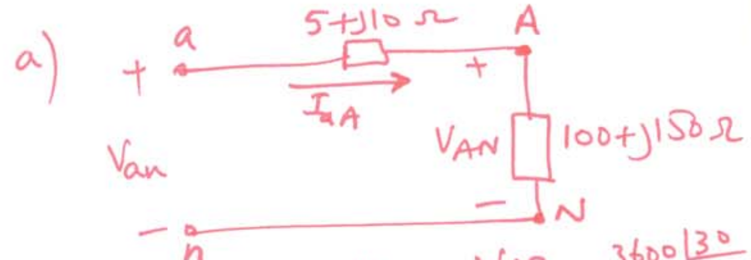
EE305 MJI

Problem 1: [15 points]
A balanced three-phase load, **Y**-connected is $\mathbf{100 + j150\,\Omega/\Phi}$. The load is fed from a Y-connected source through a **line** that has an **impedance** of $\mathbf{5 + j10\ \Omega/\Phi}$. The Line-to-Line voltage $\mathbf{V_{AB}}$ at the Load terminals is $\mathbf{3600\angle 30^0}$ **V**. Assume a **positive** sequence.

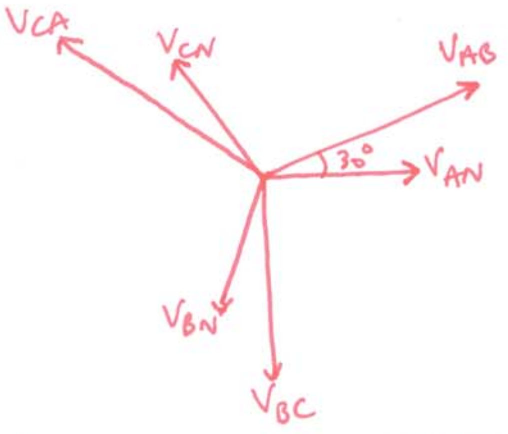
a) Construct the a-phase equivalent circuit of the system. Show all the details. **[2pts]**
b) Draw the phasor diagram showing all phase voltages and Line voltages at the Load side. **[3pts]**
c) Using the single phase equivalent circuit, calculate the **line currents** I_{aA}, I_{bB}, I_{cC}?**[3pts]**
d) Calculate the **line voltage** $\mathbf{V_{ab}}$ at the **source**? **[3pts]**
e) Find the load complex power. **[2pts]**
f) What is the efficiency of the system? (i.e. the ratio of the load real power to the source real power.) **[2pts]**

a)



$$V_{AN} = \frac{V_{AB}}{\sqrt{3}\angle 30} = \frac{3600\angle 30}{\sqrt{3}\angle 30} = 2078.5\ V$$

b)



c) $$I_{aA} = \frac{V_{AN}}{Z_\phi} = \frac{2078.5\angle 0}{100+j150} = \frac{2078.5}{180.28\angle 56.31}$$

$$= 11.53\angle -56.31\ A$$

$$I_{bB} = 11.53\angle -176.31^\circ\ A$$

$$I_{cC} = 11.53\angle 63.69^\circ\ A$$

d) $$V_{ab} = \sqrt{3}\angle 30\ V_{an}$$

$$= \sqrt{3}\angle 30\left(V_{AN} + I_{aA}(5+j10)\right)$$

or

$$V_{an} = I_{aA}(100+5+j150+j10)$$

$$= 11.53\angle -56.31\,(105+j160)$$

$$= 11.53 \times 191.38\angle -56.31+56.73$$

$$= 2206.57\angle 0.42^\circ\ V$$

$$V_{ab} = \sqrt{3}\angle 30 \cdot 2206.57\angle 0.42$$

$$= \underline{3821.89\angle 30.42^\circ\ V}$$

e) $$S_\phi = V_{AN} I_{aA}^* = 2078.5\angle 0 \cdot 11.53\angle 56.31$$

$$= 23965.11\angle 56.31$$

$$= 13293.4 + j19940.2\ VA$$

f) $$\eta = \frac{P_{\phi L}}{P_{\phi s}}100\% = \frac{13293.4}{P_{\phi s}} \cdot 100\%$$

$$S_{\phi s} = V_{an} I_{aA}^* = 2206.57\angle 0.42 \cdot 11.53\angle 56.31$$

$$= 25441.75\angle 56.73^\circ$$

$$= 13956.96 + j21271.71$$

f) $$\eta = \frac{13293.4}{13956.96}100\% = \underline{\underline{95.25\%}}$$

Problem 2: [15 points]

