KFUPM-EE DEPT.
EE205: Circuits II-082
HW # 2: Solution

Problem 1:

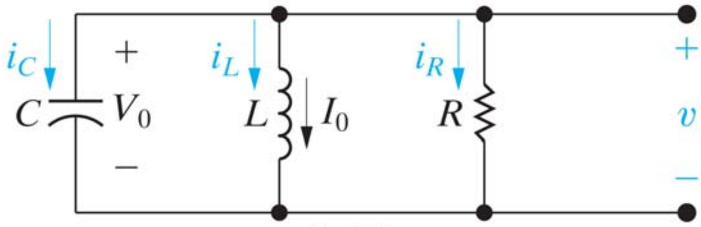


Figure: 08-01

The resistance, inductance, and capacitance in a parallel RLC circuit are 5000 Ω, 1.25 H, and 8 nF, respectively.

a) Calculate the roots of the characteristic equation that describe the voltage response of the circuit.
b) Will the response be over-, under-, or critically damped?
c) What value of R will yield a damped frequency of 6 krad/s?
d) What are the roots of the characteristic equation for the value of R found in (c) ?
e) What value of R will result in a critically damped response?

[a] $\alpha = \frac{1}{2RC} = \frac{10^9}{(10{,}000)(8)} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1.25)(8)} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(1.5625 - 1)10^8} = -12{,}500 \pm 7500$$

$$s_1 = -5000 \text{ rad/s}$$

$$s_2 = -20{,}000 \text{ rad/s}$$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 10^8 - 36 \times 10^6 = 0.64 \times 10^8$$

$$\alpha = 0.8 \times 10^4 = 8000$$

$$\frac{1}{2RC} = 8000; \qquad \therefore \quad R = \frac{10^9}{(16{,}000)(8)} = 7812.5\,\Omega$$

[d] $s_1 = -8000 + j6000$ rad/s; $\qquad s_2 = -8000 - j6000$ rad/s

[e] $\alpha = 10^4 = \frac{1}{2RC}; \qquad \therefore \quad R = \frac{1}{2C(10^4)} = 6250\,\Omega$

Problem 2:

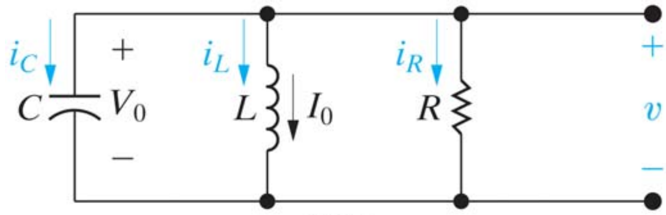


Figure: 08-01

The initial value of the voltage *v* in the above circuit is zero, and the initial value of the capacitor current, $i_c(0^+)$ is 15 mA. The expression for the capacitor current is known to be:

$$i_c(t) = A_1 e^{-160t} + A_2 e^{-40t}, \qquad t \geq 0^+$$

Where R is 200 Ω.

a) Find the value of $\alpha,\ \omega_0,\ ,L,\ C,\ A_1,\ and\ \ A_2$

$$\left(H\,\text{int}: \frac{di_C(0)}{dt} = -\frac{di_L(0)}{dt} - \frac{di_R(0)}{dt} = \frac{v(0)}{L} - \frac{1}{R}\frac{i_C(0^+)}{C} \right)$$

b) Find the expression for $v(t),\ \ t \geq 0$

c) Find the expression for $i_R(t),\ \ t \geq 0$

d) Find the expression for $i_L(t),\ \ t \geq 0$

[a] $2\alpha = 200; \qquad \alpha = 100\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 120; \qquad \omega_o = 80\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{200(200)} = 25\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(80)^2(25)} = 6.25\,\text{H}$$

