King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE205: Electric Circuits II (031)

Major Exam II

Dec 17, 2003

7:0PM-8:30PM

Building 19-416

Serial #

Name: KEY

ID:

Sec. (1) 8:00-8:50 (2) 9:00-9:50

Question	Mark
1	/10
2	/10
3	/10
Total	/30

Good luck

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Problem 1:

For the circuit shown :

$\cos(\theta - 90^{o}) = \sin\theta$

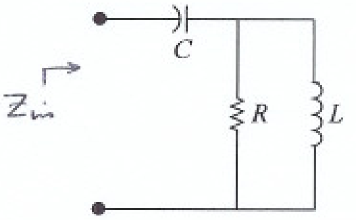
$\sin^2\theta = \frac{1}{2}\left[1-\cos(2\theta)\right]$

$\cos^2\theta = \frac{1}{2}\left[1+\cos(2\theta)\right]$

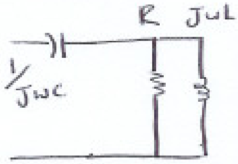
$\sin\theta - \cos\theta = \sqrt{2}\sin\left(\theta - 45^{o}\right)$

1. Find the resonance frequency. (3 points)
2. Find the Quality factor given that R=12Ω, L=2H, and $C=\frac{1}{36}$F. (6 points)

   (Hint: you may assume that you have a current source $i(t) = \cos(\omega t)$ or $1\angle 0^{o}$)
3. *Frequency scaling:* what values of R, L, and C will result in doubling the resonance frequency of the same circuit. (1 point)



1. $\underline{Z_{in}}$



$$Z_{in} = \frac{1}{J\omega c} + \frac{J\omega LR}{R + J\omega L}$$

$$= \frac{-J}{\omega c} + \frac{J\omega LR(R - J\omega L)}{R^2 + \omega^2 L^2}$$

$$= \frac{-J}{\omega C} + \frac{J\omega LR^2}{R^2 + \omega^2 L^2} + \text{Real}$$

at resonance $\text{imag}(Z_{in}) = 0$

$$\frac{-1}{\omega_r C} + \frac{\omega_r LR^2}{R^2 + \omega_r^2 L^2} = 0$$

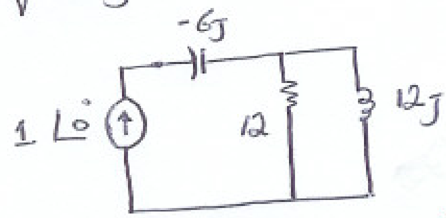
$$R^2 + \omega_r^2 L^2 = \omega_r^2 LCR^2$$

$$\omega_r^2(-L^2 + LCR^2) = R^2$$

$$\omega_r^2 = \frac{R^2}{-L^2 + LCR^2}$$

$$\omega_r = \frac{1}{\sqrt{LC - \frac{L^2}{R^2}}}$$

2. In the frequency domain



the quality factor is calculated at resonance.

$$\omega_r = \frac{1}{\sqrt{2\left(\frac{1}{36}\right) - \frac{(2)^2}{(12)^2}}} = 6 \text{ rad/s}$$ 0.25

$$Z_c = \frac{1}{J\omega c} = -6J$$

$$Z_L = J\omega L = 12J$$

$$V_c = 1\angle 0 \,(-6J) = 6\angle{-90^{o}}$$ 0.25

By current divider.

$$I_R = \frac{12J}{12 + 12J} 1\angle 0 = \frac{1}{\sqrt{2}}\angle{+45^{o}}$$ 0.25

$$I_L = \frac{12}{12 + 12J} 1\angle 0 = \frac{1}{\sqrt{2}}\angle{-45^{o}}$$ 0.25

continue #1.2

$$W_c = \frac{1}{2} C V_c^2 = \frac{1}{2} \frac{1}{36} (6)^2 \cos^2(6t - 90^\circ)$$

$$= \frac{1}{2} \sin^2(6t) = \frac{1}{4}[1 - \cos 12t]$$ 1

$$W_L = \frac{1}{2} L i_L^2 = \frac{1}{2}(2)\left(\frac{1}{\sqrt{2}}\right)^2 \cos^2(6t - 45^\circ)$$

$$= \frac{1}{4}[1 + \cos(12t - 90^\circ)]$$ 1

$$= \frac{1}{4}[1 + \sin 12t]$$

$$W_c + W_L = \frac{1}{4}[2 + \sin 12t - \cos 12t]$$

$$= \frac{1}{4}[2 + \sqrt{2}\sin(12t - 45^\circ)]$$

$$[W_c + W_L]_{max} = \frac{1}{4}[2 + \sqrt{2}]$$ 0.5

Power in the resistor.