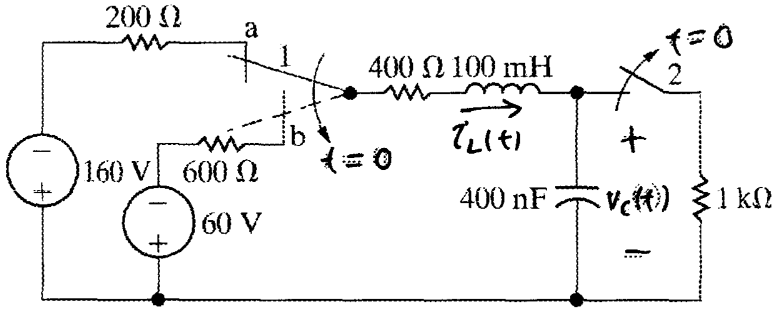


Figure: 08-21-20P8.52

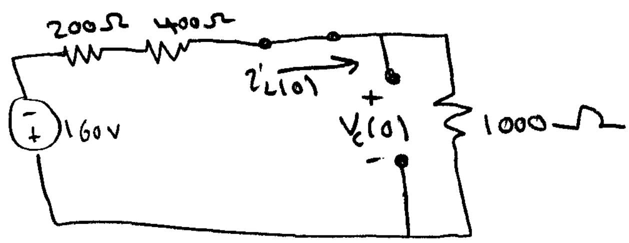
The two switches in the above figure operate synchronously. When switch **1** is in position **a**, switch **2** is closed. When switch **1** is in position **b**, switch **2** is open. Switch **1** has been in position **a** for a long time. At t = 0, it moves instantaneously to position **b**.

2.1 Find $v_c(0)$, $i_L(0)$ **[3pts]**

$i_L(0) = \frac{-160}{1600} = -100\text{ mA}$

$V_c(0) = 1000\, i_L(0) = -100\text{ V}$

$t<0$



2.2 Find $\frac{dv_c(0^+)}{dt}$, $\frac{di_L(0^+)}{dt}$ **[3pts]**

* $C\frac{dV_c(0)}{dt} = i_L(0)$

$\Rightarrow \frac{dV_c(0)}{dt} = \frac{1}{C} i_L(0)$

$= -\frac{100\text{mA}}{400\text{nF}}$

$\Rightarrow \boxed{\frac{dV_c(0)}{dt} = -250{,}000\text{ V/s}}$

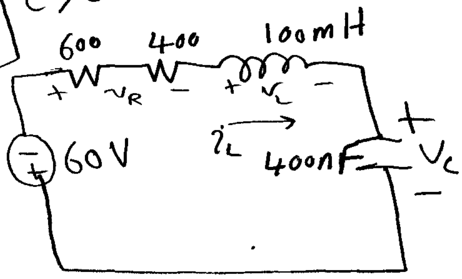
* $L\frac{di(0)}{dt} + V_c(0) + 60 + 1000\, i_L(0) = 0$

$\Rightarrow L\frac{di(0)}{dt} = 100 - 60 + 100$

$= 140$

$\Rightarrow \boxed{\frac{di(0)}{dt} = \frac{140}{100\text{m}} = 1400\text{ A/s}}$

$t>0$



2.3 Find $\alpha, \omega_0, s_1, s_2$ **[2pts]**

* $\alpha = \frac{R}{2L} = \frac{1000}{200} \times 10^3 = 5000\text{ rad/s}$

* $\omega_0^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{(100)(400)} = 25\times10^6$

