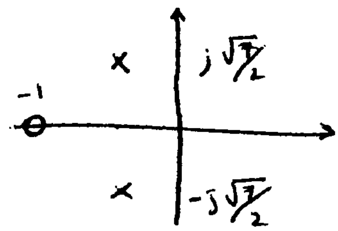
a) Sketch the pole zero plot

1

zeros. $s = -1$

poles

0.5



0.5

$$\frac{1^2 \pm \sqrt{1 - 8}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

The response is underdamped because the poles are two complex conjugate

b) if the input signal is $v_1(t) = 10\, e^{-t} \cos(t + \frac{\pi}{2})$

find the output voltage

1 $s = -1 + j$ $\quad \vec{V}_1 = 10 \angle \pi/2 = 10 \angle 90^\circ$

1 $\vec{V}_2 = H(s)\Big|_{s=-1+j} 10\angle 90^\circ = (-1+j)\ 10\angle 90^\circ = \sqrt{2}\angle 135^\circ\ 10\angle 90^\circ$

$= 14.14 \angle 225^\circ$ or $\angle -135^\circ$

$V_2(t) = 14.14\, e^{-t} \cos(t + 225^\circ)$

c) if the input is now $v_1(t) = \sqrt{19}\, e^{-\frac{1}{2}t} \cos(\frac{\sqrt{7}}{2}t + 109^\circ)$

what would be the output voltage.

Justify your answer.

1 $s = -\frac{1}{2} + j\frac{\sqrt{7}}{2}$ this the location where

the transfer function has a pole

the output is Expected to be ∞ (theoritically).