$$P_R = \frac{1}{2} |I_L|^2 R = \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2 (12) = 3 \text{ W}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ s}$$ 1

$$P_R T = (3)\left(\frac{\pi}{3}\right) = \pi \text{ J}$$

$$Q = 2\pi\left(\frac{W_{max}}{P_R T}\right) = 2\pi\left(\frac{\frac{1}{4}[2+\sqrt{2}]}{\pi}\right)$$

$$= \frac{1}{2}[2 + \sqrt{2}] = 1 + \frac{1}{\sqrt{2}}$$

0.5

1

3. Frequency scaling.

$k_f = 2$ (double)

$R \to R$ $\quad R = 12\ \Omega$

$L \to \frac{L}{k_f}$ $\quad L = 1$ H

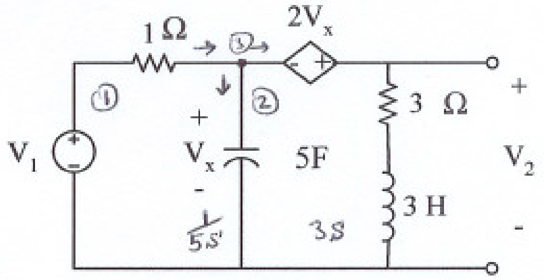
$C \to \frac{C}{k_f}$ $\quad C = \frac{1}{72}$ F

1

Problem 2:

For the following circuit:

1) Find the transfer function $H(s)=\frac{V_2}{V_1}$ (5 points)
2) Draw the pole-zero plot of $H(s)$ (3 points)
3) If the input voltage is $v_1(t)=10e^{-6t}\cos 3t$ V, what is the output voltage $v_2(t)$ (2 points)



1)

By KVL in loop ②

$$-V_x - 2V_x + V_2 = 0$$

$$\Rightarrow V_2 = 3V_x \Rightarrow \boxed{V_x = \frac{V_2}{3}}$$

By KCL at node ③

$$\frac{V_1 - V_x}{1} = \frac{V_x}{1/5s} + \frac{V_2}{3+3s}$$

$$V_1 - \frac{1}{3}V_2 = 5s\left(\frac{V_2}{3}\right) + \left(\frac{V_2}{3}\right)\frac{1}{1+s}$$

$*3$

$$3V_1 = \left(5s + \frac{1}{1+s} + 1\right)V_2$$

$$\frac{3V_1}{V_2} = \frac{5s + 5s^2 + 1 + 1 + s}{1+s}$$

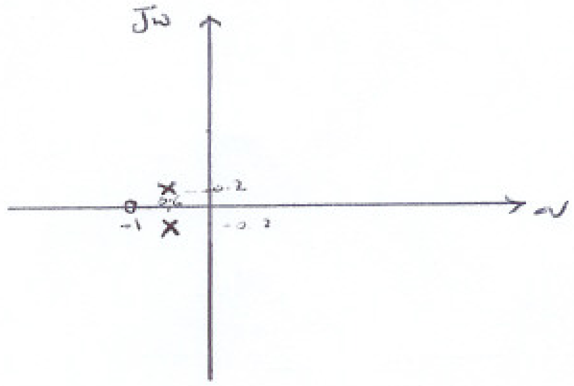
$$\frac{V_2}{V_1} = \frac{3(s+1)}{5s^2 + 6s + 2}$$