$$i_C(0^+) = A_1 + A_2 = 15\,\text{mA}$$

$$\frac{di_C}{dt} + \frac{di_L}{dt} + \frac{di_R}{dt} = 0$$

$$\frac{di_C(0)}{dt} = -\frac{di_L(0)}{dt} - \frac{di_R(0)}{dt}$$

$$\frac{di_{\rm L}(0)}{dt} = \frac{0}{6.25} = 0\,{\rm A/s}$$

$$\frac{di_{\rm R}(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_{\rm C}(0)}{C} = \frac{15\times 10^{-3}}{(200)(25\times 10^{-6})} = 3\,{\rm A/s}$$

$$\therefore \quad \frac{di_{\rm C}(0)}{dt} = -3\,{\rm A/s}$$

$$\therefore \quad 160A_1 + 40A_2 = 3$$

$$4A_1 + A_2 + = 75\times 10^{-3}; \qquad \therefore \quad A_1 = 20\,{\rm mA}; \qquad A_2 = -5\,{\rm mA}$$

$$\therefore \quad i_{\rm C} = 20e^{-160t} - 5e^{-40t}\,{\rm mA}, \qquad t \geq 0$$

[b] By hypothesis

$$v = A_3e^{-160t} + A_4e^{-40t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{15\times 10^{-3}}{25\times 10^{-6}} = 600\,{\rm V/s}$$

$$-160A_3 - 40A_4 = 600; \qquad \therefore \quad A_3 = -5\,{\rm V}; \qquad A_4 = 5\,{\rm V}$$

$$v = -5e^{-160t} + 5e^{-40t}\,{\rm V}, \qquad t \geq 0$$

[c] $i_{\rm R}(t) = \dfrac{v}{200} = -25e^{-160t} + 25e^{-40t}\,{\rm mA}, \qquad t \geq 0^+$

[d] $i_{\rm L} = -i_{\rm R} - i_{\rm C}$

$$i_{\rm L} = 5e^{-160t} - 20e^{-40t}\,{\rm mA}, \qquad t \geq 0$$

Problem 3:

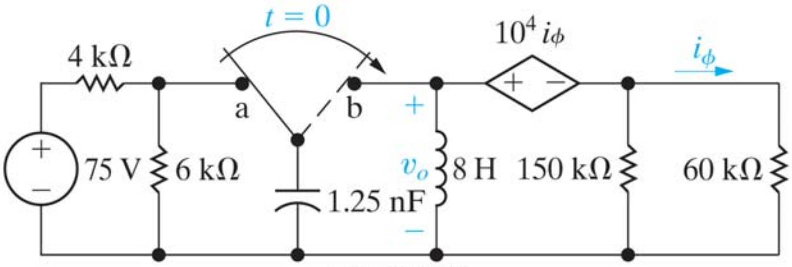
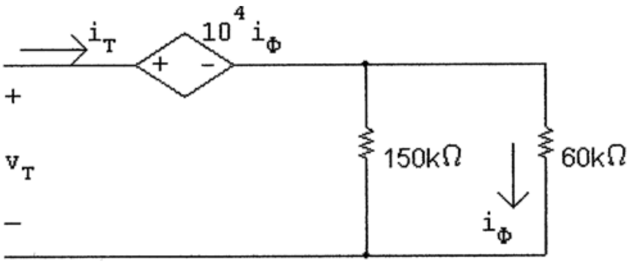


Figure: 08-21-01P8.16

The switch in the above circuit has been in position for a long time. At t = 0, the switch moves instantaneously to position b. Find $v_0(t)$ for $t \geq 0$.



$$v_T = 10^4 \frac{i_T(150 \times 10^3)}{210 \times 10^3} + \frac{(150)(60)10^6}{210 \times 10^3} i_T$$

$$\frac{v_T}{i_T} = \frac{1500 \times 10^3}{210} + \frac{9000 \times 10^3}{210} = \frac{10{,}500}{210} \times 10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-0.9}{1.25} \times 10^6 = -720 \times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(8)(1.25)} = 10^8; \qquad \omega_o = 10^4\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(1.25)\times 10^3} = 8000 \text{ rad/s}$$

$$\omega_d = \sqrt{(100-64)\times 10^6} = 6000 \text{ rad/s}$$

$$v_o = B_1 e^{-8000t}\cos 6000t + B_2 e^{-8000t}\sin 6000t$$

$$v_o(0) = B_1 = 45\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 6000B_2 - 8000B_1 = -720\times 10^3$$

$$\therefore\quad 6000B_2 = 8000(45) - 720\times 10^3; \qquad \therefore\quad B_2 = -60\,\text{V}$$

$$v_o = 45e^{-8000t}\cos 6000t - 60e^{-8000t}\sin 6000t\,\text{V}, \qquad t \geq 0$$

