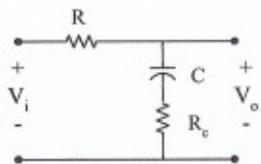


## Quiz 8

Name: KEY

For the circuit shown:



- a) Derive an expression for the transfer function  $H(s)$  where  $H(s) = V_o/V_i$  (3 points)

b) At what frequency will the magnitude of  $H(j\omega)$  be maximum. (1 point)

c) What is the maximum value of  $H(j\omega)$ ? (1 point)

d) At what frequency will the magnitude of  $H(j\omega)$  equal its maximum value divided by  $\sqrt{2}$ ? (4 points)

e) what is the minimum value of the magnitude of  $H(j\omega)$  and at what frequency does it occur?

a) By voltage divider.

$$H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{R_c} + \frac{1}{sC}}{\frac{1}{R_c} + \frac{1}{sC} + \frac{1}{R}} = \frac{R_c C s + 1}{(R_c + R) C s + 1}$$

$$b) |H(j\omega)| = \frac{\sqrt{(R_c \omega)^2 + 1}}{\sqrt{(R_c + R)^2 \omega^2 + 1}} = \frac{\sqrt{R_c^2 \omega^2 + 1}}{\sqrt{(R_c C + CR)^2 \omega^2 + 1}}$$

$$c) \text{ at } \omega = 0 \quad |H(j\omega)|_{\text{max}} = 1 \quad \text{otherwise always } \frac{\sqrt{R_c^2 \omega^2 + 1}}{\sqrt{(R_c C + CR)^2 \omega^2 + 1}} > \frac{\sqrt{R_c^2 \omega^2}}{\sqrt{(R_c C + CR)^2 \omega^2}} \quad \& |H(j\omega)| < 1$$

$$d) \frac{\sqrt{R_c^2 \omega^2 + 1}}{\sqrt{(R_c^2 C^2 + 2R_c R C^2 + R^2 C^2) \omega^2 + 1}} = \frac{1}{2} \Rightarrow 2\sqrt{R_c^2 \omega^2 + 1} = \sqrt{(R_c^2 C^2 + 2R_c R C^2 \omega^2 + R^2 C^2 \omega^2 + 1)}$$

$$\Rightarrow R_c^2 \omega^2 - 2R_c R C^2 \omega^2 - R^2 C^2 \omega^2 = -1$$

$$\Rightarrow \omega^2 = \frac{-1}{C^2 (R_c^2 - 2R_c R - R^2)} \Rightarrow \omega = \frac{1}{\sqrt{R_c^2 + 2R_c R - R^2}}$$

$$e) |H(j\omega)_{\text{min}}| = \frac{R_c}{R_c + R} \quad \text{voltage divider as } \omega \rightarrow \infty$$