2) one zero $s = -1$

two poles

$$s_1, s_2 = \frac{-6 \pm \sqrt{36 - 40}}{10}$$

$$= \frac{-6 \pm j2}{10} = -0.6 \pm j0.2$$



3) $s = -6 + 3j$ $\quad V_1 = 10\angle 0°$

$$\frac{V_2}{V_1} = \frac{3(3j-5)}{5(-6+3j)^2 + 18j - 36 + 2}$$

$$= \frac{9j-15}{5(36-9-36j)+18j-34}$$

$$= \frac{9j-15}{135-180j+18j-34}$$

$$= \frac{9j-15}{101-162j} = \frac{17.49\angle 149°}{190.9\angle -58.06°}$$

$$= 0.0916\angle 207.1$$

if $V_1 = 10\angle 0$

$$\Rightarrow V_2 = 10 * 0.0916\angle 207.1° = 0.916\angle 207.1°$$

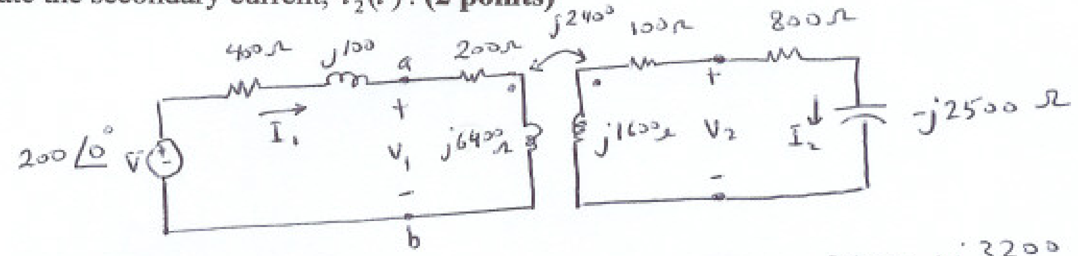
$$\Rightarrow v_2(t) = 0.916\, e^{-6t}\cos(3t + 207.1)\ V$$

Problem 3:

The parameters of a certain linear transformer are R_1=200Ω, R_2=100Ω, L_1=16 H, L_2=4 H, and k=0.75 The transformer couples an impedance consisting of an 800Ω resistor in series with a 1μF capacitor to a sinusoidal voltage source. The 200 V (rms) source has an internal impedance of 400+j100 Ω and a frequency of 400 rad/s.

1) Construct the frequency-domain equivalent circuit of the system. **(2 points)**
2) Calculate the self-impedance of the primary circuit. **(0.5 point)**
3) Calculate the self-impedance of the secondary circuit. **(0.5 point)**
4) Calculate the impedance reflected into the primary winding. **(2 points)**
5) Calculate the impedance seen into the primary terminals of the transformer. **(1 point)**
6) Calculate the primary current, $i_1(t)$. **(2 points)**
7) Calculate the secondary current, $i_2(t)$. **(2 points)**

1.)



$j\omega L_1 = j(400)(16) = j\,6400\,\Omega$

$j\omega L_2 = j(400)(4) = j\,1600\,\Omega$

$M = 0.75\sqrt{(18)(4)} = 6\text{ H}$

$j\omega M = j(400)(6) = j\,2400\,\Omega$

$\dfrac{1}{j\omega C} = \dfrac{10^6}{j\,400} = -j\,2500\,\Omega$

2) $Z_{11} = 400 + j100 + 200 + j6400$

$= 600 + j\,6500\,\Omega$

3) $Z_{22} = 100 + 800 + j1600 - j\,2500$

$= 900 - j\,900\,\Omega$

4) $Z_r = \dfrac{\omega^2 M^2}{|Z_{22}|^2}\,Z_{22}^*$

$= \dfrac{(400)^2(6)^2}{|900 - j900|^2}\,(900 + j900)$

$= \dfrac{32}{9}(900 + j900) = 3200 + j3200\,\Omega$

5) $Z_{ab} = 200 + j\,6400 + 3200 + j\,3200$

$= 3400 + j\,9600\,\Omega$

6) $I_1 = \dfrac{200\angle 0^\circ}{400 + j100 + Z_{ab}}$

$I_1 = \dfrac{200\angle 0^\circ}{3800 + j\,9700} = 19.2\angle -68.61\ \text{mA}$

$I_{1\,amplitude} = \sqrt{2}\,I_{1\,rms} = 27.15\text{ mA}$

$\Rightarrow i_1(t) = 27.15\cos(400t - 68.61^\circ)\ \text{mA}$

7) By KVL in the secondary:

$I_2 Z_{22} - I_1 j\omega M = 0$

$\Rightarrow I_2 = \dfrac{I_1 j\omega M}{Z_{22}} = \dfrac{(400)(6)\angle 90^\circ}{900 - j900}\,I_1$

$I_2 = 1.886\angle 90^\circ + 45^\circ\ I_1$

$I_2 = 36.2\angle 66.39^\circ\ \text{mA}$

$I_{2\,amplitude} = \sqrt{2}\,I_{2\,rms} = 51.2\text{ mA}$

$\Rightarrow i_2(t) = 51.2\cos(400t + 66.39^\circ)\ \text{mA}$