Problem 4:

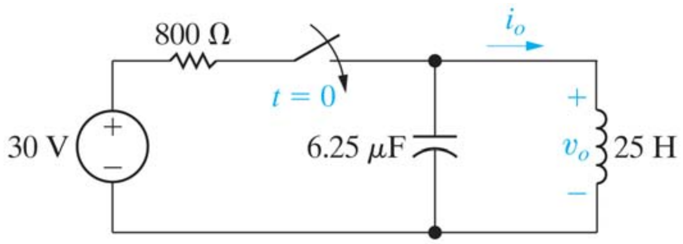
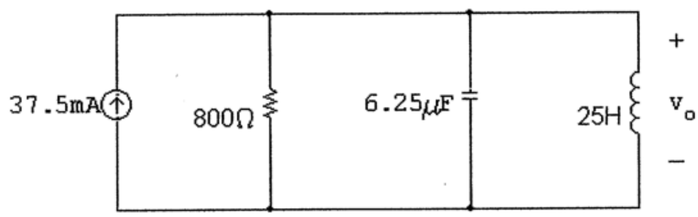


Figure: 08-21-05P8.30

There is no energy stored in the circuit in the above Figure when the switch is closed at t=0. Find $v_0(t)$ for $t \geq 0$.

For $t > 0$



$$\alpha = \frac{1}{2RC} = 100; \qquad \frac{1}{LC} = 6400$$

$$s_{1,2} = -100 \pm 60$$

$$s_1 = -40\,\text{rad/s}; \qquad s_2 = -160\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-40t} + A_2' e^{-160t}$$

$$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_C(0^+) = 37.5\,\text{mA}$$

$$\therefore \quad A_1' + A_2' = 0$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 6000\,\text{V/s}$$

$$\frac{dv_o(0^+)}{dt} = -40A_1' - 160A_2'$$

$$-40A_1' - 160A_2' = 6000$$

$$A_1' + 4A_2' = -150$$

$$A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 50\,\text{V}; \qquad A_2' = -50\,\text{V}$$

$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

Problem 5:

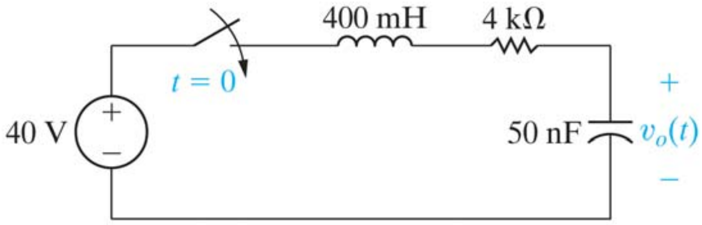


Figure: 08-21-16P8.45

The initial energy stored in the circuit in the above Figure is zero, Find $v_0(t)$ for $t \geq 0$.

$$\alpha = \frac{R}{2L} = 5000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{20} = 50 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 50 \times 10^6} = -5000 \pm j5000\,\text{rad/s}$$

$$v_o = V_f + B_1' e^{-5000t} \cos 5000t + B_2' e^{-5000t} \sin 5000t$$

$$v_o(0) = 0 = V_f + B_1'$$

$$v_o(\infty) = 40\,\text{V}; \qquad \therefore \quad B_1' = -40\,\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = 5000B_2' - 5000B_1'$$

$$\therefore \quad B_2' = B_1' = -40\,\text{V}$$

$$v_o = 40 - 40e^{-5000t} \cos 5000t - 40e^{-5000t} \sin 5000t\,\text{V}, \quad t \geq 0$$