$\Rightarrow \omega_0 = 5000\text{ rad/s}$

$s_1 = s_2 = -\alpha = -5000\text{ rad/s}$

2.4 What is the type of the response for $t \geq 0$ **[1pts]**

Since $\alpha^2 = \omega_0^2 \Rightarrow$ Critically Damped

2.5 Find $v_c(t)$ for $t \geq 0$ **[3pts]**

$V_c(t) = V_f + D_1 t e^{-5000t} + D_2 e^{-5000t}$

* Find D_1 and D_2 from initial Conditions

* $V_c(0) = V_f + D_2 = -100V$

$\Rightarrow D_2 = V_c(0) - V_f = -100 + 60$

$\boxed{D_2 = -40V}$

* $\frac{dV_c(0)}{dt} = D_1 - 5000 D_2 = -250{,}000 \text{ V/s}$

$\Rightarrow D_1 = 5000 D_2 - 250{,}000$

$= 5000(-40) - 250{,}000$

$\boxed{D_1 = -450{,}000}$

$\therefore$

$V_c(t) = -60 - 450{,}000 t e^{-5000t} - 40 e^{-5000t} \text{ V}$

for $t \geq 0$

2.6 Find $i_L(t)$ for $t \geq 0$ **[3pts]**

$$i_L(t) = C \frac{dv_c(t)}{dt}$$

$$= C\left[D_1 e^{-5000t} - 5000 D_1 t e^{-5000t} - 5000 D_2 e^{-5000t}\right]$$

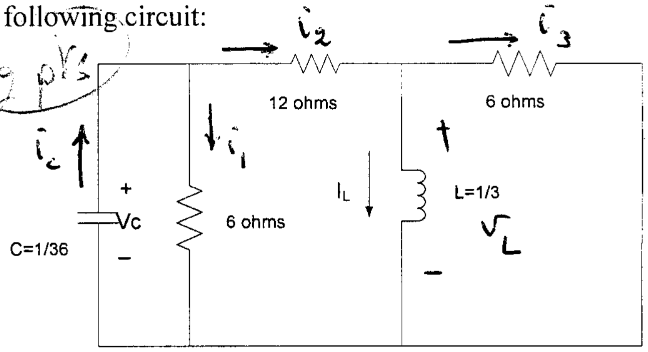
$$= -5000 C D_1 t e^{-5000t} + \left[C D_1 - 5000 C D_2\right] e^{-5000t}$$

$$= -(5\times10^3)(400\times10^{-9})(-45\times10^4) t e^{-5000t} + \left[(400\times10^{-9})(-45\times10^4) - (5\times10^3)(400\times10^{-9})(-40)\right] e^{-5000t}$$

$$= 900 t e^{-5000t} + [-0.18 + 0.08] e^{-5000t}$$

$$i_L(t) = 900 t e^{-5000t} - 0.1 e^{-5000t} \text{ A}, \; t \geq 0$$

Problem 3: [15 points]
Part A: [9pts]
Write down the state equations for the following circuit:



First, we have $\begin{cases} L \dfrac{di_L}{dt} = v_L \\ C \dfrac{dv_c}{dt} = -i_c \end{cases}$ 9 pts

KVL: $\begin{cases} v_c = 6 i_1 \\ v_L = 6 i_3 \\ v_c = 12 i_2 + 6 i_3 \end{cases}$ 3pts $\rightarrow i_3 = \frac{1}{18} v_c - \frac{2}{3} i_L$, so that $L \dfrac{di_L}{dt} = 6 i_3 = -4 i_L + \frac{1}{3} v_c$

KCL: $\begin{cases} i_2 = i_3 + i_L \\ i_c = i_1 + i_2 \end{cases}$

$\rightarrow -i_c = -i_1 - i_2 = -i_1 - i_3 - i_L = -\frac{1}{6} v_c - i_L - \frac{1}{18} v_c + \frac{2}{3} i_L$

so that $C \dfrac{dv_c}{dt} = -\frac{1}{3} i_L - \frac{4}{18} v_c$.

Finally $\dfrac{d}{dt}\begin{bmatrix} i_L \\ v_c \end{bmatrix} = \begin{bmatrix} -12 & 1 \\ -12 & -8 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix}$ 4pts using $L = 1/3$, $C = 1/36$

Part B: [6pts]

Consider the following state equation with state vector $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$:

$$\frac{d}{dt} X(t) = AX(t), \quad \text{where } A = \begin{bmatrix} -12 & 1 \\ -12 & -8 \end{bmatrix}$$

Assume the initial conditions are $x_1(0) = 1$, and $x_2(0) = 1$.

Use the Euler numerical method to find $x_1(\Delta t), x_2(\Delta t)$, and $x_1(2\Delta t), x_2(2\Delta t)$, for $\Delta t = 0.1$

We have: $x_1[(k+1)\Delta t] \approx x_1(k\Delta t) + [a_{11} x_1(k\Delta t) + a_{12} x_2(k\Delta t)] \cdot \Delta t$

$x_2[(k+1)\Delta t] \approx x_2(k\Delta t) + [a_{21} x_1(k\Delta t) + a_{22} x_2(k\Delta t)] \cdot \Delta t$ 2pts

with $\Delta t = 0.1$, $x_1(0) = x_2(0) = 1$ & $a_{11} = -12$ $a_{12} = 1$ $a_{21} = -12$ $a_{22} = -8$.

we get $x_1(0.1) = 1 + [-12 + 1] \times 0.1 = -0.1$

$x_2(0.1) = 1 + [-12 - 8] \times 0.1 = -1$ 2pts

$x_1(0.2) = -0.1 + [-12 \times -0.1 - 1] \times 0.1 = -0.08$ 2pts

$x_2(0.2) = -1 + [1.2 + 8] \times 0.1 = -0.08$