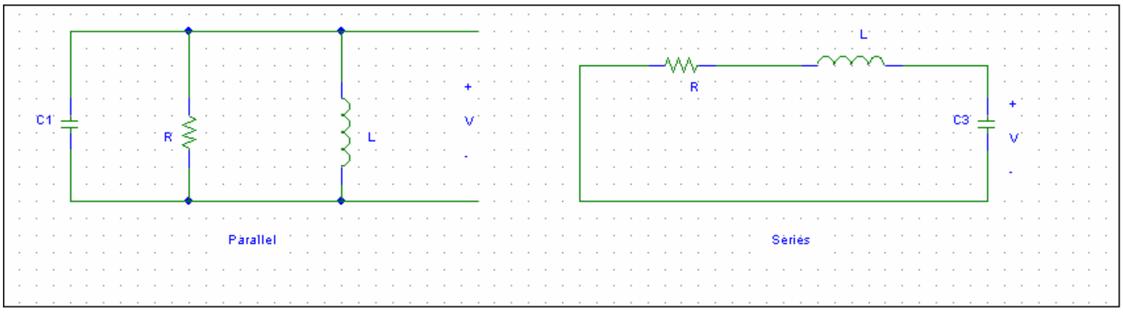
Summary for CH#8

Natural and Step Responses of RLC Circuits

- The **characteristic equation** of the following RLC circuits,



is:

$$s^2 + 2\alpha s + \omega_o^2 = 0$$

where,

$$\boxed{\alpha = \frac{1}{2RC}} (parallel)$$

$$\boxed{\alpha = \frac{R}{2L}} (series)$$

$$\boxed{\omega_o = \frac{1}{\sqrt{LC}}} (for\ parallel\ \&\ series)$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

- Depending on the values of α^2 & ω_o^2 the natural and step response of the series & parallel RLC circuits can be classified as follow:

Type	Roots ($S_{1,2}$)	Condition
Overdamped	Real, distinct	$\alpha^2 > \omega_o^2$
Underdamped	Complex	$\alpha^2 < \omega_o^2$
Critically damped	Real, repeated	$\alpha^2 = \omega_o^2$

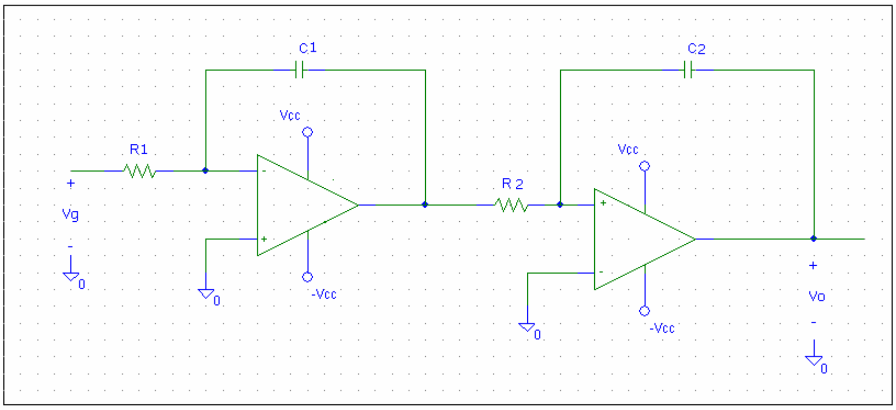
- **<u>For natural response</u>**:

Type	Equations	Initial Conditions
Overdamped	$x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$x(0) = A_1 + A_2;$ $dx/dt\,(0) = \dfrac{i_c(0)}{C} = A_1 s_1 + A_2 s_2$
Underdamped	$x(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t) e^{-\alpha t}$	$x(0) = B_1;$ $dx/dt\,(0) = \dfrac{i_c(0)}{C} = -\alpha B_1 + \omega_d B_2,$ $where: \boxed{\omega_d = \sqrt{\omega_0^2 - \alpha^2}}$ $(Warning\ !!)$
Critically damped	$x(t) = (D_1 t + D_2) e^{-\alpha t}$	$x(0) = D_2$ $dx/dt\,(0) = \dfrac{i_c(0)}{C} = D_1 - \alpha D_2$

- **<u>For step response</u>**: (See examples 8.6-8.10)

Type	Equations	Initial Conditions
Overdamped	$x(t) = X_f + A'_1 e^{s_1 t} + A'_2 e^{s_2 t}$	$x(0) = X_f + A'_1 + A'_2;$ $dx/dt\,(0) = \dfrac{i_c(0)}{C} = A'_1 s_1 + A'_2 s_2$
Underdamped	$x(t) = X_f + (B'_1 \cos \omega_d t + B'_2 \sin \omega_d t$	$x(0) = X_f + B'_1;$ $dx/dt\,(0) = \dfrac{i_c(0)}{C} = -\alpha B'_1 + \omega_d B'_2,$
Critically damped	$x(t) = X_f + (D'_1 t + D'_2) e^{-\alpha t}$	$x(0) = X_f + D'_2$ $dx/dt\,(0) = \dfrac{i_c(0)}{C} = D'_1 - \alpha D'_2$

<u>Two Integrator Amplifier</u>: (*see* example 8.13)



Applying KCL at the inverting terminals result in the following:

$$\boxed{\frac{dv_o}{dt} = -\frac{1}{R_1C_1}v_g} \qquad 1$$

$$\boxed{\frac{dv_o}{dt} = -\frac{1}{R_2C_2}v_{o1}} \overset{Differentiating}{\Leftrightarrow} \boxed{\frac{d^2v_o}{dt^2} = -\frac{1}{R_2C_2}\frac{dv_{o1}}{dt}} \qquad 2$$

From 1 & 2 :

$$\boxed{\frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1}\frac{1}{R_2C_2}v_g}$$

- **<u>Two Integrating Amplifiers with Feedback Resistors</u>**:

The reason for adding the feedback resistors is the fact that the op amp in the integrating amplifier saturates because of the feedback capacitor's accumulation of charge. A resistor is placed in parallel with each feedback capacitor (C1 and C2) to overcome this problem.

$$\boxed{\frac{d^2v_o}{dt^2} + \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)\frac{dv_o}{dt} + \left(\frac{1}{\tau_1\tau_2}\right)v_o = \frac{v_g}{R_aC_1R_bC_2}}$$

The characteristic equation :

$$s^2 + \left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)s + \frac{1}{\tau_1\tau_2} = 0$$

The roots :

$$s_1 = -\frac{1}{\tau_1}; s_2 = -\frac{1}{\tau_2}$$

Example 8.14 illustrates the analysis of the step response of two cascaded integrating amplifiers when the feedback capacitors are shunted with feedback resistors.