Rules for Making Bode Plots

Term	Magnitude	Phase
Constant: K	$20 \cdot \log_{10}(\lvert K \rvert)$	$K > 0: \quad 0^\circ$ $K < 0: \quad \pm 180^\circ$
Real Pole: $\dfrac{1}{\frac{s}{\omega_0}+1}$	• Low freq. asymptote at 0 dB • High freq. asymptote at -20 dB/dec • Connect lines at break freq.	• Low freq. asymptote at 0^o • High freq. asymptote at -90^o • Connect with straight line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$
Real Zero*: $\dfrac{s}{\omega_0}+1$	• Low freq. asymptote at 0 dB • High freq. asymptote at +20 dB/dec. • Connect lines at break freq	• Low freq. asymptote at 0^o • High freq. asymptote at $+90^o$ • Connect with line from $0.1 \cdot \omega_0$ to $10 \cdot \omega_0$
Pole at Origin: $\dfrac{1}{s}$	-20 dB/dec; through 0 dB at $\omega=1$	-90°
Zero at Origin*: s	+20 dB/dec; through 0 dB at $\omega=1$	$+90^\circ$
Underdamped Poles: $\dfrac{1}{\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right)+1}$	• Low freq. asymptote at 0 dB • High freq. asymptote at -40 dB/dec. • Draw peak† at freq. $\omega_r = \omega_0\sqrt{1-2\zeta^2}$ with amplitude • $H(j\omega_r) = -20 \cdot \log_{10}\left(2\zeta\sqrt{1-\zeta^2}\right)$ • Connect lines	• Low freq. asymptote at 0^o • High freq. asymptote at -180^o • Connect with straight line from ‡ $\omega = \omega_0 \dfrac{\log_{10}\left(\frac{2}{\zeta}\right)}{2}$ to $\omega = \omega_0 \dfrac{2}{\log_{10}\left(\frac{2}{\zeta}\right)}$
Underdamped Zeros*: $\left(\frac{s}{\omega_0}\right)^2 + 2\zeta\left(\frac{s}{\omega_0}\right)+1$	• Draw low freq. asymptote at 0 dB • Draw high freq. asymptote at +40 dB/dec. • Draw dip† at freq. $\omega_r = \dfrac{\omega_0}{\sqrt{1-2\zeta^2}}$ with amplitude $H(j\omega_r) = +20 \cdot \log_{10}\left(2\zeta\sqrt{1-\zeta^2}\right)$ • Connect lines	• Low freq. asymptote at 0^o • Draw high freq. asymptote at -180^o • Connect with a straight line from ‡ $\omega = \omega_0 \dfrac{\log_{10}\left(\frac{2}{\zeta}\right)}{2}$ to $\omega = \omega_0 \dfrac{2}{\log_{10}\left(\frac{2}{\zeta}\right)}$

Notes:

* Rules for drawing zeros create the mirror image (around 0 dB) of those for a pole with the same break freq

† For underdamped poles and zeros peak exists for $0 < \zeta < 0.707 = \frac{1}{\sqrt{2}}$ and peak freq. is typically very near the break freq

‡ For underdamped poles and zeros If ζ<0.02 draw phase vertically from 0 to -180 degrees at break freq

For n^{th} order pole or zero make asymptotes and peaks n times higher than shown (i.e., second order asymptote is -40 dB/dec, and phase goes from 0 to -180^o). Don't change frequencies, only plot values and slopes.

For more detail go to web page http://www.swarthmore.edu/NatSci/echeeve1/Ref/Bode/Bode.html

Quick Reference for Making Bode Plots

If starting with a transfer function of the form (some of the coefficients b_i, a_i may be zero).

$$H(s) = C\frac{s^n + \cdots + b_1 s + b_0}{s^m + \cdots + a_1 s + a_0}$$

Factor polynomial into real factors and complex conjugate pairs (j can be positive, negative, or zero).

$$H(s) = C \cdot s^q \frac{(s+\omega_{z1})(s+\omega_{z2})\cdots(s^2+2\zeta_{z1}\omega_{0z1}s+\omega_{0z1}^2)(s^2+2\zeta_{z2}\omega_{0z2}s+\omega_{0z2}^2)\cdots}{(s+\omega_{p1})(s+\omega_{p2})\cdots(s^2+2\zeta_{p1}\omega_{0p1}s+\omega_{0p1}^2)(s^2+2\zeta_{p2}\omega_{0p2}s+\omega_{0p2}^2)\cdots}$$

Put polynomial into standard form for Bode Plots (q can be positive, negative, or zero).

$$H(s) = C\frac{\omega_{p1}\omega_{p2}\cdots\omega_{0p1}^2\omega_{0p2}^2\cdots}{\omega_{z1}\omega_{z2}\cdots\omega_{0z1}^2\omega_{0z2}^2\cdots} \cdot s^q \frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{(s+\omega_{p1})(s+\omega_{p2})\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

$$= K \cdot s^q \frac{\left(\frac{s}{\omega_{z1}}+1\right)\left(\frac{s}{\omega_{z2}}+\right)\cdots\left(\left(\frac{s}{\omega_{0z1}}\right)^2+2\zeta_{z1}\left(\frac{s}{\omega_{0z1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0z2}}\right)^2+2\zeta_{z2}\left(\frac{s}{\omega_{0z2}}\right)+1\right)\cdots}{(s+\omega_{p1})(s+\omega_{p2})\cdots\left(\left(\frac{s}{\omega_{0p1}}\right)^2+2\zeta_{p1}\left(\frac{s}{\omega_{0p1}}\right)+1\right)\left(\left(\frac{s}{\omega_{0p2}}\right)^2+2\zeta_{p2}\left(\frac{s}{\omega_{0p2}}\right)+1\right)\cdots}$$

Take the terms (constant, real poles and zeros, origin poles and zeros, complex poles and zeros) one by one and plot magnitude and phase according to rules on previous page. Add up resulting plots.

For more detail go to web page http://www.swarthmore.edu/NatSci/echeeve1/Ref/Bode/Bode.html