KFUPM-Electrical Engineering Department
EE205: Electric Circuits II
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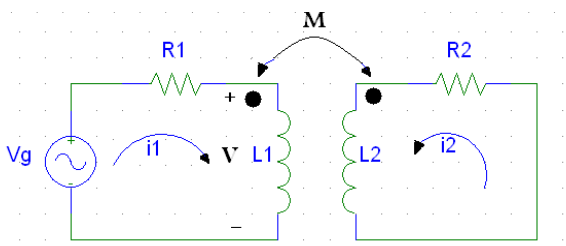
Summary of (corrected ver. 2.0)

Mutual Inductance & Transformers

- The inductance, *L*, is the parameter that relates a voltage of a time-varying current in the same circuit; thus *L* is more precisely referred to as "self-inductance".
- However, in the situation where two circuits are linked by a magnetic field, the voltage induced in the second circuit can be related to the time-varying current in the first circuit by a parameter known as "mutual-inductance", *M*.



Two magnetically coupled coils

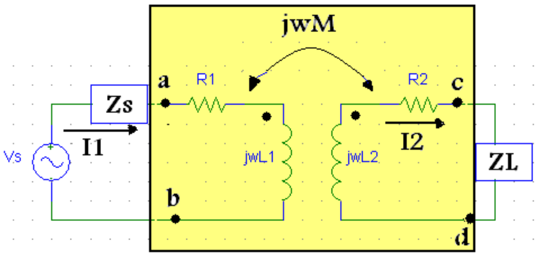
- For the circuit above, the self-induced voltage across the coil on the left equals to: $L_1(\frac{di_1}{dt})$, and the mutually induced voltage across the same coil is $M\,(\frac{di_2}{dt})$.
- Sign convention for the self-induced voltage: the self-induced voltage is a voltage drop in the direction of the current producing the voltage. "Passive sign convention"
- Sign convention for the "mutually induced voltage: the polarity of this voltage is assigned by method known as the "Dot Convention" which states that:
(When the reference direction for a current enters the dotted terminal of a coil; the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal)

$$V = L_1(\frac{di_1}{dt}) + M\,(\frac{di_2}{dt}) \qquad \text{(for the shown circuit).}$$

- The Coefficient of Coupling (*k*) is a measure of the degree of magnetic coupling. By definition, $0 \le k \le 1$.
- The relationship between the self-inductance of each winding and the mutual inductance between windings is $M = k\sqrt{L_1 L_2}$.
- The dot marking can be done using the right hand rule (if we have access to the coils) or experimentally using a test voltage source.
- The energy stored in magnetically coupled coils is related to the coils currents and inductances by the relationship: $W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$, with + if the two currents are the same relative to the dots (both enter or both leave the coils at the dotted terminals)
- Some physical relations are important when understanding the physics of inductance:

$v = \frac{d\lambda}{dt}$, $\lambda = N_v \Phi$, $\Phi = \rho N_i i$, $L = \rho N^2$, for nonmagnetic (linear) materials $M_{12}=M_{21}=M$

$M_{21} = N_2 N_1 \rho_{21}$, $M_{12} = N_1 N_2 \rho_{12}$,



The frequency domain circuit model for a transformer used to connect a load to a source

- The impedance seen by the internal voltage source V_S is:

$$Z_{\text{int}} = \frac{V_S}{I_1} = \frac{Z_{11}Z_{22} + \omega^2 M^2}{Z_{22}} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$$

Where: Z_{11}, the total "self impedance" of the mesh containing the primary winding (Left here) of the transformer.
Z_{22}, the total "self impedance" of the mesh containing the secondary winding (Right here) of the transformer.

- The impedance at the terminals of the source is $Z_{int} - Z_S$:

$$Z_{ab} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}} - Z_S = R_1 + j\omega L_1 + \frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}$$

- The term $\frac{\omega^2 M^2}{(R_2 + j\omega L_2 + Z_L)}$ is called the "Reflected Impedance", which is equal to the second coil impedance and the load impedance *reflected* to the primary side of the transformer. It is also can be written as: $Z_r = \frac{\omega^2 M^2}{|Z_{22}|^2} Z_{22}^* = \frac{\omega^2 M^2}{Z_{22} Z_{22}^*} Z_{22}^* = \frac{\omega^2 M^2}{Z_{22}}$

- For an ideal transformer :

$$\left|\frac{V_1}{N_1}\right| = \left|\frac{V_2}{N_2}\right| \qquad\qquad |I_1 N_1| = |I_2 N_2|$$

The polarity is determined as follows:

1- If the coil voltages V_1 and V_2 are both positive or negative at the dot-marked terminal, use plus sign in the first equation, otherwise use a negative sign.
2- If the coil current i_1 and i_2 are both directed into or out of the dot-marked terminal, use a minus sign in the second equation, otherwise use a plus sign.

- $a = N_2 / N_1$ (Turns ratio)
- Ideal transformers can be used for Impedance Matching:

$$Z_{in} = \frac{1}{a^2} Z_L$$

This is not a comprehensive summary. It is meant to help you summarize the main ideas.
Please send me an e-mail if you find any mistake.
This summary was made with the help of Mr. Adel Al-Ghamdi

Regards, Dr. Muqaibel